Black-hole accretion disks
theory, observations, open questions

Jiří Horák

July 19, 2021

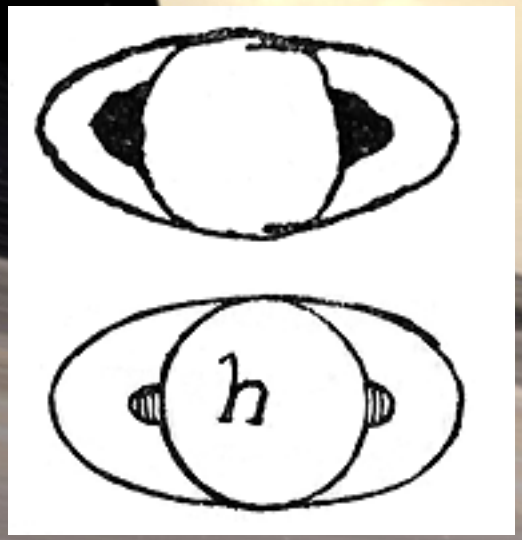
Outline

- Introduction: disks in astrophysics
  → Black-hole accretion disks as a powerful energy sources
- Quick summary of GR
  → Relativity without relativity
- Accretion: what is going on?
  → Basic principles of a cold thin-disk accretion
  → Standard model with questions
- Advanced modeling
  → Angular momentum transport
  → Advection and transonicity
  → ADAFs, slim disks, *etc*
- Global picture
  → Accretion spectra
  → Coronae

INTRODUCTION

Disks in astrophysics

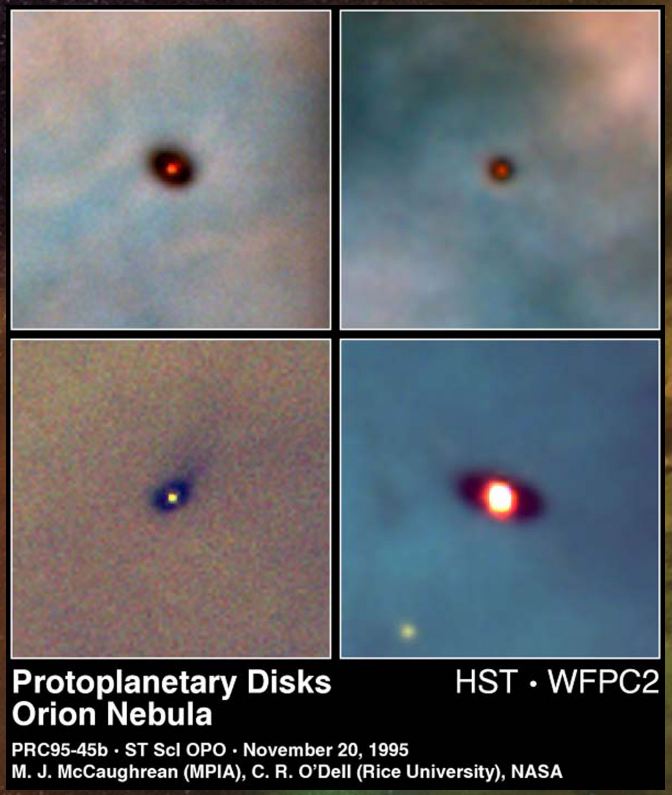
Saturn rings



credit: Galieo Galilei

Galactic disks

Protoplanetary disks



Basic properties:

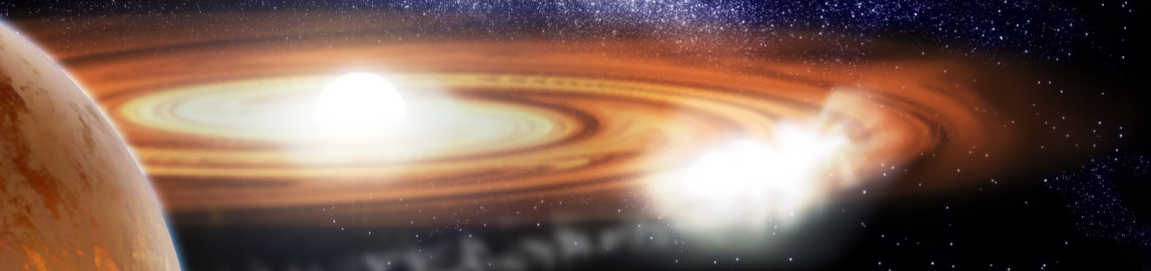
- Low luminosity
- Solar-mass stars: $M_* \sim M_\odot$
- Temperature: $T \sim (10 - 10^3)\mathrm{K}$
- Self-gravitating $\rightarrow$ planet formation
- Disk size: $R_{\mathrm{out}} \sim (10^{11} - 10^{15})\mathrm{cm}$

...Artist's impression & HST

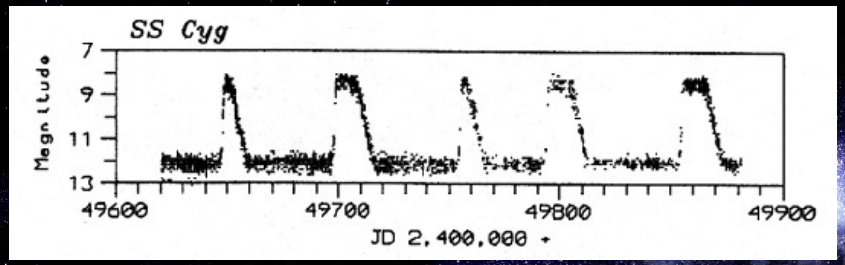
Cataclysmic variables

Basic properties:

- Close stellar binaries (primary is white dwarf)
- $M_{\rm WD} \sim M_\odot$, $R_{\rm out} \sim (10^9 - 10^{10})\,{\rm cm}$, $T \sim (10^3 - 10^7)\,{\rm K}$

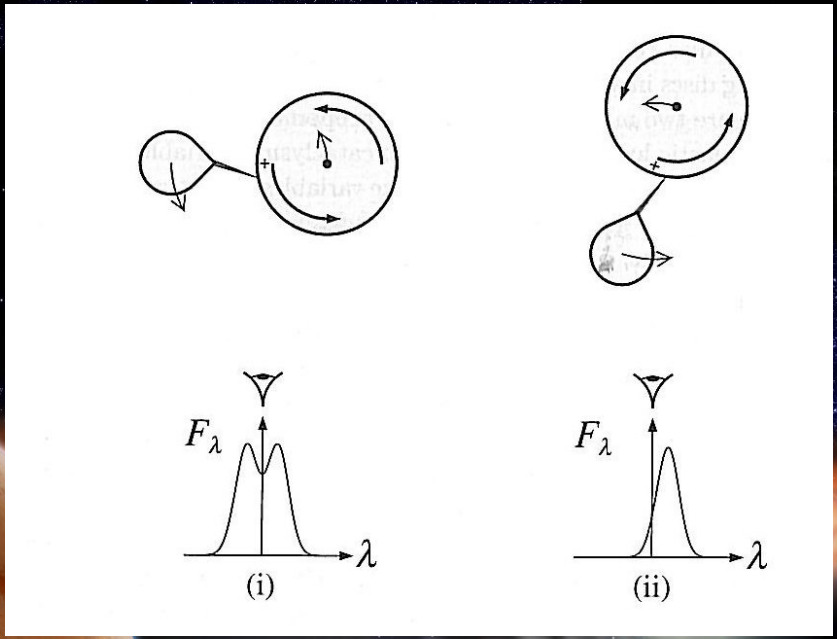


outbursts → Dwarf novae

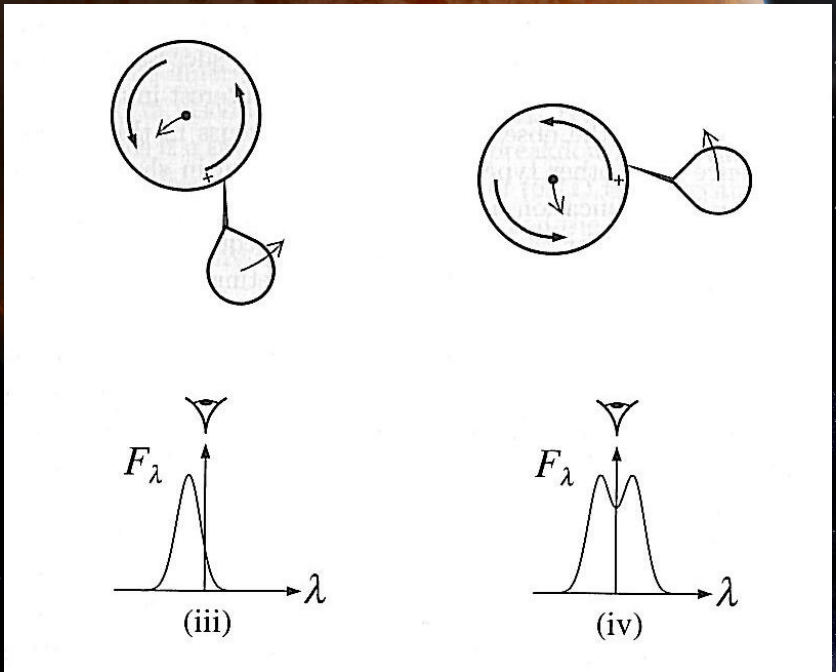


...Artist's impression

Cataclysmic variables



Observational evidence:

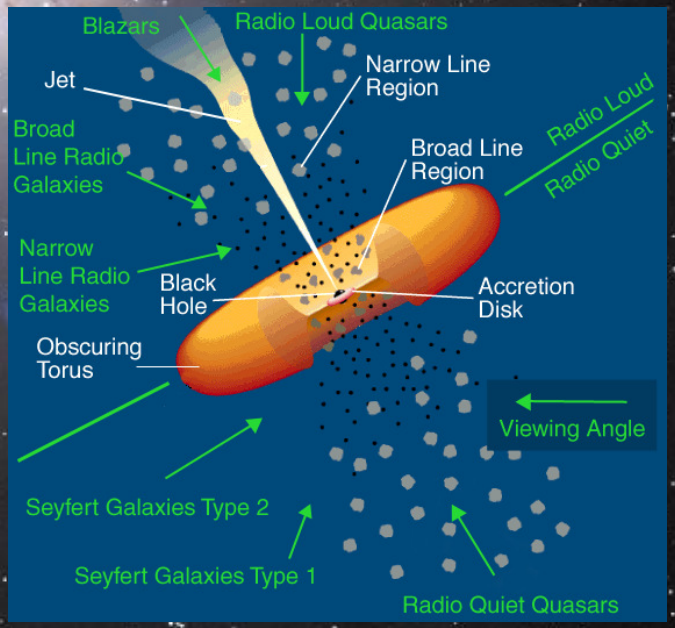


Frank, King & Raine

Active Galactic Nuclei & Quasars

Basic properties:

- Supermassive black-holes: $M_\bullet \sim (10^8 - 10^{12}) M_\odot$
- Largest accretion disks: $R_{\rm out} \sim (10^6 - 10^{11})(M_\bullet / M_\odot)\,{\rm cm}$
- intraday variability
- Temperature: $T \sim (10^2 - 10^5)\,{\rm K}$
- Cosmologic distances

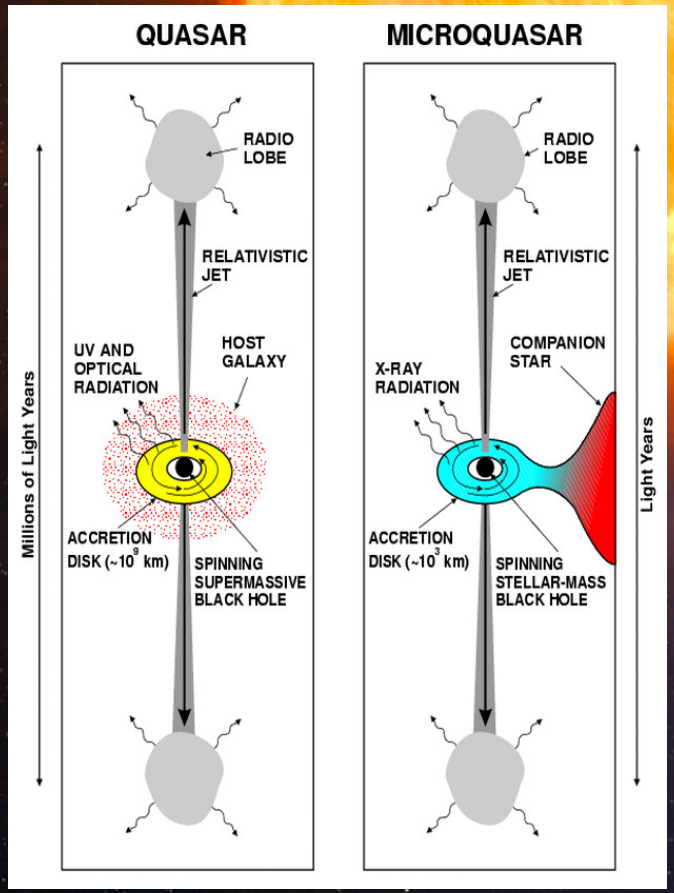


Unified model:

- Radio galaxies
- Seyfert galaxies I, II
- Blazars

... Centaurus A, Urry & Padovanni

X-ray binaries & μquasars

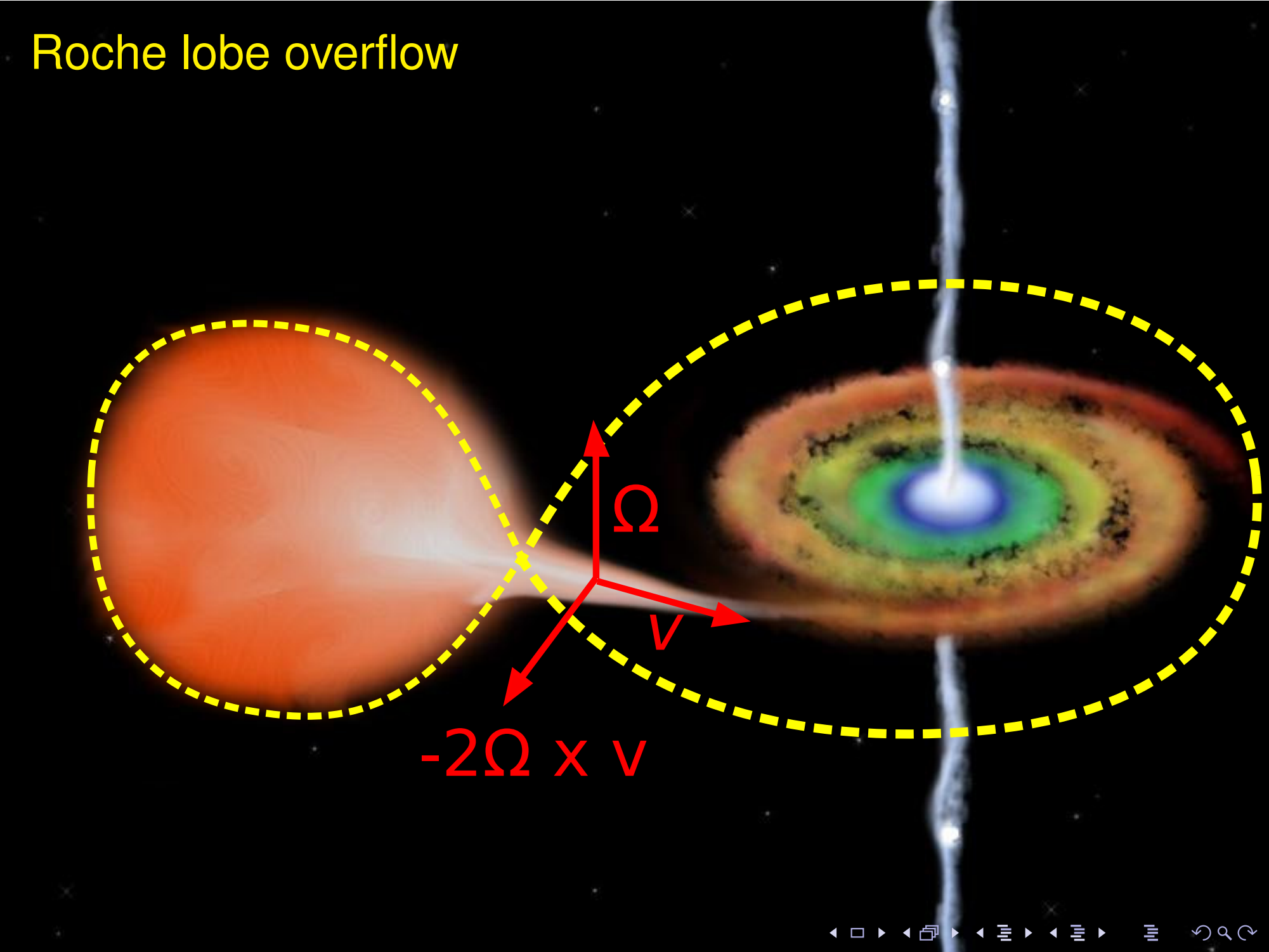


... Artist's impression & Mirabel et al.

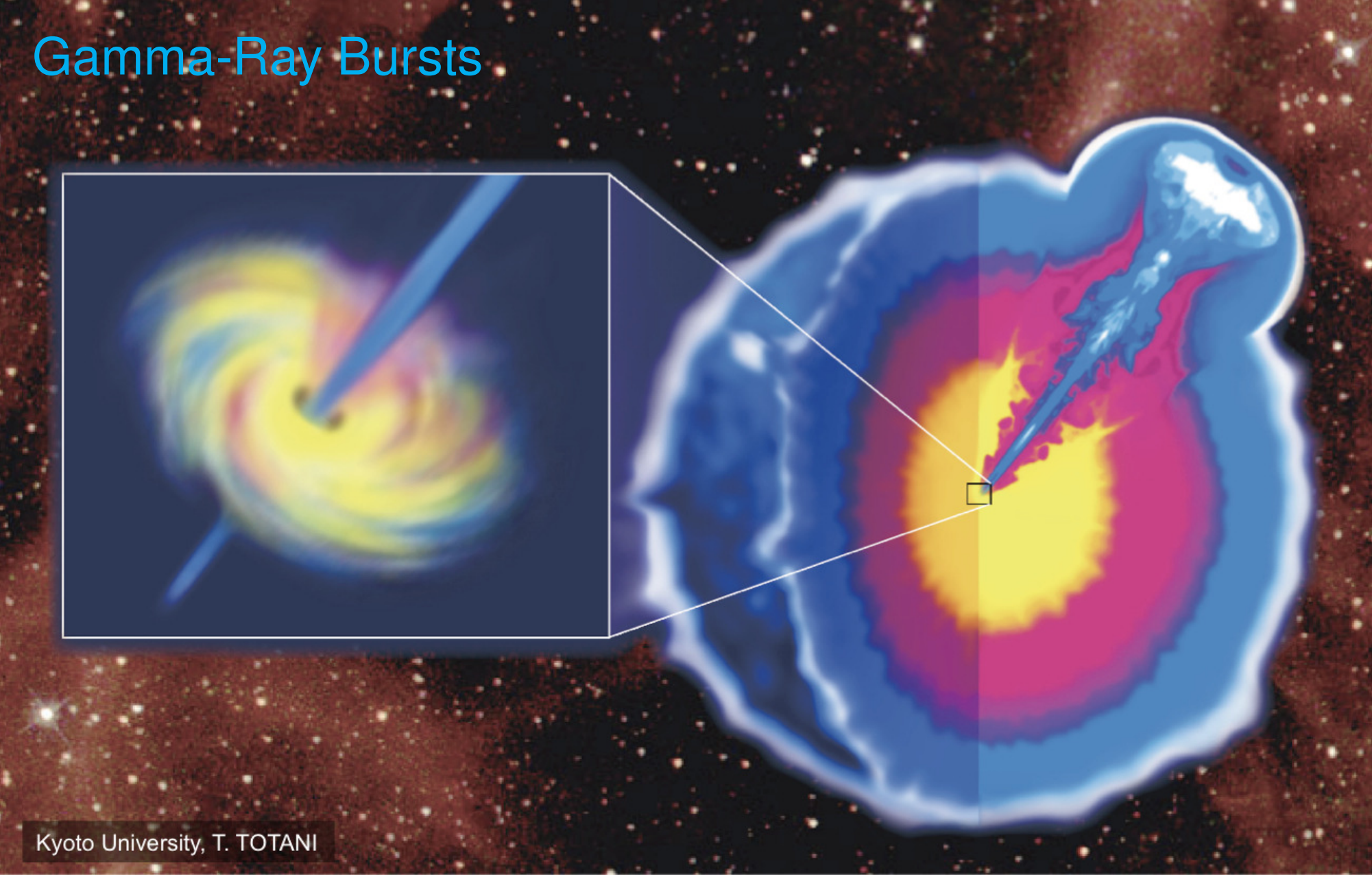
Basic properties:

- Stellar mass black-holes: $M_\bullet \sim (3 - 20) M_\odot$
- Disk sizes: $R_{\rm out} \sim (10^6 - 10^{11})\,{\rm cm}$
- ms variability
- Temperature: $T \sim (10^3 - 10^7)\,{\rm K}$
- Relativistic jets

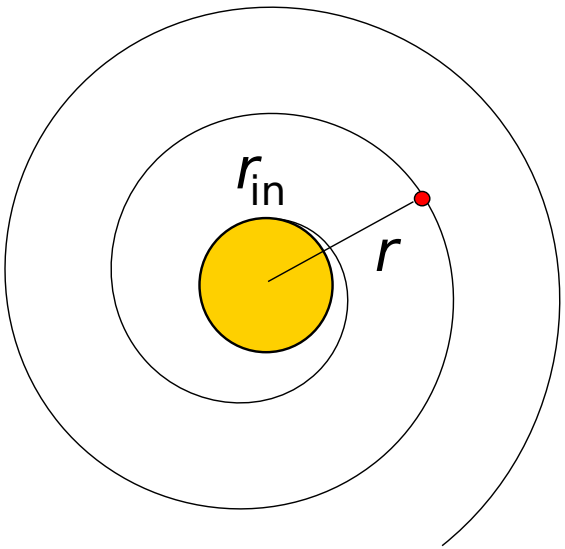
Roche lobe overflow



Gamma-Ray Bursts



Accretion power



- Force balance during inspiral:

$$\frac{mv^2}{r} \approx G\frac{mM}{r^2} \quad \Rightarrow \quad v^2 \approx \frac{GM}{r}$$

- Energy balance:

$$E = E_{\rm kin} + E_{\rm pot} = \frac{1}{2}mv^2 - G\frac{mM}{r} \approx -\frac{GMm}{2r}$$

- Energy loses during accretion:

$$\Delta E = E(\infty) - E(r_{\rm in}) \approx \frac{GMm}{2r_{\rm in}}$$

- Black holes: $r_{\rm in} \approx r_{\rm g} \sim GM/c^2 \Rightarrow \boxed{\Delta E \sim mc^2}$

Realistically: $\Delta E = \eta mc^2$, where η is accretion efficiency.

Accretion power

Efficiency of energy generators in nature:

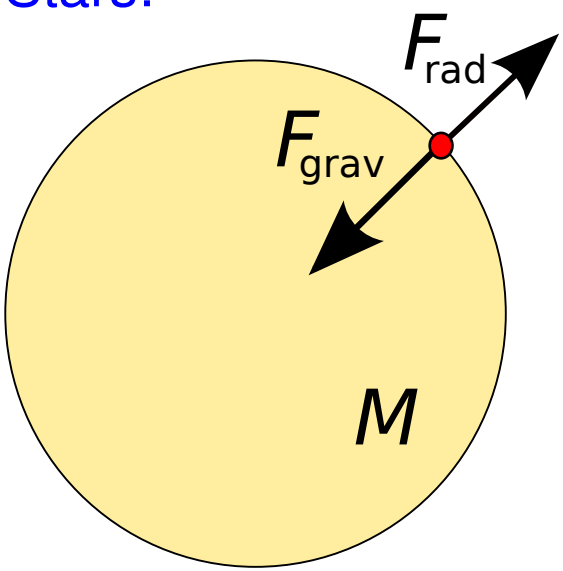
- Chemical burning: $\eta < 0.000\ 00001$
- Nuclear burning: $\eta < 0.01$
- Disk accretion onto counter-rotating BH: $\eta = 0.04$
- Disk accretion onto non-rotating BH: $\eta = 0.06$
- Disk accretion onto rotating BH: $\eta = 0.42$

Black hole accretion is a POWERFUL source of energy!

Luminosity

Is there a limit on the luminosity of an object of a given mass?

Stars:

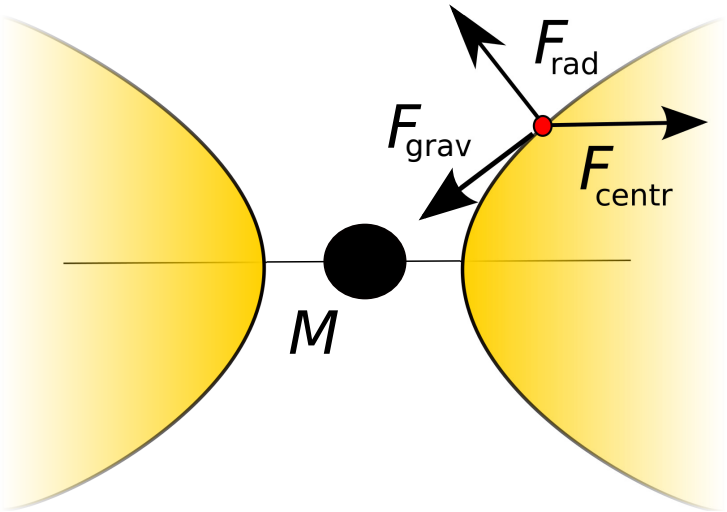


$$F_{\rm rad} \approx F_{\rm grav}, \quad F_{\rm rad} = \frac{\sigma}{c}\frac{L}{4\pi R^2}, \quad F_{\rm grav} = \frac{GMm}{R^2}$$

$$\boxed{L \leq \frac{4\pi GMmc}{\sigma} \equiv L_{\rm Edd}}$$

Disks:

- Rotation acts against radiation
- No principal limit



Accretion disks are the most LUMINOUS objects in the universe!

GENERAL RELATIVITY AND BLACK HOLES

i.e.,

what you need to know for accretion...

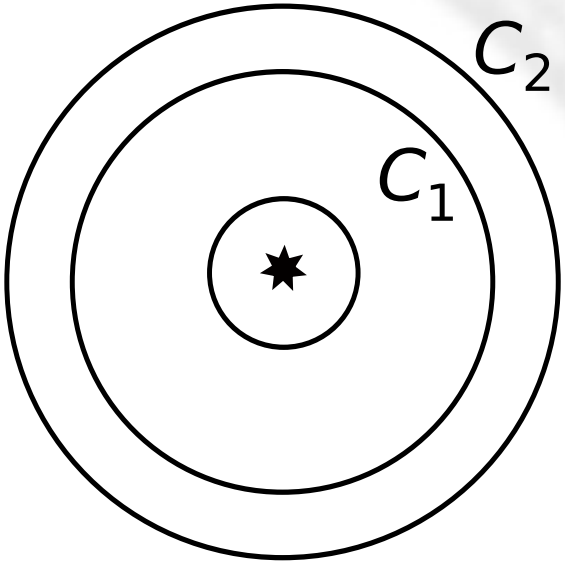
Black holes

In strong gravitational fields the space time is warped and twisted

- Space time geometry of non-rotating BHs is described by the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

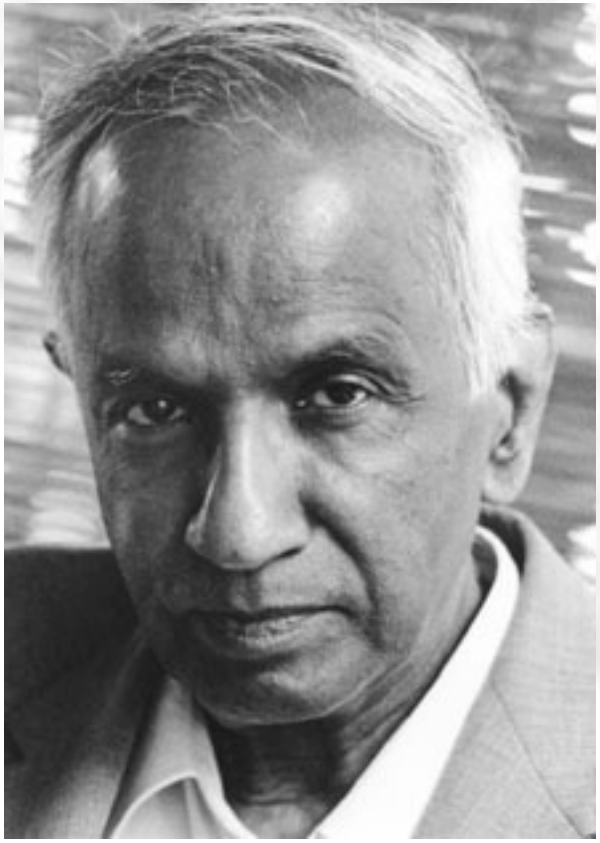
- Singular at $r = 2M$ (horizon) and $r = 0$ (singularity)



- Euclidean geometry does not apply:

$$\boxed{\Delta r > \frac{C_2}{2\pi} - \frac{C_1}{2\pi}}$$

Black holes

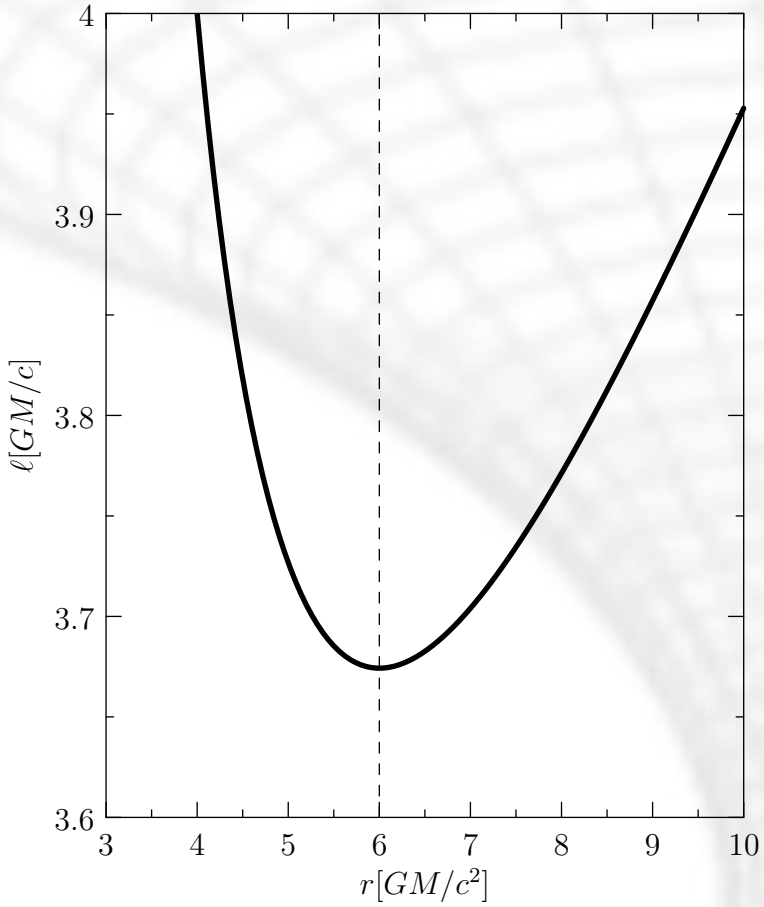


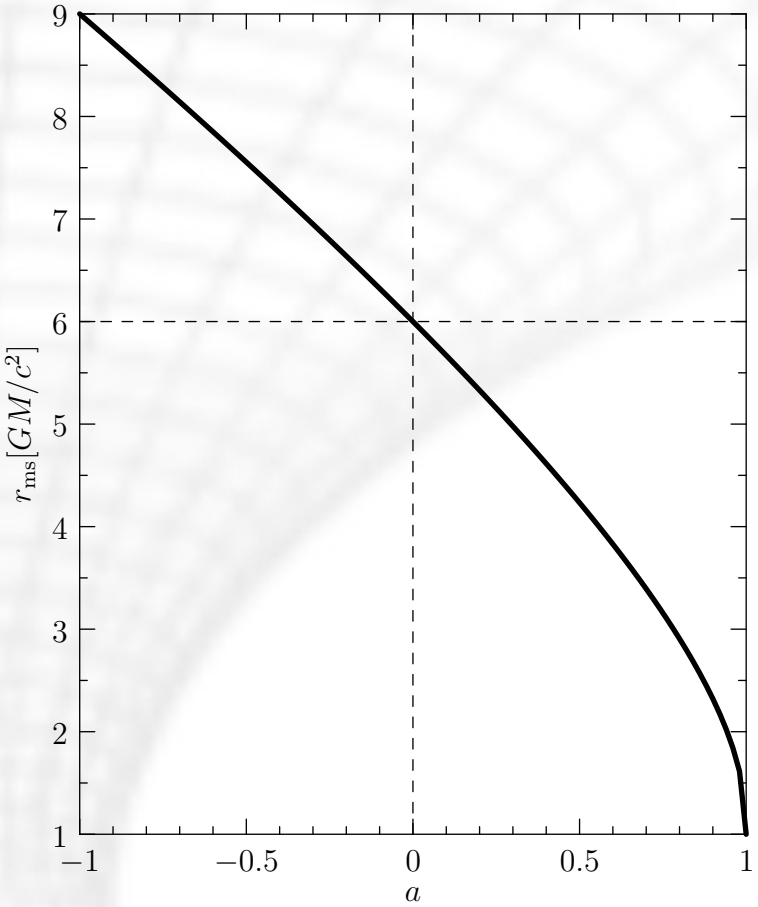
"The black holes of nature are the most perfect macroscopic objects there are in the universe:
the only elements in their construction are our concepts of space and time.
And since the general theory of relativity provides only a single unique family of solutions for their descriptions, they are the simplest objects as well."

S. Chandrasekhar, The Mathematical Theory of Black holes

Circular orbits in black-hole spacetimes

Innermost stable circular orbit (ISCO) at $r_{\rm ms} = 6M$





$\rightarrow d\ell_{\rm K}/dr < 0$ for $r < 6M \Rightarrow$ Region of unstable orbits.

Relativity without relativity

Paczynski's trick: pseudo-Newtonian potential

Can we invent a Newtonian potential that mimics GR effects?

Even a small 'perturbation' of a Newtonian potential gives a big change:

$$\boxed{\Phi(r) = -\frac{GM}{r - r_{\mathrm{Schw}}}, \quad r_{\mathrm{Schw}} = \frac{2GM}{c^2}}$$

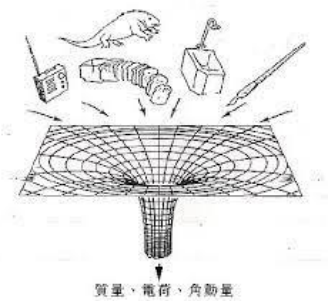
- Gives a correct formula for ℓ_{K} in Schwarzschild spacetime:

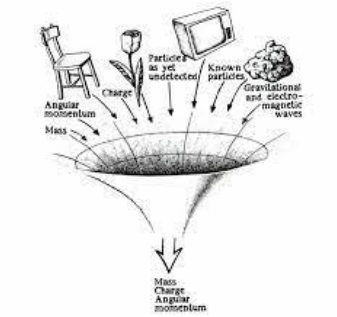
$$\ell_{\mathrm{K}} = \frac{\sqrt{GMr}}{1 - \frac{r_{\mathrm{Schw}}}{r}}$$

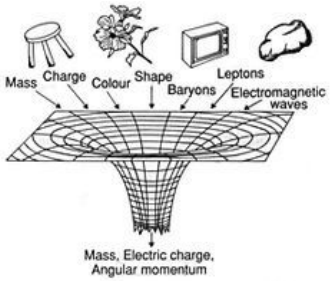
- Gives acceptable values for the epicyclic frequencies.
- Unsuitable for rotating BHs (gravito-magnetism, ergosphere)

Central question:

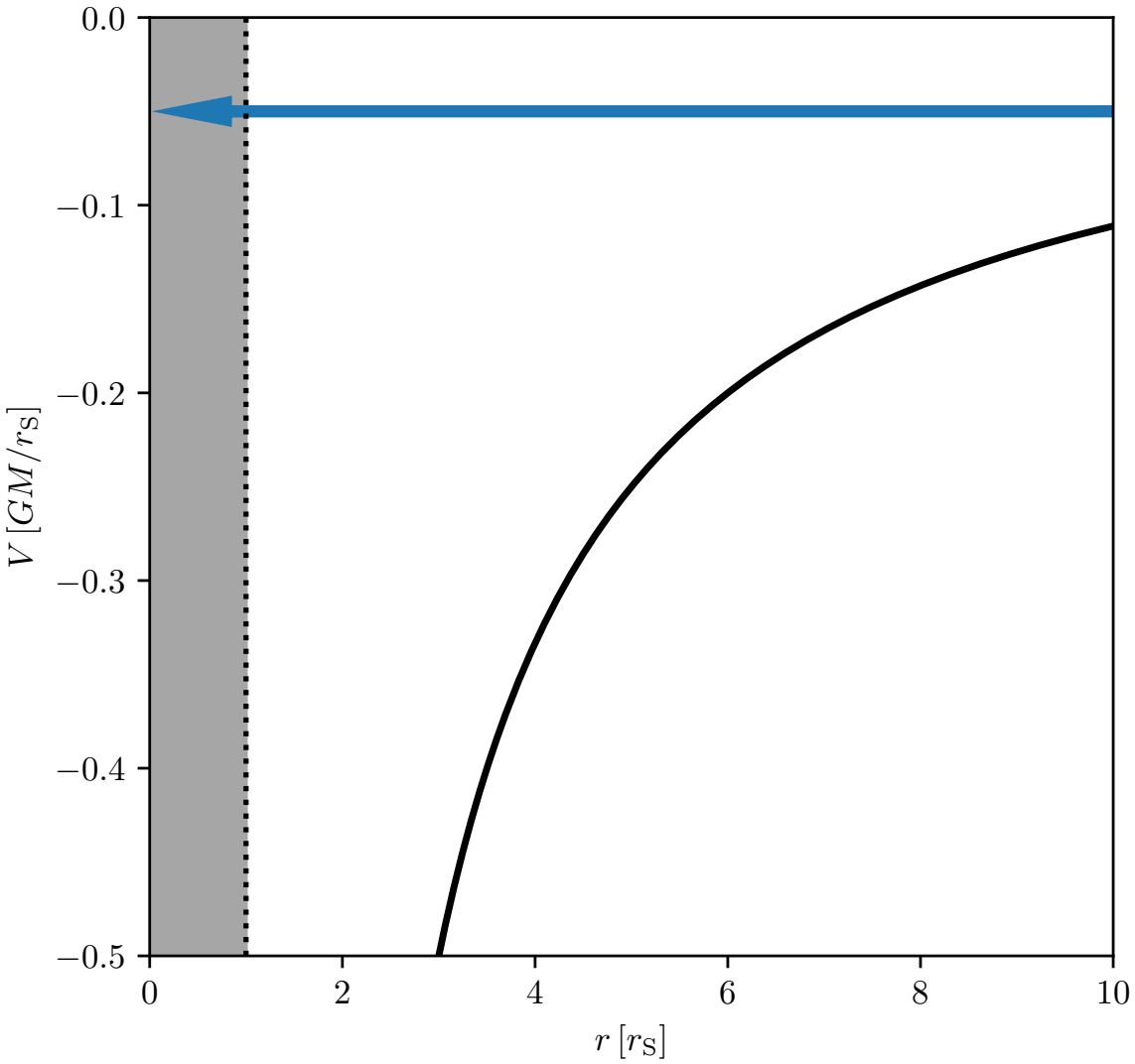
How to feed black hole?





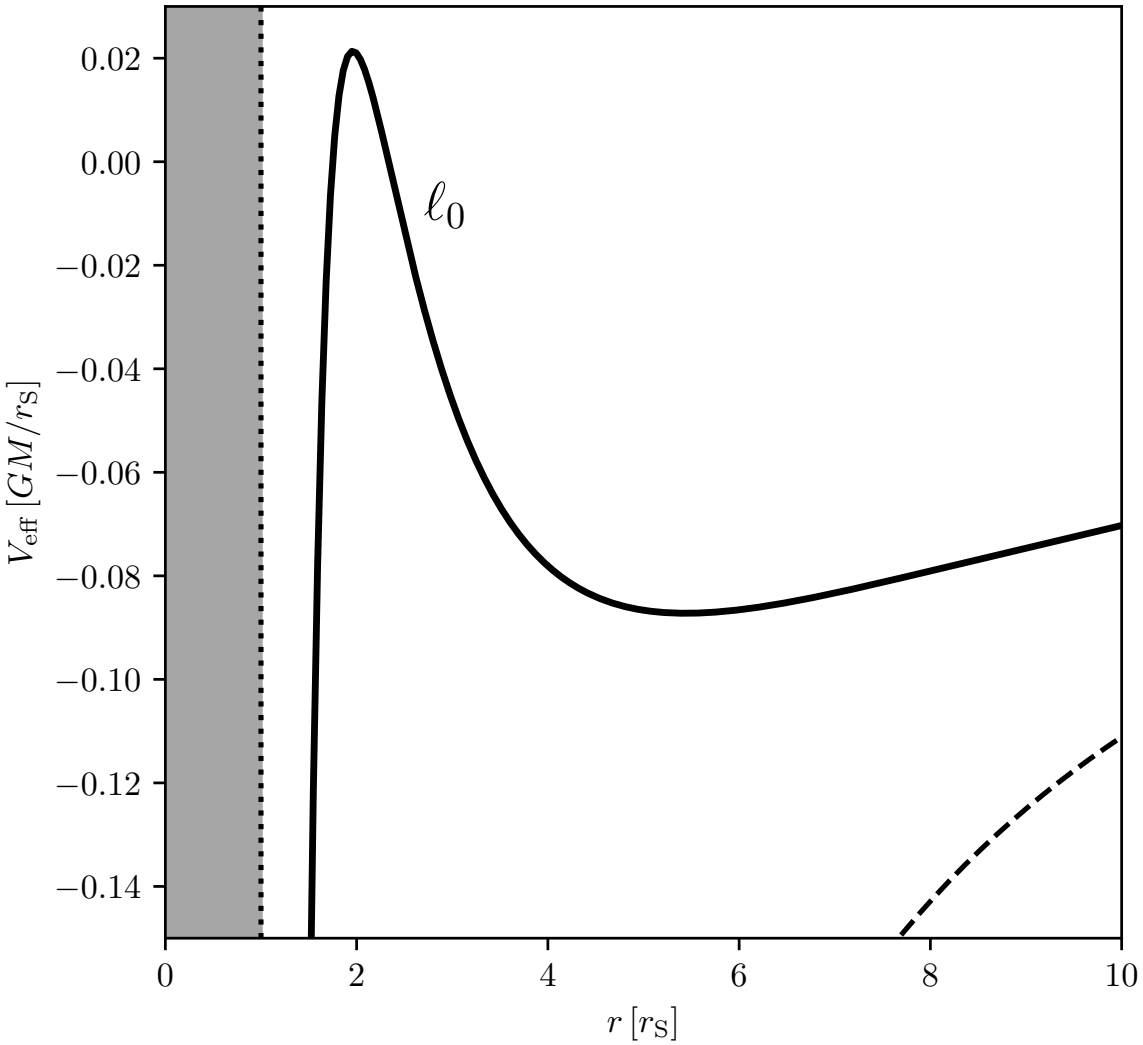


Objects with zero angular momentum



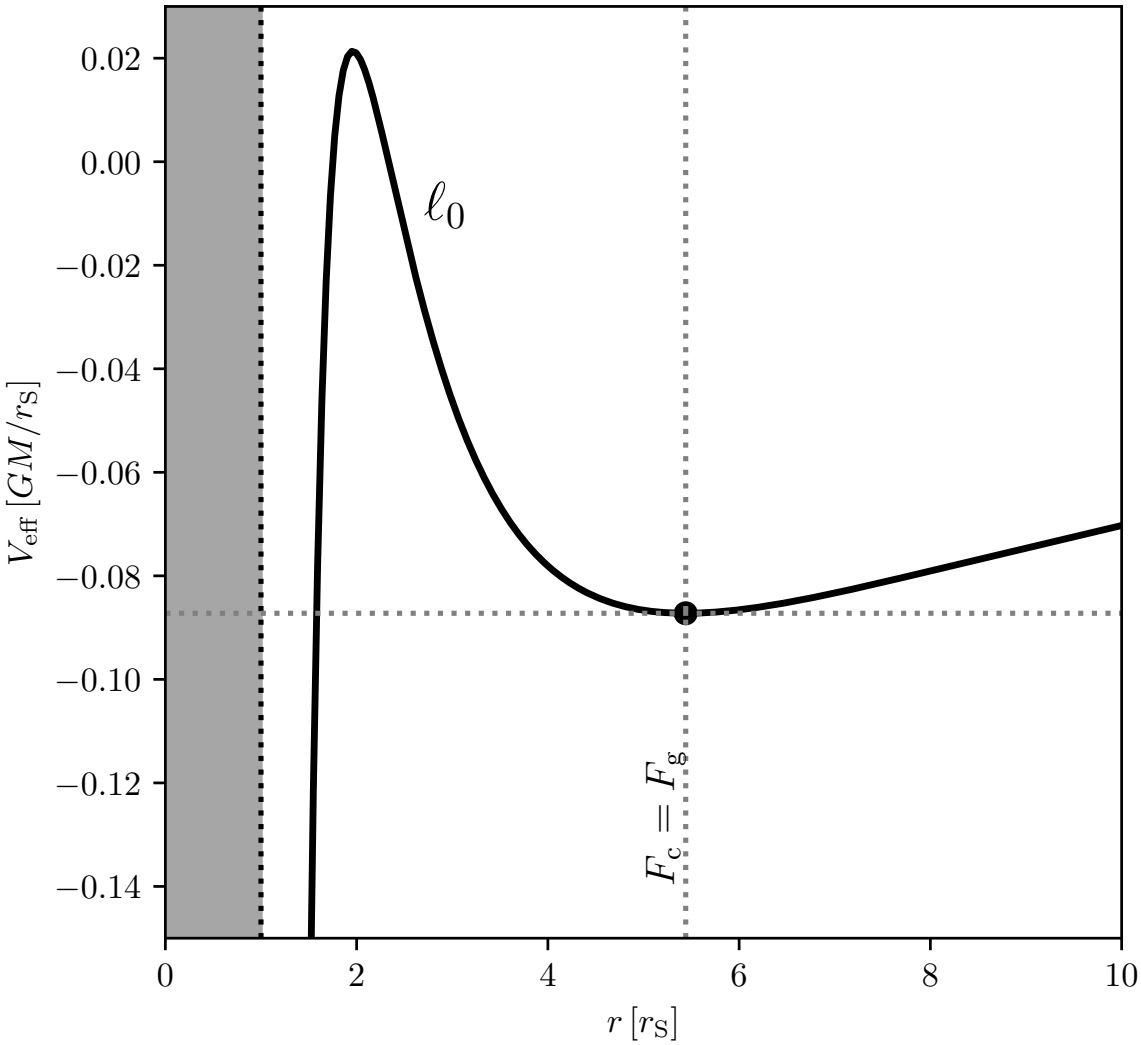
Sending an object with zero angular momentum is easy

Rotating objects



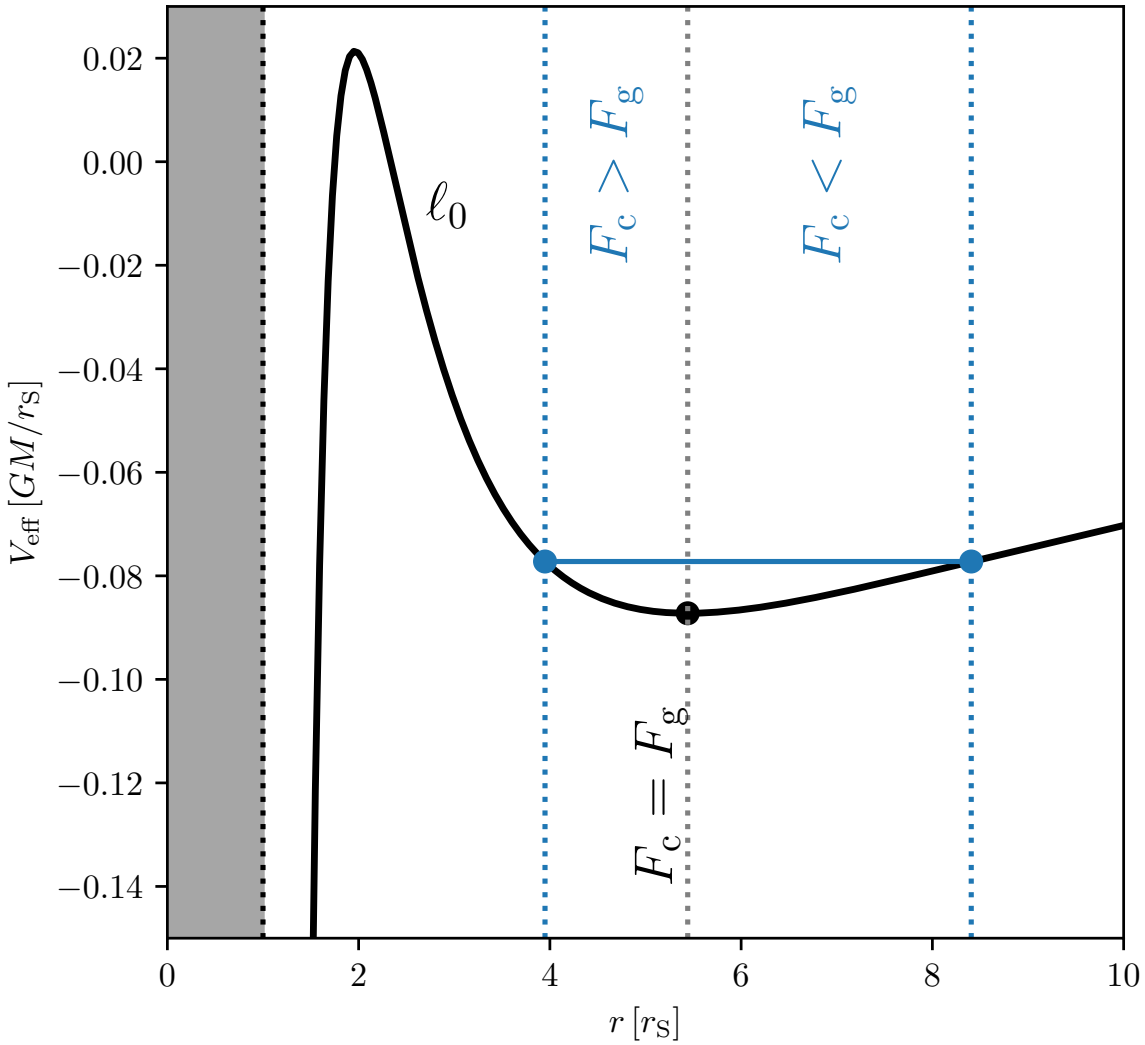
Effective potential: Gravity + centrifugal force

Rotating objects



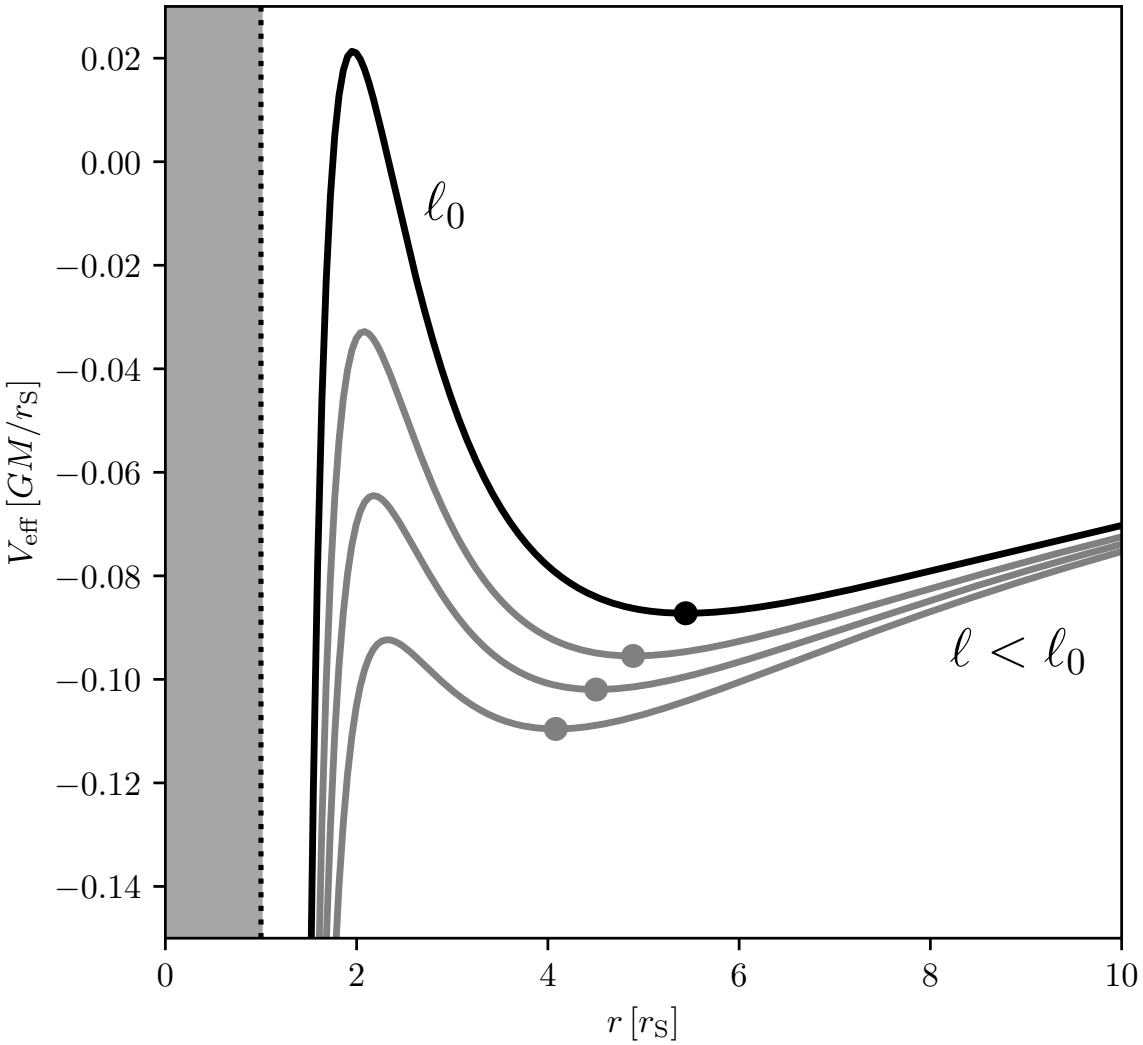
Minimum the energy corresponds to circular orbit.

Rotating objects



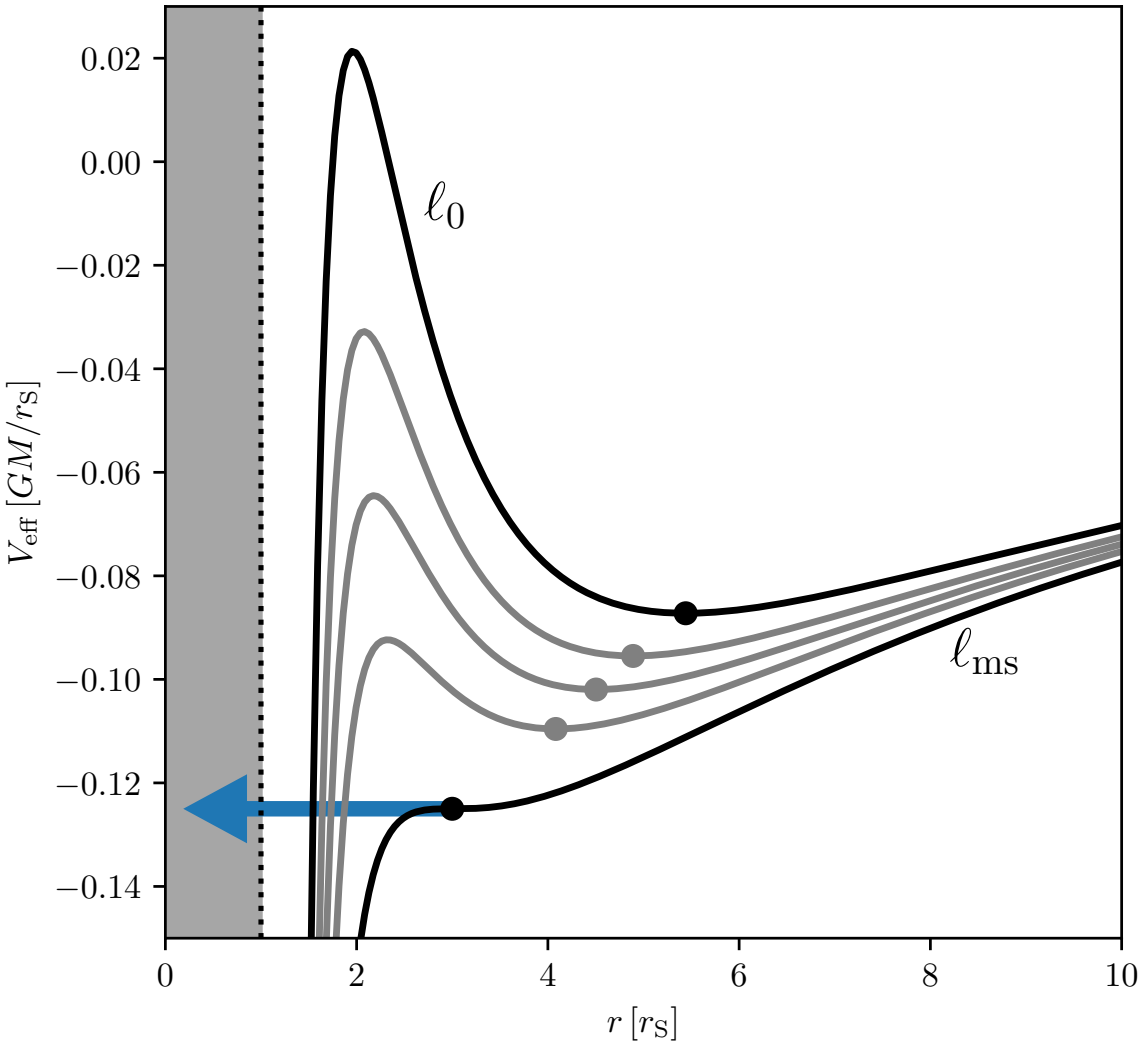
Increase of energy does not help (centrifugal barrier)

Rotating objects



Lowering the angular momentum helps

Rotating objects

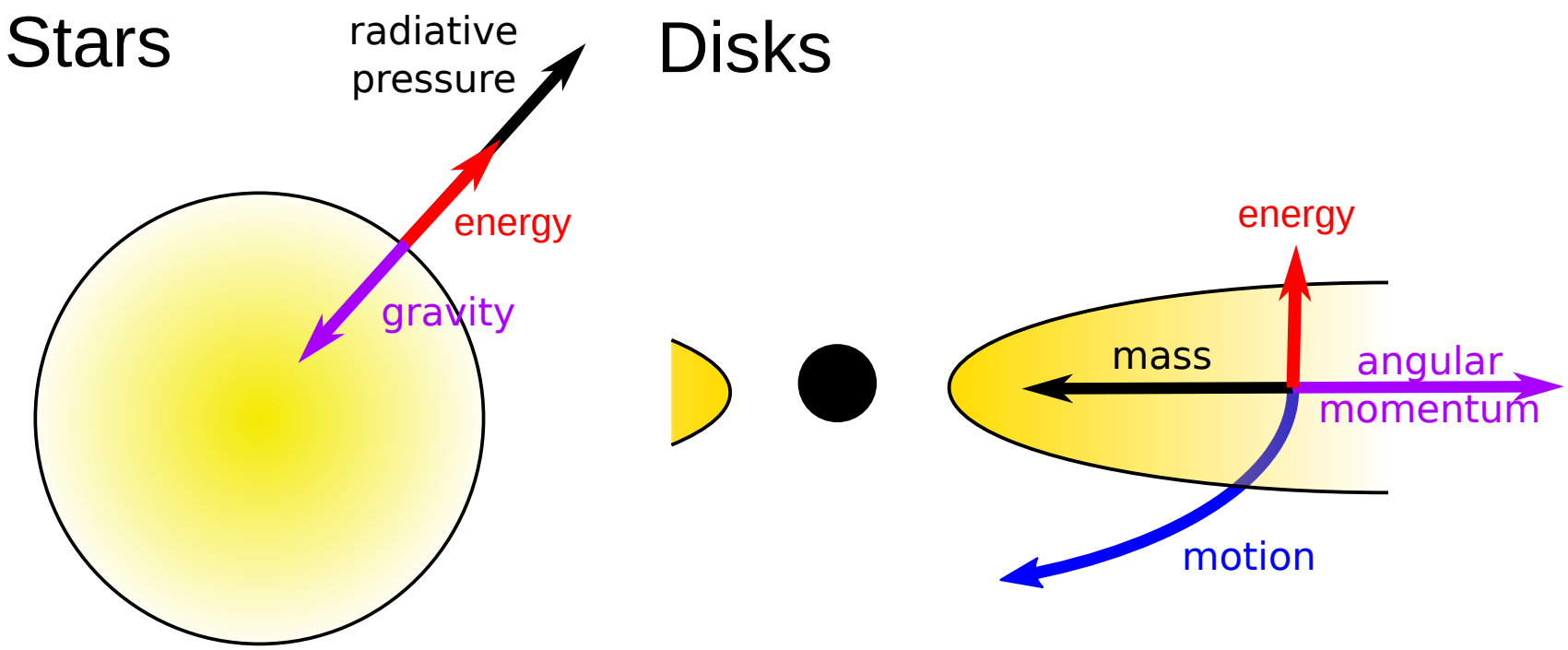


This process finish at marginally stable orbit ⇒ free fall

Essential ingrediences

- Fluid (presure)
- Angular momentum transport (viscosity)
- Energy balance (removing mechanical energy)

⇒ Disk accretion may be described by Navier-Stokes equations



⇒ Disk accretion is essentially 3D problem

STANDARD PICTURE

Shakura-Sunyaev cold thin disk model

Navier-Stokes equations

Radial momentum equation:

$$v^r \frac{\partial v^r}{\partial r} + v^z \frac{\partial v^r}{\partial z} - r(\Omega^2 - \Omega_{\rm K}^2) + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{2}\frac{d\Omega_{\rm K}}{dr} z^2 = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(2r\eta\partial_r \frac{\partial v^r}{\partial r}\right) - 2\frac{\eta v^r}{\rho r^2} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial r}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial r}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial r}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the azimuthal one:

$$\frac{v^r}{r^2}\frac{\partial}{\partial r}\left(r^2\Omega\right) + v^z\frac{\partial\Omega}{\partial z} = \frac{1}{r^3\rho}\frac{\partial}{\partial r}\left(r^3\eta\frac{\partial\Omega}{\partial r}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial\Omega}{\partial z}\right),$$

the vertical one:

$$v^r \frac{\partial v^z}{\partial r} + v^z \frac{\partial v^z}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + \Omega_{\rm K}^2 z = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^z}{\partial r}\right) + \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(2\eta\frac{\partial v^z}{\partial z}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial z}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v^r) + \frac{\partial}{\partial z}(\rho v^z) = 0$$

and the energy equation and equation of state:

$$Tv^r\frac{\partial s}{\partial r} = Q^+_{\rm visc} - Q^-_{\rm rad}, \quad p = p_{\rm gas} + p_{\rm rad} = \mathcal{R}_{\rm m}\rho T + aT^4.$$

Shakura-Sunyaev disk model

Radial momentum equation (Keplerian flow):

$$v^r \frac{\partial v^r}{\partial r} + v^z \frac{\partial v^r}{\partial z} - \underline{r(\Omega^2 - \Omega_{\mathrm{K}}^2)} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{2}\frac{d\Omega_{\mathrm{K}}}{dr} z^2 = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(2r\eta\partial_r \frac{\partial v^r}{\partial r}\right) - 2\frac{\eta v^r}{\rho r^2} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial r}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial r}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial r}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the azimuthal one (radial transport of angular momentum):

$$\underline{\frac{v^r}{r^2}\frac{\partial}{\partial r}\left(r^2\Omega\right)} + v^z\frac{\partial\Omega}{\partial z} = \underline{\frac{1}{r^3\rho}\frac{\partial}{\partial r}\left(r^3\eta\frac{\partial\Omega}{\partial r}\right)} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial\Omega}{\partial z}\right),$$

the vertical one (hydrostatic balance):

$$v^r \frac{\partial v^z}{\partial r} + v^z \frac{\partial v^z}{\partial z} + \underline{\frac{1}{\rho}\frac{\partial p}{\partial z}} + \underline{\Omega_{\mathrm{K}}^2 z} = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^z}{\partial r}\right) + \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(2\eta\frac{\partial v^z}{\partial z}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial z}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the continuity equation (mass conservation):

$$\underline{\frac{1}{r}\frac{\partial}{\partial r}(r\rho v^r)} + \frac{\partial}{\partial z}(\rho v^z) = 0$$

and the energy equation (local balance) and equation of state:

$$Tv^r\frac{\partial s}{\partial r} = \underline{Q_{\mathrm{visc}}^+} - \underline{Q_{\mathrm{rad}}^-}, \quad p = p_{\mathrm{gas}} + p_{\mathrm{rad}} = \mathcal{R}_{\mathrm{m}}\rho T + aT^4.$$

Shakura & Sunyaev (1973)

Easy solution

Two equations are algebraic:

- Radial Euler equation:

$$\boxed{\Omega(r) = \Omega_{\rm K}(r)}$$

- Vertical Euler equation:

$$\boxed{H(r) = \frac{c_{\rm s}(r)}{r\Omega_{\rm K}(r)}}$$

Two equations are easily integrable:

- Continuity equation:

$$\frac{d}{dr}\left(r\Sigma v^r\right) = 0 \quad \Rightarrow \quad \boxed{r\Sigma v^r = \text{const} \equiv -\frac{\dot{M}}{2\pi}}$$ *... accretion rate*

- Azimuthal equation:

$$\frac{d}{dr}\left(r\Sigma v^r \ell\right) - \underbrace{\frac{d}{dr}\left(r^3 \nu \Sigma \frac{d\Omega}{dr}\right)}_{\mathcal{G}/(2\pi)} = 0 \quad \Rightarrow \quad \boxed{-\dot{M}\ell_{\rm K} - \mathcal{G} = \text{const} \equiv -\dot{M}\ell_*}$$

When $\ell = \ell_* \Rightarrow \mathcal{G} = 0$ *... zero torque*

Viscous dissipation & radiative cooling

Energy balance:

- Rate of energy production (viscous heating):

$$Q^+ = \frac{\mathcal{G}}{8\pi r}\frac{d\Omega}{dr}$$

- Rate of energy loss (radiative cooling):

$$Q^- = \sigma_{\rm SB} T^4$$

Thermal equilibrium $Q^+ = Q^-$ gives the local temperature:

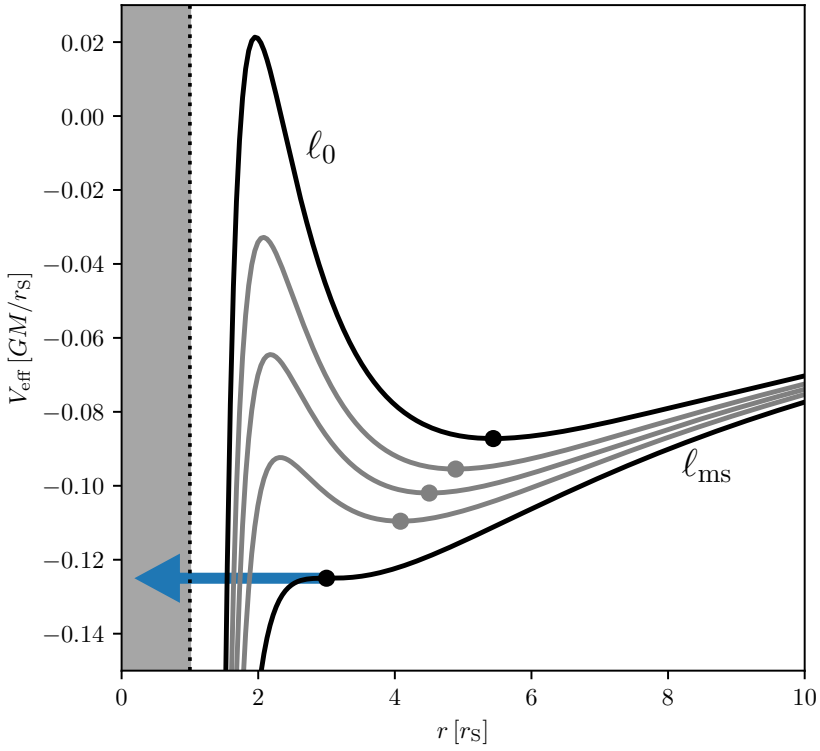
$$\boxed{T(r) = \left[-\frac{\dot{M}}{4\pi\sigma_{\rm SB}}\frac{1}{r}\frac{d\Omega}{dr}\left(\ell_{\rm K} - \ell_*\right)\right]^{1/4}}$$

Local temperature in the disk is independent of the *form* of the viscosity

Where is the zero-torque point (what is ℓ_*)?

Accretion torques are mediated *throuhout* the disk

(inner annuli act on the outer ones)



After reaching $r_{\rm ms}$:

- no more centrifugal support (plunging)
- flow becomes *quickly* transonic
- no causal connection from supersonic flow
- no mediated viscous torques

Assumption: at $r_{\rm ms}$ the applied torque is zero.

$$\boxed{\ell_* = \ell_{\rm K}(r_{\rm ms})}$$ *... zero-torque boundary condition*

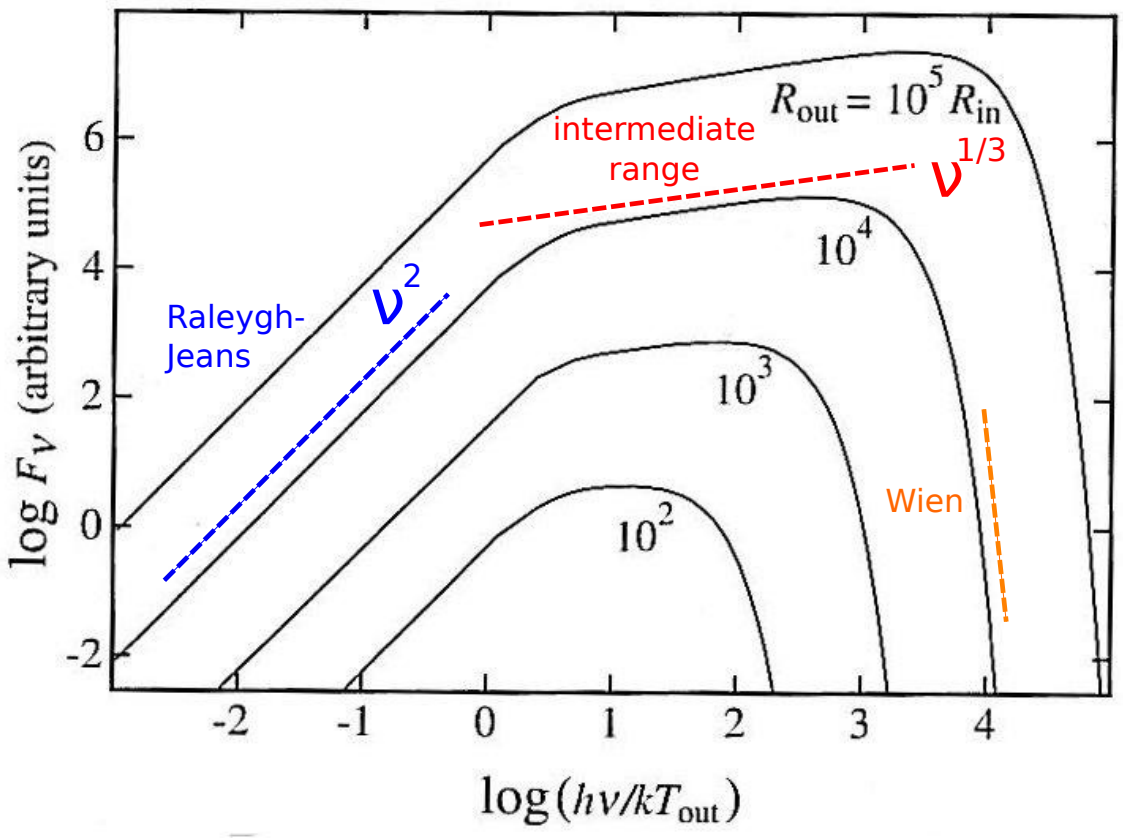
BUT: Other torques, causality in viscous flow, *etc...*

Spectra

Local spectra of the disk are given by the Planck law:

$$I_\nu(r) = B_\nu\left[T(r)\right] = \frac{2h\nu^3}{c^2\left[e^{h\nu/kT(r)} - 1\right]}$$

Disk spectra after integrating over the surface:



... see talk of Michal Bursa

Radial structure

For other physical quantities viscosity prescription needed.

Shakura & Sunyaev (1973):

$$\boxed{\tau^{\hat{r}\hat{\phi}}_{\rm visc} = \alpha p}$$ *...α-viscosity prescription*

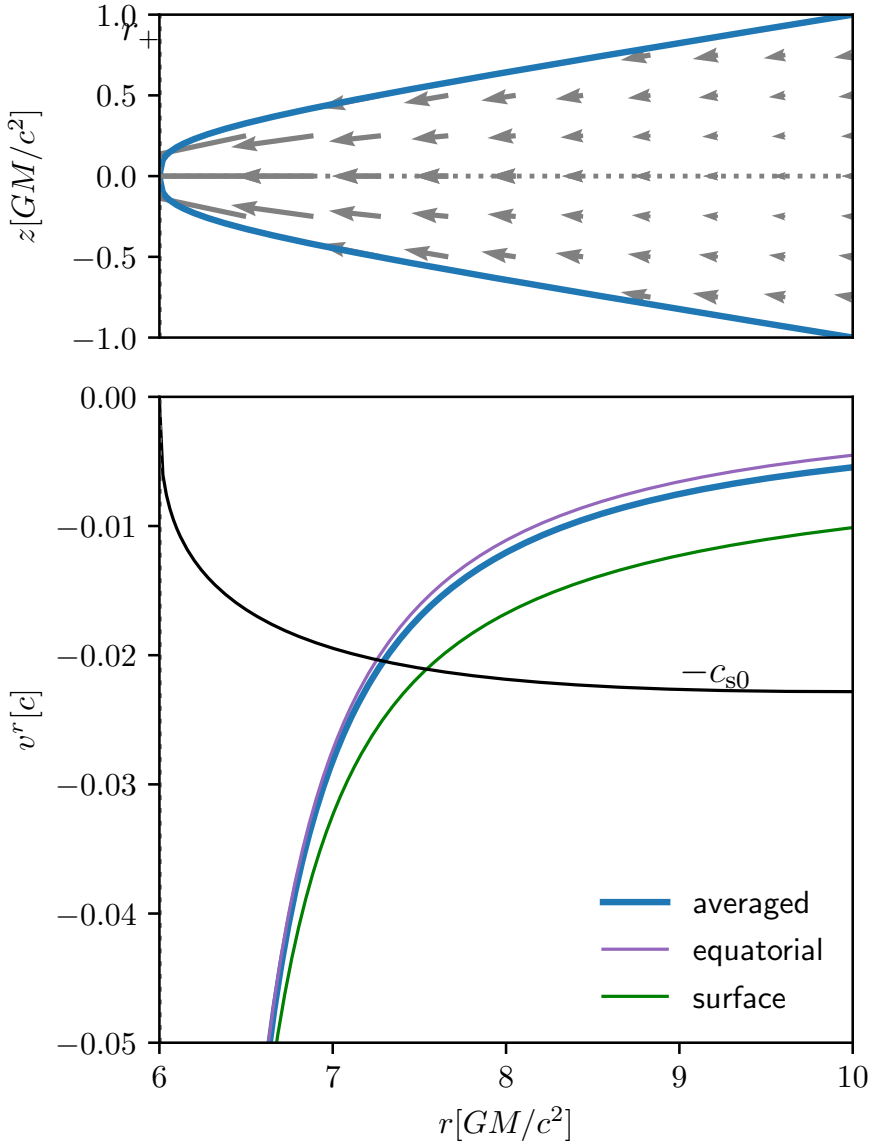
Radial structure (radiation-pressure dominated disk):

$$\begin{aligned} h &= (1.59\mathrm{km})\,(f\dot{m}) \\ v^r &= (1.16\times 10^8\mathrm{m/s})\,\alpha m^{-2}(f\dot{m})^2\hat{r}^{-5/2} \\ \rho &= (0.0291\mathrm{kg/m^3})\,\alpha^{-1}m(f\dot{m})^{-2}\hat{r}^{3/2} \\ T_{\rm c} &= (4.96\times 10^7\mathrm{K})\,\alpha^{-1/4}m^{-1/4}\hat{r}^{-3/8} \end{aligned}$$

where

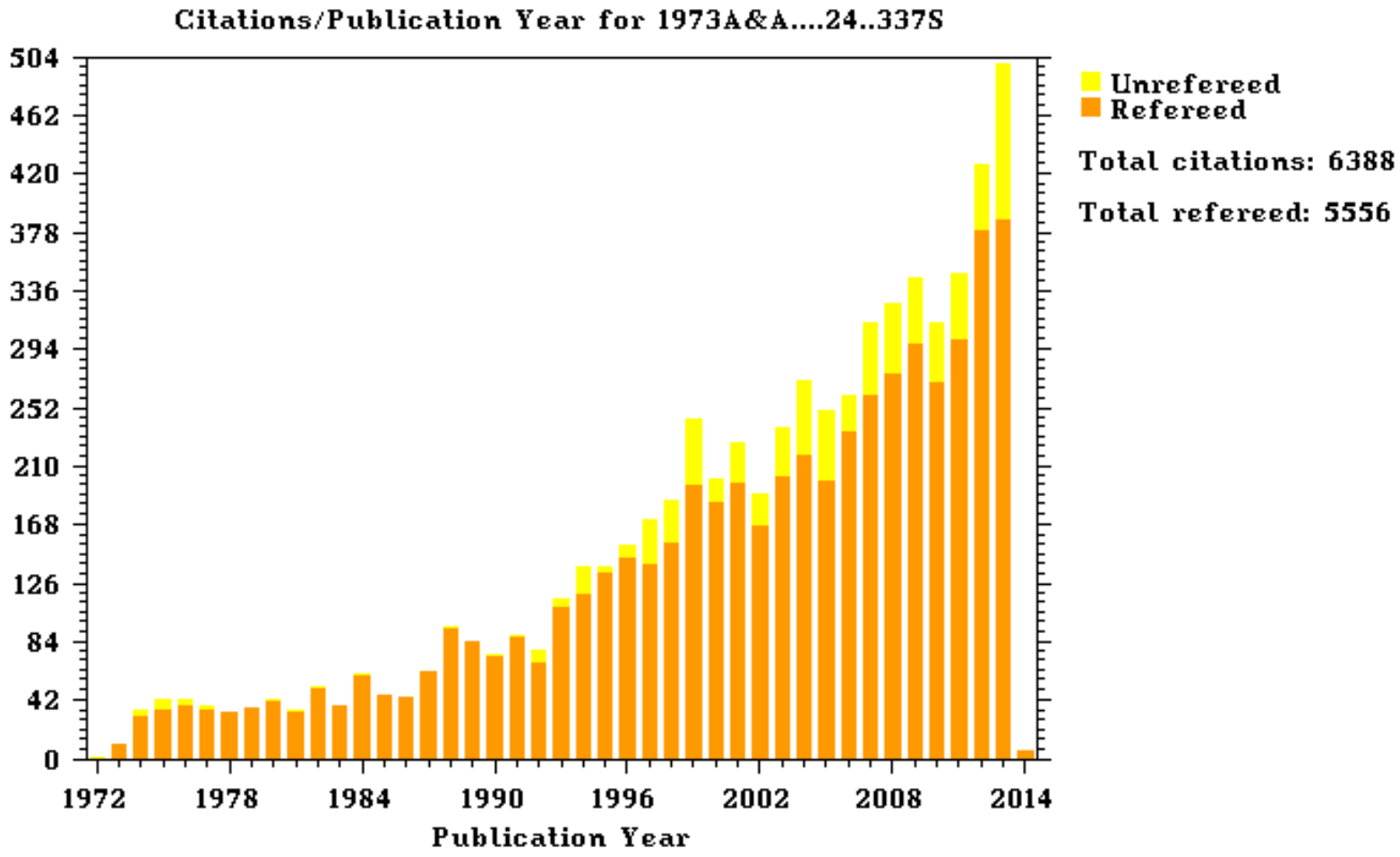
$$m \equiv \frac{M}{M_\odot}, \quad \dot{m} \equiv \frac{\dot{M}}{10^{14}kg/s}, \quad \hat{r} \equiv \frac{r}{r_{\rm g}}, \quad f = 1 - \sqrt{\frac{r_{\rm ms}}{r}}$$

Unphysical singularities at inner edge



... Horak & Kluzniak (2021), Penna+2012: Regularization $v^r = c_s|_{\rm ms}$

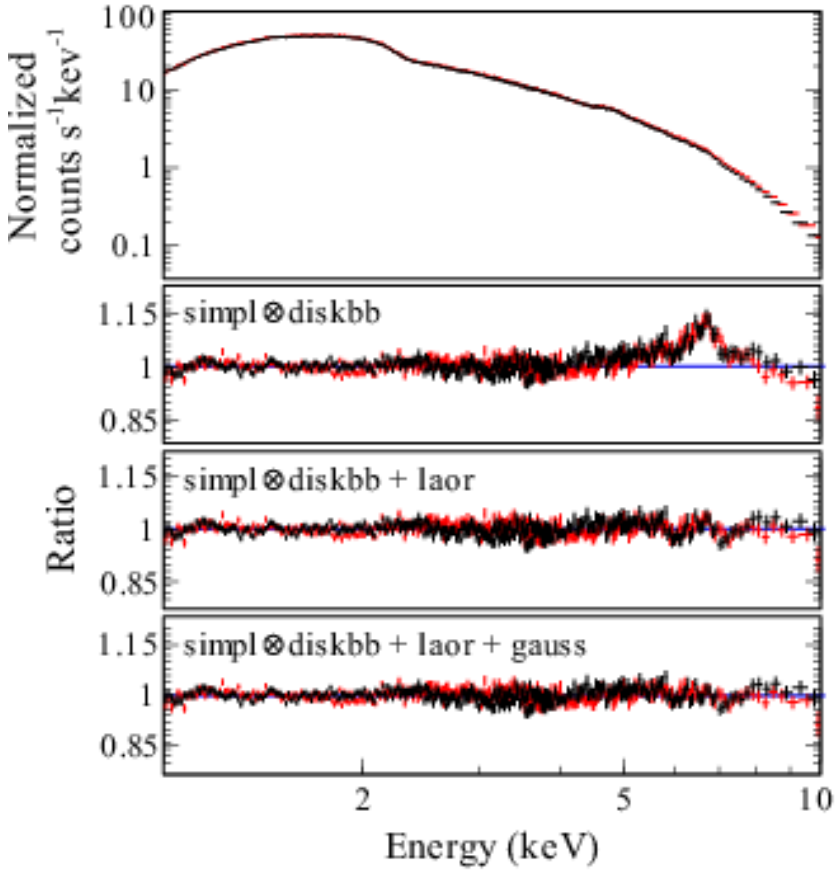
Shakura-Sunyaev (1973) model



A fully relativistic version by Novikov & Thorne in the same year

Applications

- Protoplanetary disks, layered disks with dead zones,...
- Measuring black-hole parameters (kerrBB, diskBB)

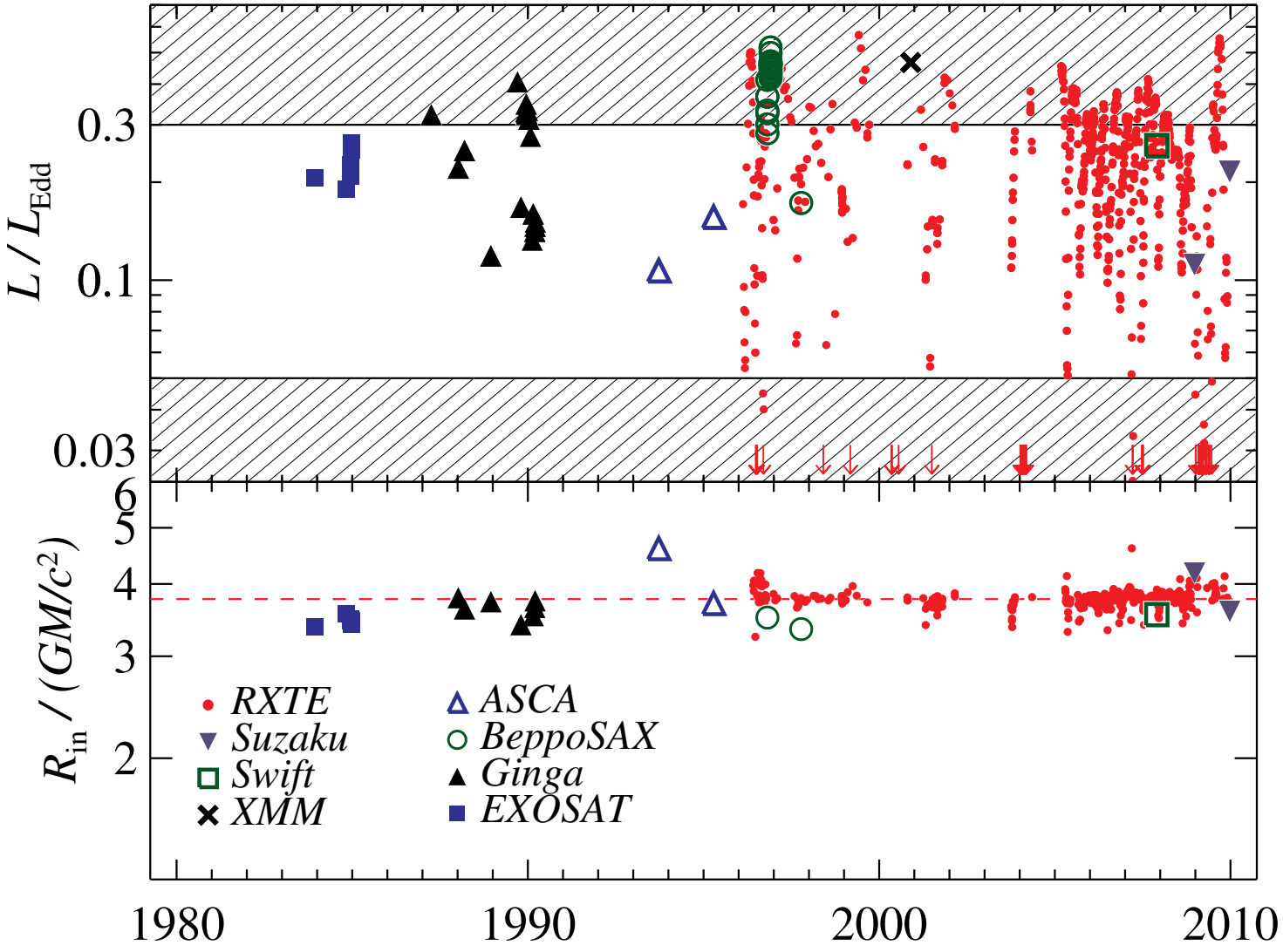


McClintock et al. (2013)

System	a_*	$M/M_\odot$
Persistent		
Cyg X-1	> 0.95	14.8 ± 1.0
LMC X-1	$0.92^{+0.05}_{-0.07}$	10.9 ± 1.4
M33 X-7	0.84 ± 0.05	15.65 ± 1.45
Transient		
GRS 1915+105	$> 0.95^b$	10.1 ± 0.6
4U 1543–47	0.80 ± 0.10^b	9.4 ± 1.0
GRO J1655–40	0.70 ± 0.10^b	6.3 ± 0.5
XTE J1550–564	$0.34^{+0.20}_{-0.28}$	9.1 ± 0.6
H1743–322	0.2 ± 0.3	$\sim 8^c$
LMC X-3	$< 0.3^d$	7.6 ± 1.6
A0620–00	0.12 ± 0.19	6.6 ± 0.25

Applications

Consistent result in a reasonable luminosity range!



...

measuring the inner of the disk in LMC X3 (McClintock et al. 2013)

More advanced models

Angular momentum transport

Nature of the angular momentum transport

Viscosity?

Conditions in the accretion disks at $r \sim 10^{10}\mathrm{cm}$:

- Temperature: $T \sim 10^{10}\mathrm{K}$
- Density: $n \sim 10^{16}\mathrm{cm}^{-3}$
- Thermal velocity: $\langle v^2 \rangle^{1/2} \sim 10^6\mathrm{cm}$
- Particle mean-free path (ionized gas): $\lambda = k^2 T^2/(\pi e^4 n) \sim 10^3\mathrm{cm}$
- Kinematic viscosity: $\nu \sim 10^3\mathrm{cm}^2\mathrm{sec}^{-1}$

This gives an inflow radial velocity of $v^r \sim 3\nu/2R \sim 5\mathrm{cm/year}$.

Molecular viscosity too low

Nature of the angular momentum transport

Turbulence!

Enhanced momentum transport in turbulent flows:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = -\boldsymbol{\nabla} p - \rho \boldsymbol{\nabla} \Phi,$$

Separation of mean and fluctuating quantities:

$$\rho = \bar{\rho} + \tilde{\rho}, \quad \bar{\rho} \equiv \langle \rho \rangle, \quad \boldsymbol{v} \equiv \bar{\boldsymbol{v}} + \tilde{\boldsymbol{v}}, \quad \bar{\boldsymbol{v}} \equiv \frac{1}{\bar{\rho}} \langle \rho \boldsymbol{v} \rangle.$$

Equations for mean flow:

$$\frac{\partial \bar{\rho}}{\partial t} + \boldsymbol{\nabla} \cdot (\bar{\rho} \bar{\boldsymbol{v}}) = 0,$$

$$\frac{\partial}{\partial t}(\bar{\rho} \bar{\boldsymbol{v}}) + \boldsymbol{\nabla} \cdot (\bar{\rho} \bar{\boldsymbol{v}} \bar{\boldsymbol{v}}) = -\boldsymbol{\nabla} \bar{p} - \boldsymbol{\nabla} \cdot \mathsf{R} - \bar{\rho} \boldsymbol{\nabla} \Phi,$$

where

$$\mathsf{R} \equiv \langle \rho \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}} \rangle \quad \textit{... Reynolds stress}$$

Back to Shakura-Sunyaev prescription

α-viscosity as a prescription for Reynolds stress:

$$\mathrm{R} \equiv \langle \rho \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}} \rangle \sim \rho c_{\mathrm{s}}^2 \left(\frac{\tilde{\boldsymbol{v}}}{c_{\mathrm{s}}} \right) \left(\frac{\tilde{\boldsymbol{v}}}{c_{\mathrm{s}}} \right)$$

Now, $\rho c_{\mathrm{s}}^2 \sim p$ and $|\tilde{\boldsymbol{v}}| < c_{\mathrm{s}}$:

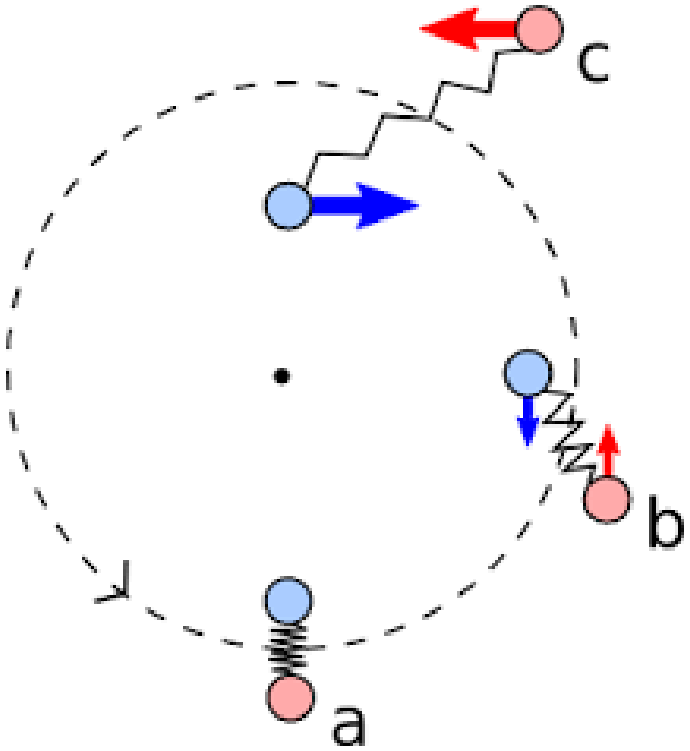
$$\boxed{R^{\hat{r}\hat{\phi}} = \alpha p}$$

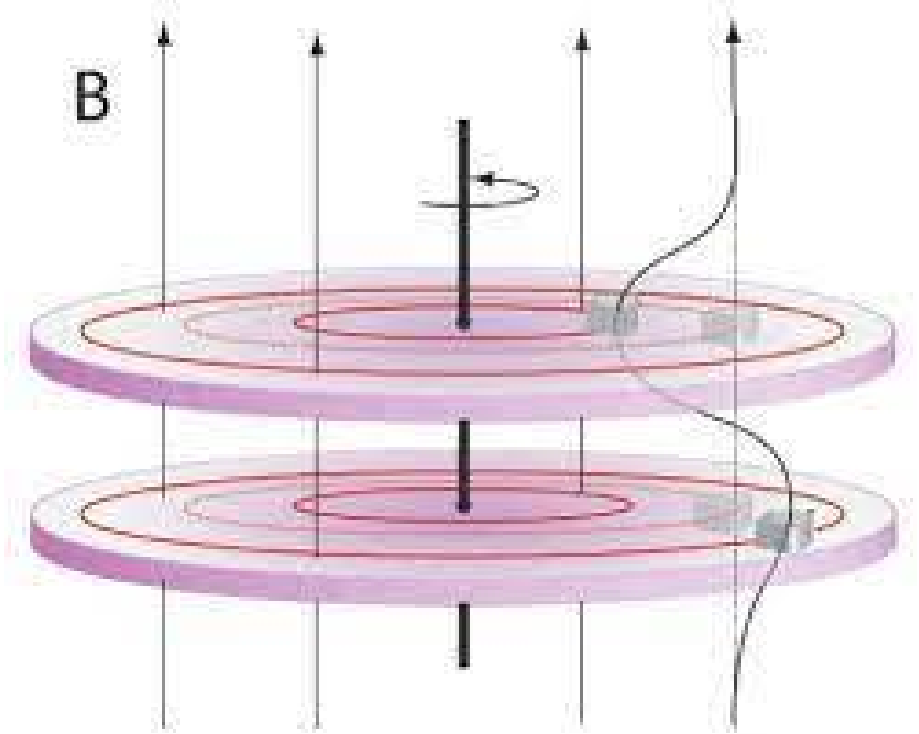
Our ignorance is parameterized by the parameter α:

- $\alpha < 1$ (subsonic turbulence)
- α may be function of r

Turbulence driving mechanism?

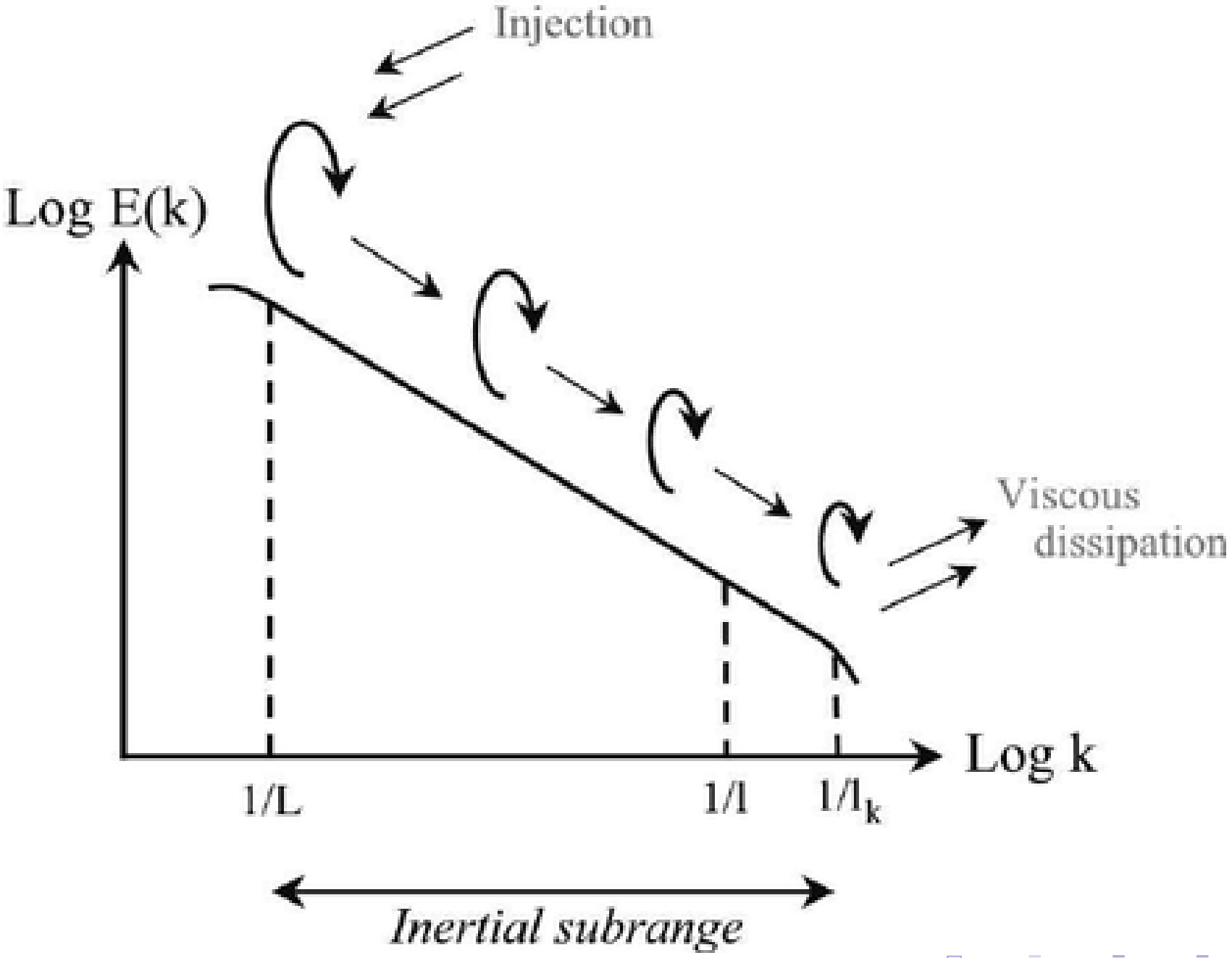
- Keplerian HDYN flows are boringly stable.
- *Local* instability discovered by Balbus & Hawley in 1993





Magneto-**R**otational **I**nstability

Turbulent cascade



More advanced models

Transonic flows with advection

$$\underline{Q^{+}_{\mathrm{visc}}} - \underline{Q^{-}_{\mathrm{rad}}} = Tv^{r}\frac{\partial s}{\partial r} \equiv \underline{Q^{-}_{\mathrm{adv}}}$$

Shakura-Sunyaev disk model

Radial momentum equation:

$$\underline{v^r \frac{\partial v^r}{\partial r}} + v^z \frac{\partial v^r}{\partial z} - \underline{r(\Omega^2 - \Omega_{\rm K}^2)} + \underline{\frac{1}{\rho}\frac{\partial p}{\partial r}} + \frac{1}{2}\frac{d\Omega_{\rm K}}{dr} z^2 = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(2r\eta\partial_r \frac{\partial v^r}{\partial r}\right) - 2\frac{\eta v^r}{\rho r^2} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial r}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial r}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial r}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the azimuthal one (radial transport of angular momentum):

$$\underline{\frac{v^r}{r^2}\frac{\partial}{\partial r}\left(r^2\Omega\right)} + v^z\frac{\partial\Omega}{\partial z} = \underline{\frac{1}{r^3\rho}\frac{\partial}{\partial r}\left(r^3\eta\frac{\partial\Omega}{\partial r}\right)} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial\Omega}{\partial z}\right),$$

the vertical one (hydrostatic balance):

$$v^r\frac{\partial v^z}{\partial r} + v^z\frac{\partial v^z}{\partial z} + \underline{\frac{1}{\rho}\frac{\partial p}{\partial z}} + \underline{\Omega_{\rm K}^2 z} = \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^z}{\partial r}\right) + \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\eta\frac{\partial v^r}{\partial z}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(2\eta\frac{\partial v^z}{\partial z}\right)$$
$$- \frac{2}{3\rho}\frac{\partial}{\partial z}\left[\frac{\eta}{r}\frac{\partial(rv^r)}{\partial r}\right] + -\frac{2}{3\rho}\frac{\partial}{\partial z}\left(\eta\frac{\partial v^z}{\partial z}\right),$$

the continuity equation (mass conservation):

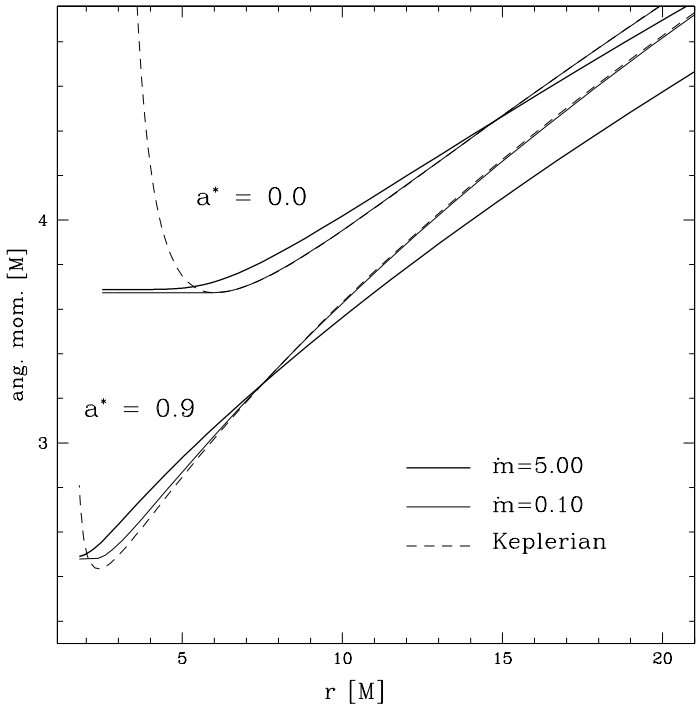
$$\underline{\frac{1}{r}\frac{\partial}{\partial r}(r\rho v^r)} + \frac{\partial}{\partial z}(\rho v^z) = 0$$

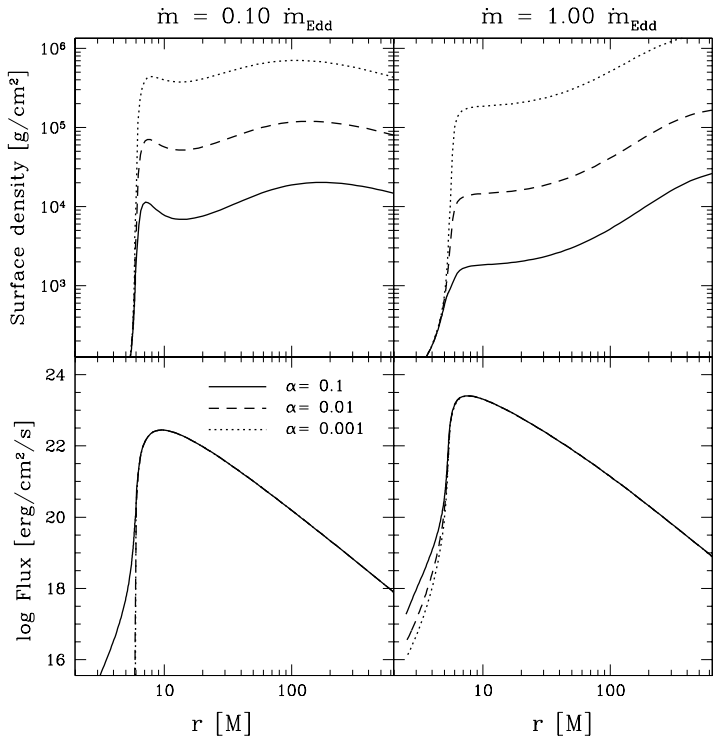
and the energy equation (including advection) and equation of state:

$$\underline{Q_{\rm visc}^+} - \underline{Q_{\rm rad}^-} = Tv^r\frac{\partial s}{\partial r} \equiv \underline{Q_{\rm adv}^-} \quad p = p_{\rm gas} + p_{\rm rad} = \mathcal{R}_{\rm m}\rho T + aT^4.$$

Slim accretion disks

- Probably suitable for higher luminosities
- Advection term included in the equations: $Q^+ = Q^-_{\rm rad} + Q^-_{\rm adv}$
- Optically thick, geometrically 'slim', $H/r \lesssim 1$
- Global transonic solution: zero-torque point r_* at black-hole horizon

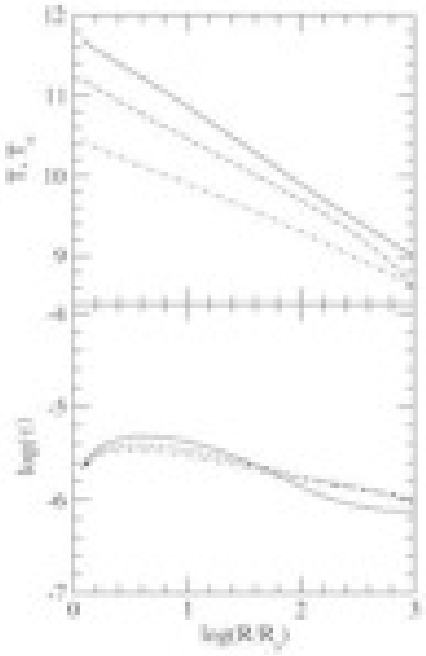


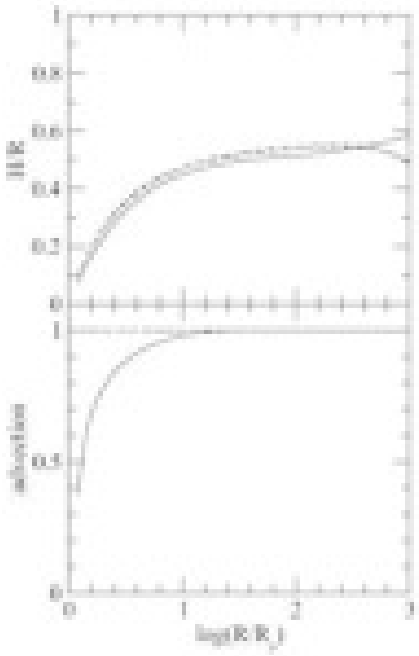


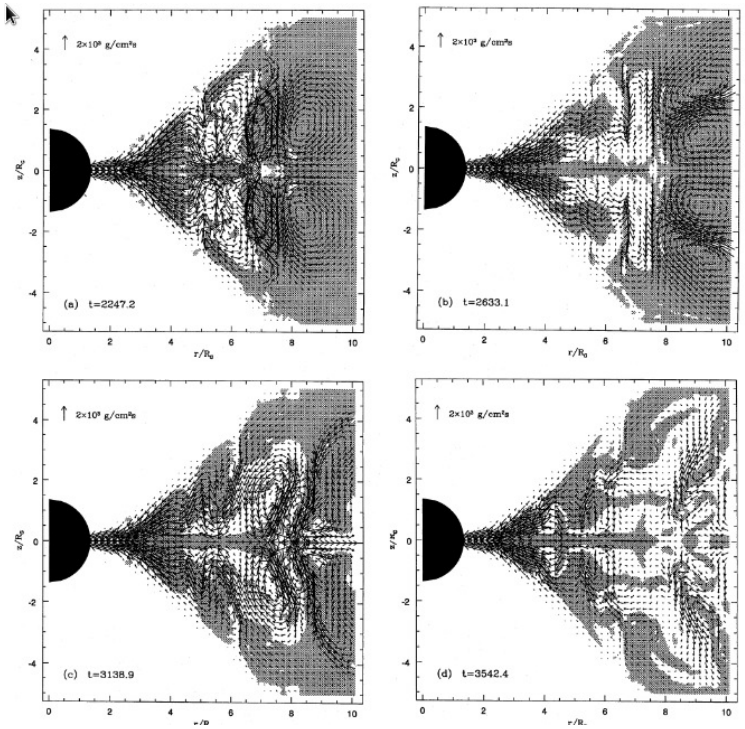
ADAFs

Advection Dominated Accretion Flows

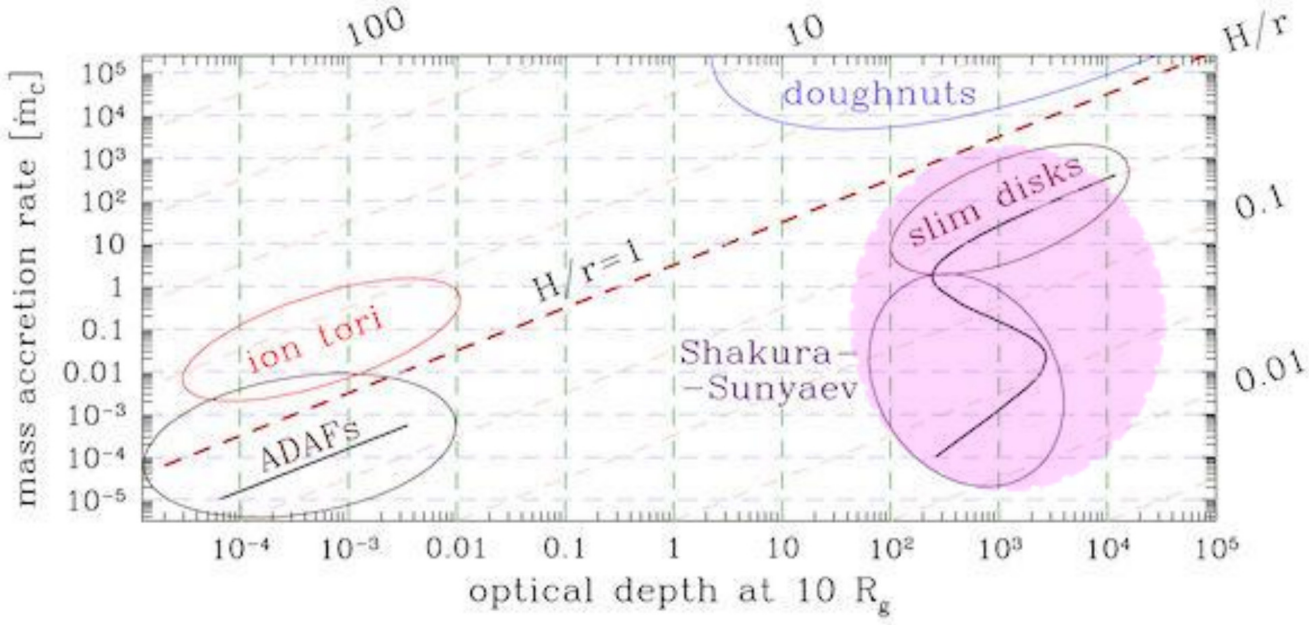
- Low-luminosity disks
- Advection term dominant in the equations $\boxed{Q^+ \approx Q^-_{\rm adv}}$
- Extended structures (almost spherical shape)
- Significant radial inflow velocity
- Many other versions correcting the original model (RIAF, ADIOS,...)

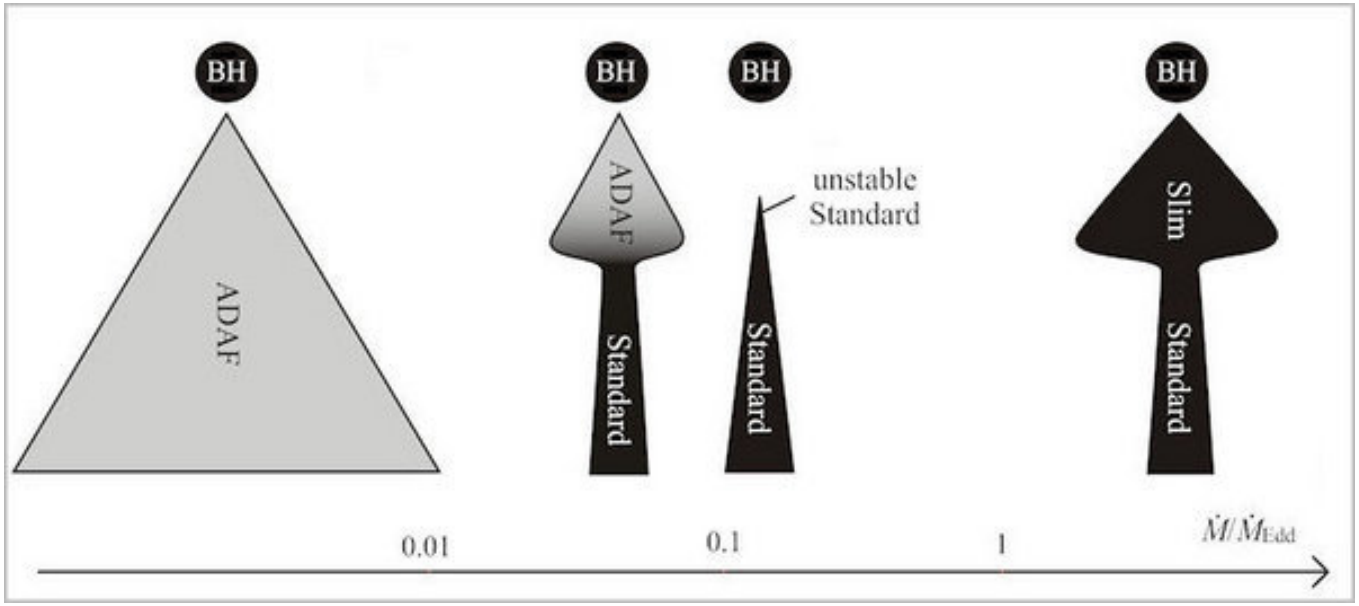






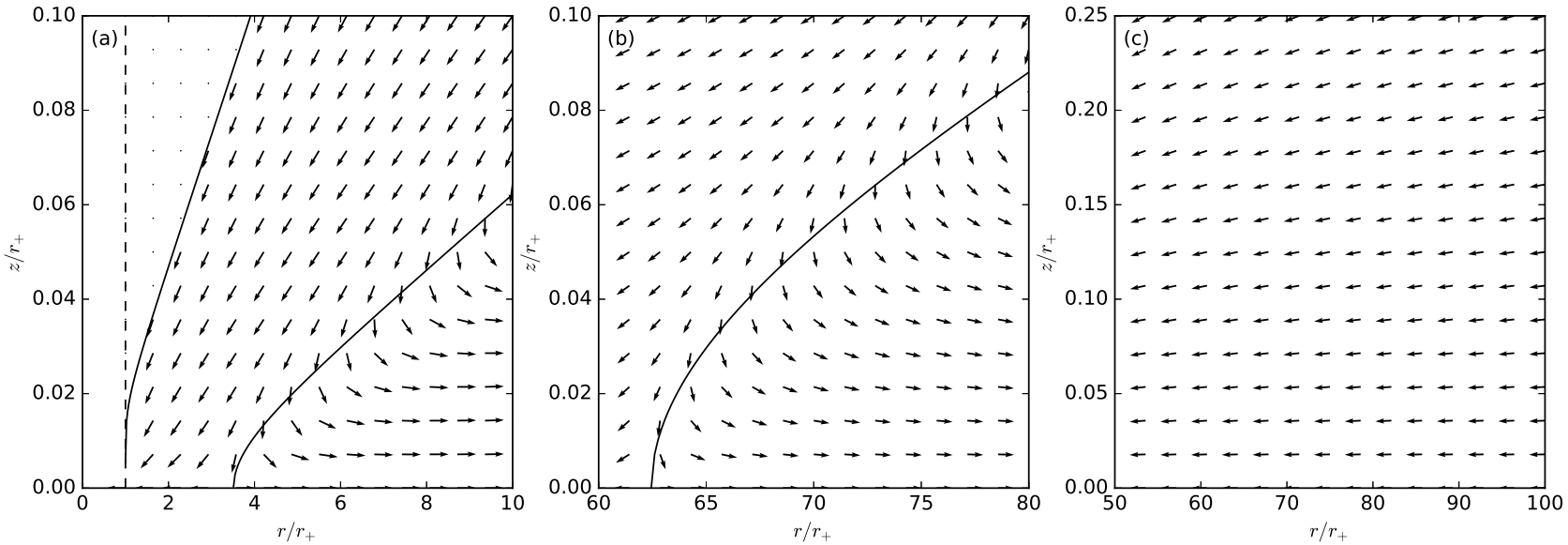
Accretion disk ZOO





BTW: What if we keep *all* the terms?

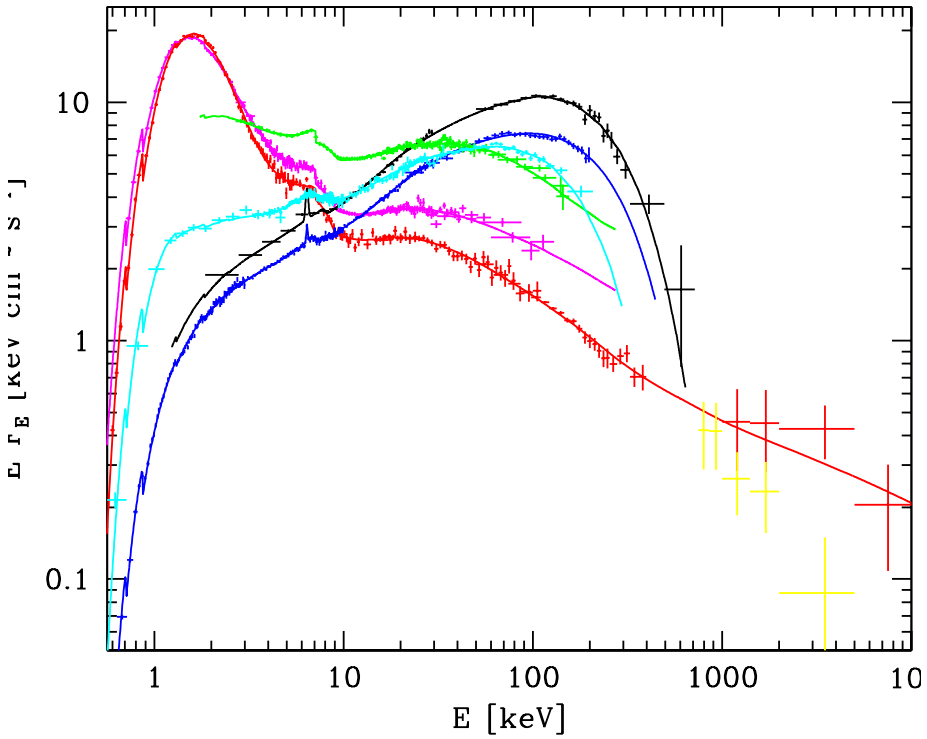
Analytic perturbative solution of Kluzniak & Kita (1995): backflows



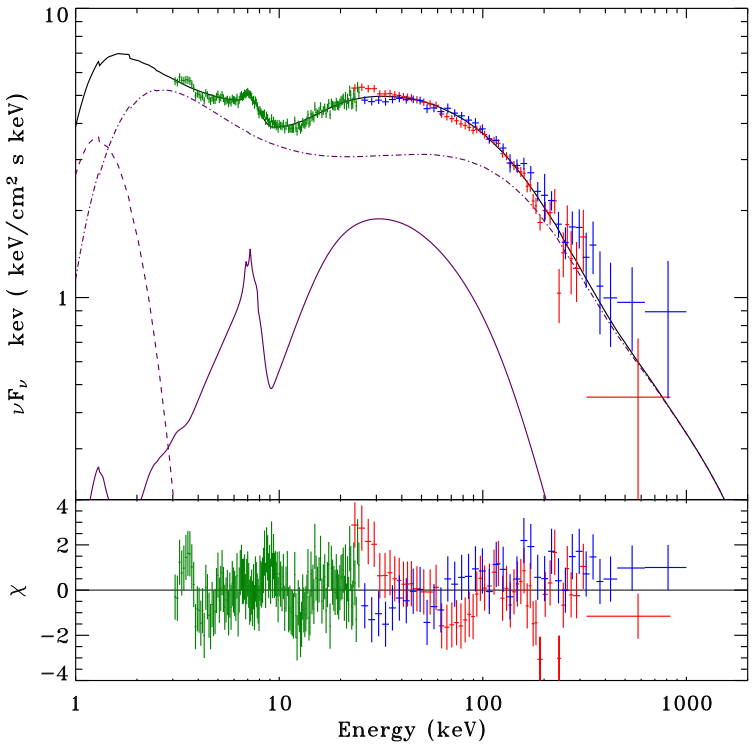
Still not complete picture

CORONAE

Corona contribution in different spectral states



Zdziarski et al. (2003)

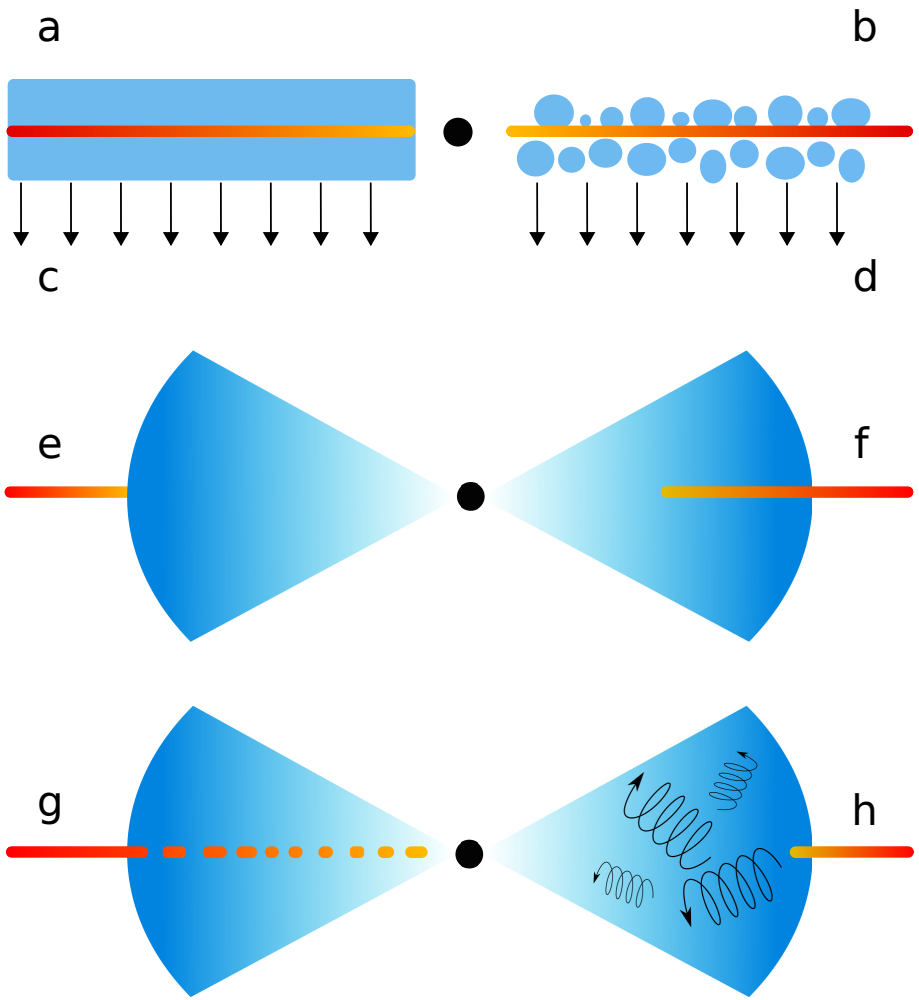


Malzac et al. (2006)

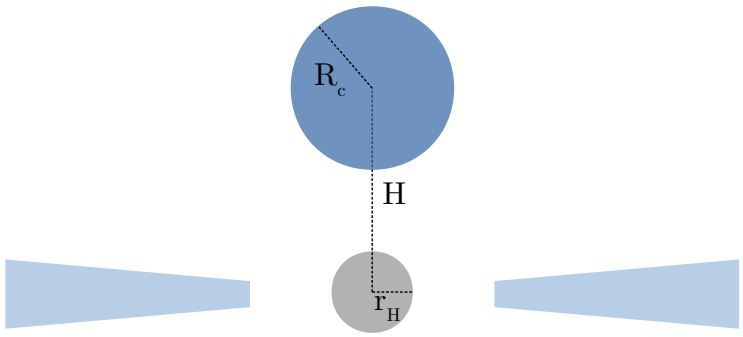
2 spectral states, 3 spectral components (disk, corona, reflection):

- LHS: Weak disk component (BB and reflection), thermal comptonization in the corona
- HSS: Strong disk component and reflection, non-thermal comptonization in the corona.

Different geometries



Poutanen et al. (2017)



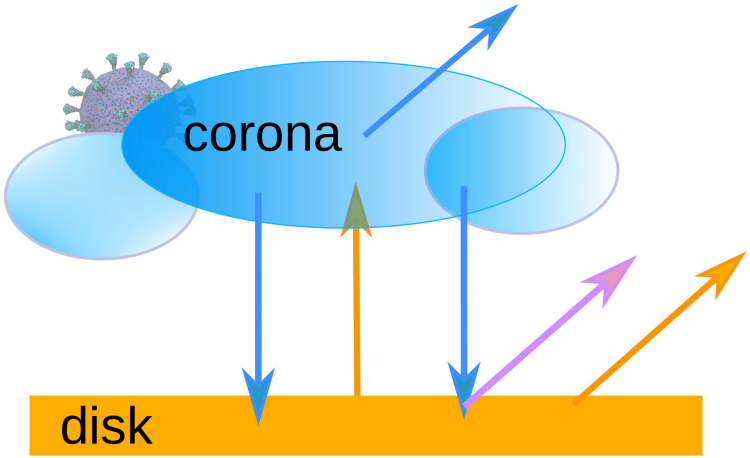
Ursini et al. (2020)

- Point-like or extended
- Patchy or continuous
- Static or variable

Self-consistent modelling

The three components are not independent:

- Disk provides seed photons for Comptonization in the corona.
- Irradiation of the corona causes cooling of corona.
- Upscattered photons from the corona are primary radiation for the reflection and (additional) heating of the disks



Energetic balance (Haardt+1991):

$$(1-f)P_g + (1-a)\eta L_C = L_s$$

$$fP_g + L_s = AL_s$$

- f is the heating of corona
- $f \lesssim 1$ to explain observations.

Considering mutual effects may put strong constraints on both, **geometry** and **variability** of the corona