GRAVITATIONALLY COUPLED ELECTROWEAK MONOPOLE

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‘WHAT HAVE MAGNETIC MONOPOLES EVER DONE FOR US?’

MONOPOLE IN HEP-TH

- Explains quantization of electric charge
- QCD confinement = dual color superconductor
- Avatar of Grand Unification of forces
- Last great prediction of the Standard Model
- SUSY, string theory, integrability
- …

MONOPOLE IN GR-QC

- Source of "hairy" black holes solution
- Inflation & cosmology
- …

THIS TALK:

- We show how using the method of *field dressing* we can estimate the *mass* of the magnetic model within Grand Unification Theory (GUT) and in Standard Model (SM).
- We explore the vast landscape of models, where the monopoles are *BPS solutions* and provide *exact solutions*.
- We describe a way of generating monopole solutions using a novel notion of duality => rescuing/reinterpreting *singular solutions*.
- We want to extend this game to General Relativity and look for new *exact solutions* describing magnetically charged compact objects.

WORK IN PROGRESS

DIRAC MONOPOLE

DIRAC MONOPOLE

Let us consider a point magnetic charge

$$\vec{\nabla} \cdot \vec{B} = \delta^3(\vec{x}) \quad \Rightarrow \quad \vec{B} \neq \vec{\nabla} \times \vec{A}$$

Can we somehow define the vector potential?

Yes. In fact, in infinitely many ways.

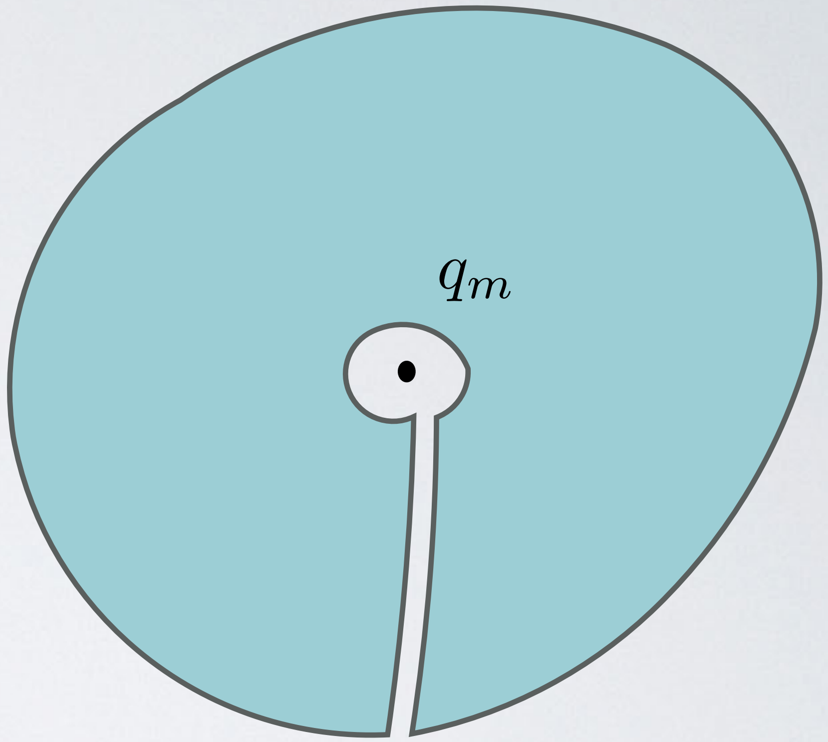
$$A_i^S = -(1 - \cos\theta)\partial_i\varphi = \varepsilon_{ij3}\partial_j \log(r + z)$$

$$A_i^N = (1 + \cos\theta)\partial_i\varphi = -\varepsilon_{ij3}\partial_j \log(r - z)$$

Notice that

$$A_i^N - A_i^S = 2\partial_i\varphi$$

$$\vec{\nabla} \times \vec{A}^N = \vec{\nabla} \times \vec{A}^S = -\frac{\vec{r}}{r^3}$$



Gauge fields can be defined everywhere *except* on a line stretching from the monopole to infinity

— **the Dirac string**

DIRAC QUANTIZATION CONDITION

The potentials leads to different Schroedinger equations

$$i\dot{\psi}^N = -\frac{1}{2m}\left(\vec{\nabla} - iq_e\vec{A}^N\right)^2\psi^N$$
$$i\dot{\psi}^S = -\frac{1}{2m}\left(\vec{\nabla} - iq_e\vec{A}^S\right)^2\psi^S$$

But the physics must be same. In particular the wave-functions must be related by a gauge transformation

$$\vec{A}^N = \vec{A}^S + \frac{q_m}{2\pi}\vec{\nabla}\varphi \qquad \psi^N = e^{i\alpha}\psi^S \qquad \alpha = \frac{q_e q_m}{2\pi}\varphi$$

The wave-function must be also single-valued

$$\psi(\varphi) = \psi(\varphi + 2\pi) \quad \Rightarrow \quad \boxed{q_e q_m = 2\pi n}$$

HOW TO GO BEYOND CLASSICAL ELECTROMAGNETISM?

EM Duality

electric monopole ⟷ magnetic monopole

Classical EM energy diverges:

$$\int d^3x \, \frac{1}{2} \vec{E}^2 = \infty \qquad \int d^3x \, \frac{1}{2} \vec{B}^2 = \infty$$

Quantum effects dominate for an electron

$$r_{\rm cl} \sim e^2 \lambda_{\rm C} \ll \lambda_{\rm C}$$

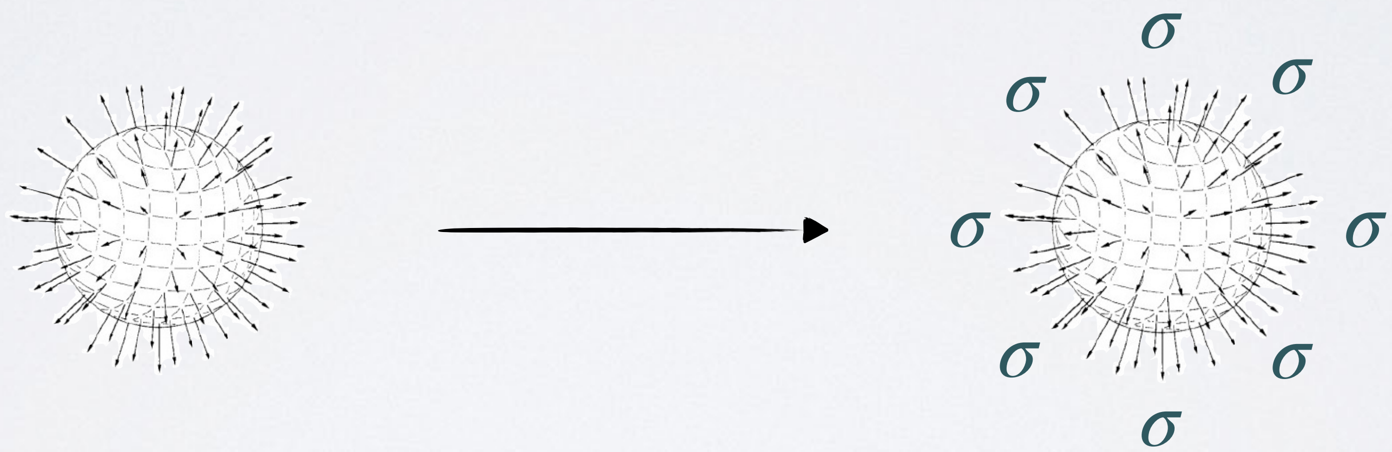
But monopoles can be treated semi-classically

$$r_{\rm cl} \sim q^2 \lambda_{\rm C} \sim \frac{1}{e^2} \lambda_{\rm C} \gg \lambda_{\rm C}$$

Natural descriptions:

QFT ⟷ (S Duality) **Solitons**

THE METHOD OF *FIELD DRESSING*



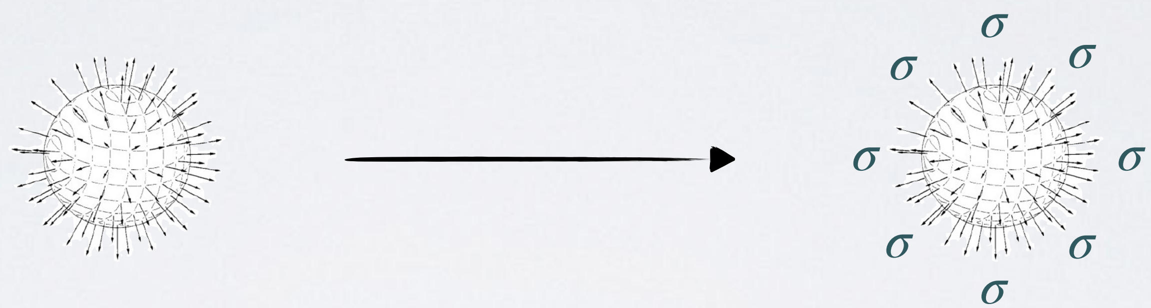
FIELD DRESSING

Theoretician's Gedanken's laboratory: coupling a magnetic monopole with a various fields through *field-dependent permittivity*:

$$\mathscr{L} = -\frac{\sigma^2}{4e^2v^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$$

This leads to a *spontaneous* condensation of the fields around the monopole

Naked monopole (unstable)



Dressed monopole (stable)

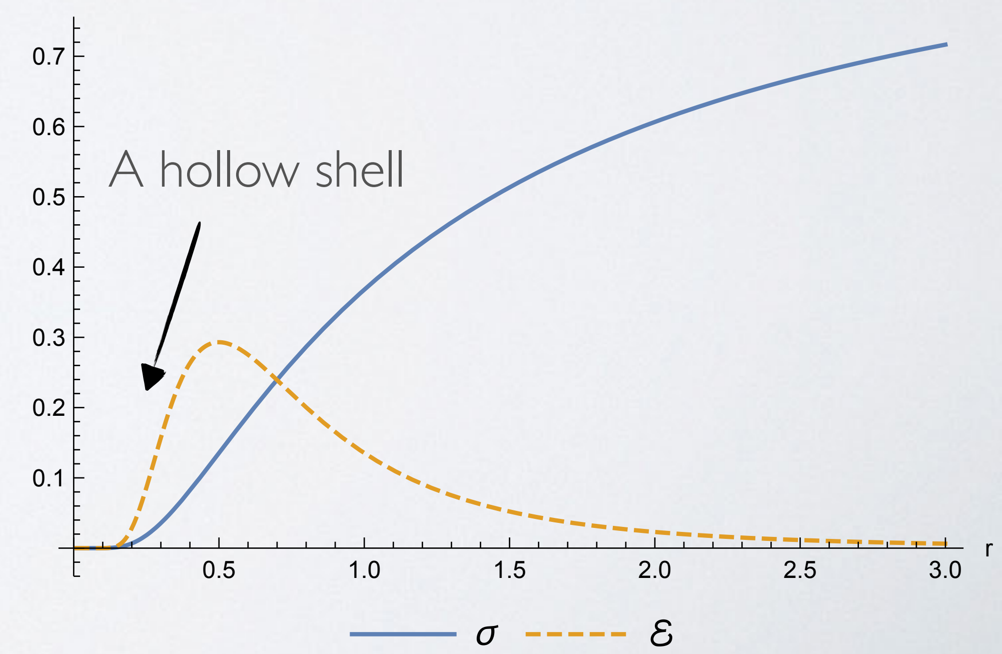
$$\sigma = v + \delta\sigma \qquad \left(-\nabla^2 - \frac{1}{e^2v^2r^4}\right)\delta\sigma = \frac{1}{e^2vr^4} \qquad \delta\sigma = -v$$

The exact solution for the condensate:

$$\sigma(r) = ve^{-\frac{1}{evr}}$$

EM energy is made *finite*!

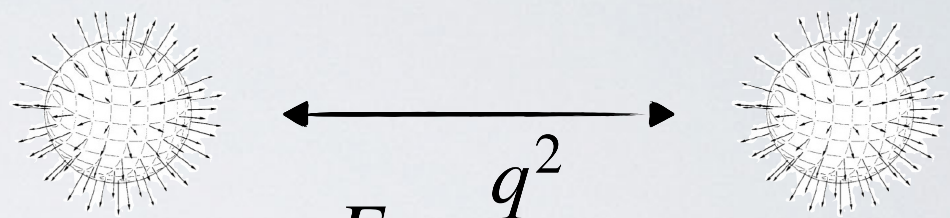
$$E_{\rm EM} = \frac{\pi}{e^2}\int_0^\infty dr\,\frac{e^{-1/evr}}{r^2} = \frac{\pi v}{2e}$$



HOW TO WEIGH A MONOPOLE LIKE A THEORETICIAN

Monopoles are not static in the vacuum (duh)

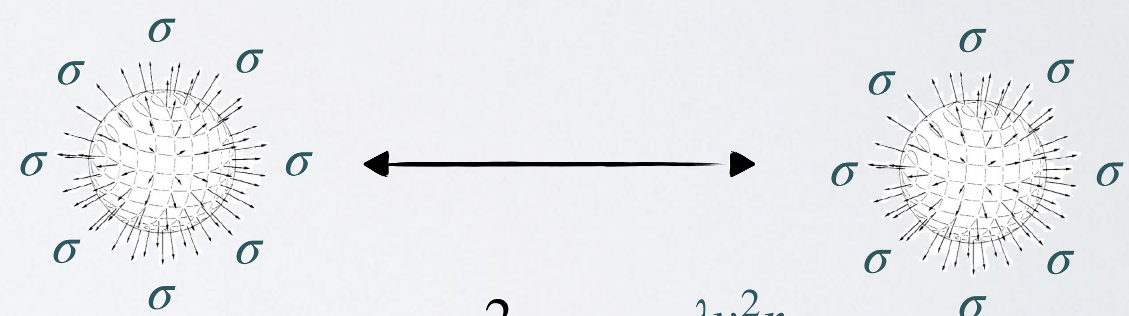
$$\mathscr{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}$$



$$F \sim \frac{q^2}{r^2}$$

However, coupling the monopole to a field via *field-dependent permittivity* will modify the charge and add an attractive force. The monopoles become `dressed'.

$$\mathscr{L} = -\frac{\sigma^2/v^2}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\lambda}{8}\left(\sigma^2 - v^2\right)^2$$



$$F \sim \frac{q^2}{r^2} - \frac{e^{-\lambda v^2 r}}{r^2}$$

`Weighing' is simply measuring the amount of the condensate necessary to strike the balance of forces as $\lambda \to 0$

A LANDSCAPE OF DRESSED MONOPOLES

$$\mathcal{L} = -\frac{1}{4e^2} h'^{\,2}(\sigma/v) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \lambda\left(\sigma^2 - v^2\right)^2$$

$h'(1) = 1 \quad h(0) = 0$

Energy density can be completed into a perfect square

$$\mathcal{E} = \frac{1}{2e^2} h'^{\,2}(\sigma/v) B_i B_i + \frac{1}{2}\partial_i \sigma \partial_i \sigma = \frac{1}{2}\left(\partial_i \sigma + \frac{1}{e} h'(\sigma/v) B_i\right)^2 - \frac{v}{e}\partial_i\left(h(\sigma/v) B_i\right)$$

Thus the mass of the dressed monopole is

$$M = -\frac{v}{e}\int d^3x\, \partial_i\left(h(\sigma/v) B_i\right) = -\frac{v}{e}\lim_{r\to\infty}\int d^2\Omega\, h(\sigma/v) r x_i B_i$$

$-\frac{x_i}{r^3}$

$\sigma(\infty) = v$

$$M = \frac{4\pi v}{e} h(1) = \frac{4\pi v}{e}\int_0^1 dt\, h'(t)$$

The punchline:
If we know the *vev* of the condensate we know the mass of the monopole:

Example:

For SM Higgs: $v = 246$ GeV

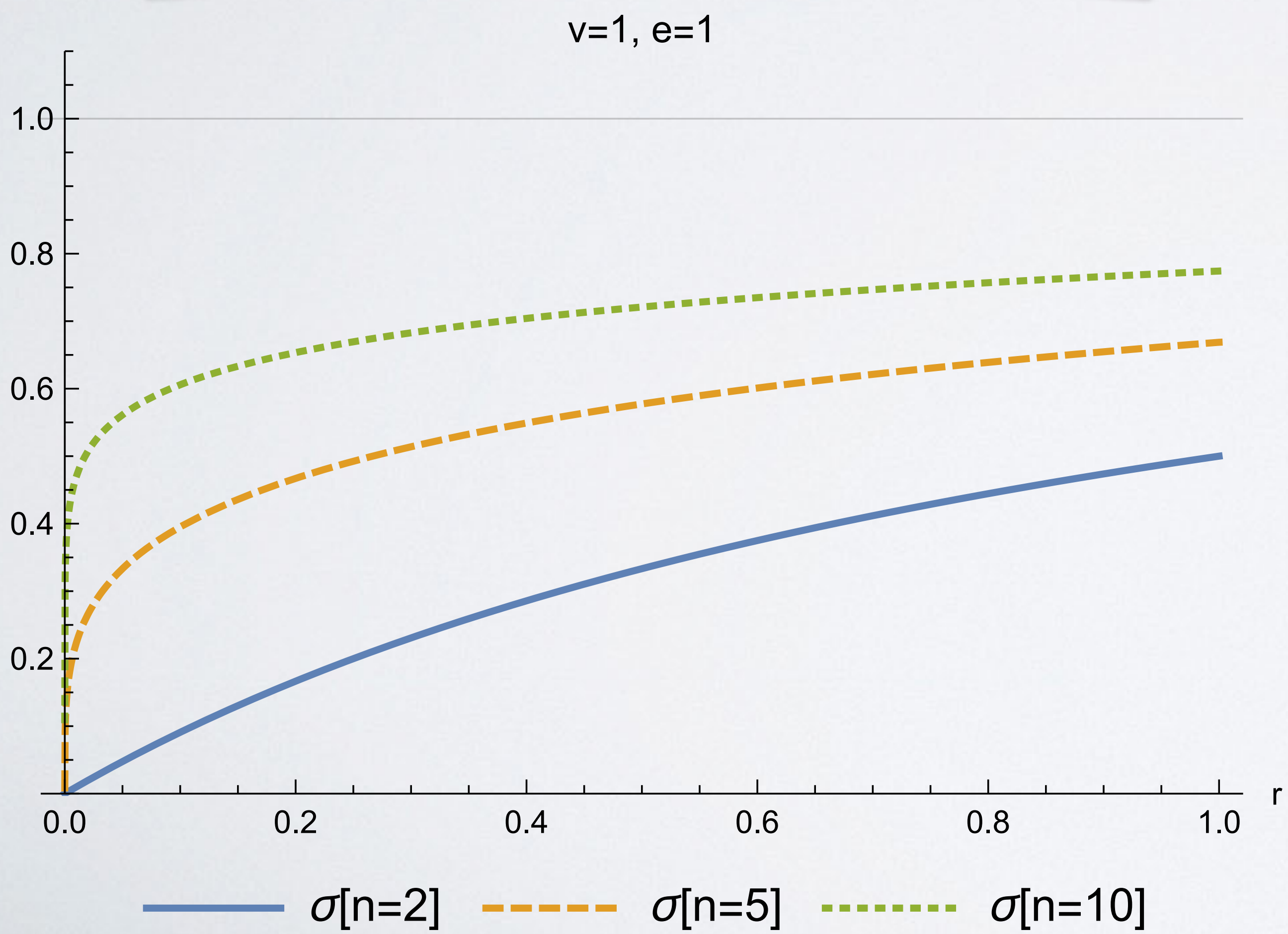
$M \approx 1 - 10$ TeV

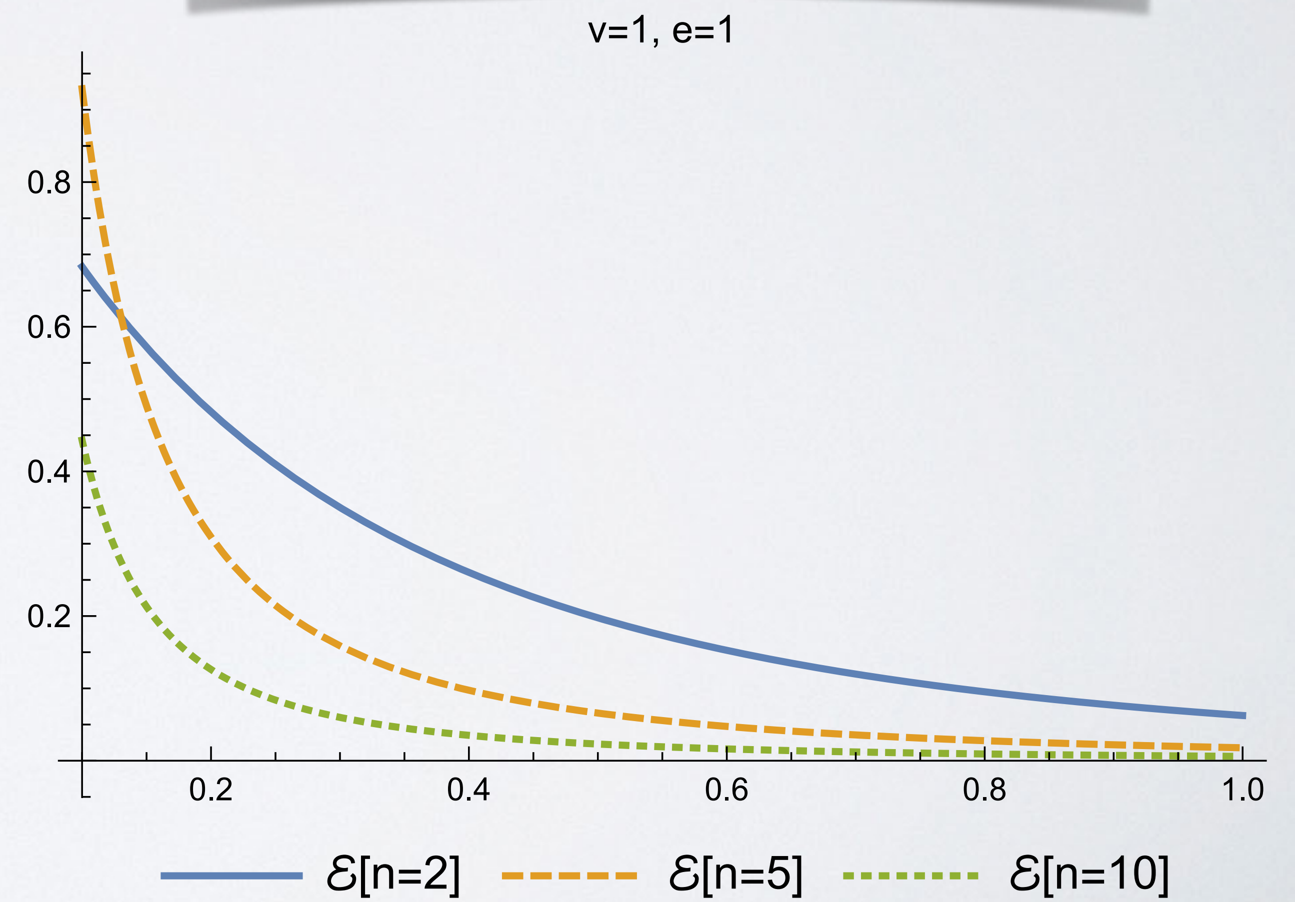
SOME OTHER EXACT SOLUTIONS

$$\partial_i \sigma + \frac{1}{e} h'(\sigma/v) B_i = 0 \qquad h'(x) = x^n \qquad B_i = -\frac{x_i}{r^3}$$

$$\sigma[n] = v\left(1 + \frac{n-1}{ver}\right)^{-\frac{1}{n-1}}$$

$$\mathcal{E}[n] = \frac{1}{e^2 r^2} \frac{r^{\frac{2}{n-1}}}{\left(r + \frac{n-1}{ve}\right)^{\frac{2n}{n-1}}}$$

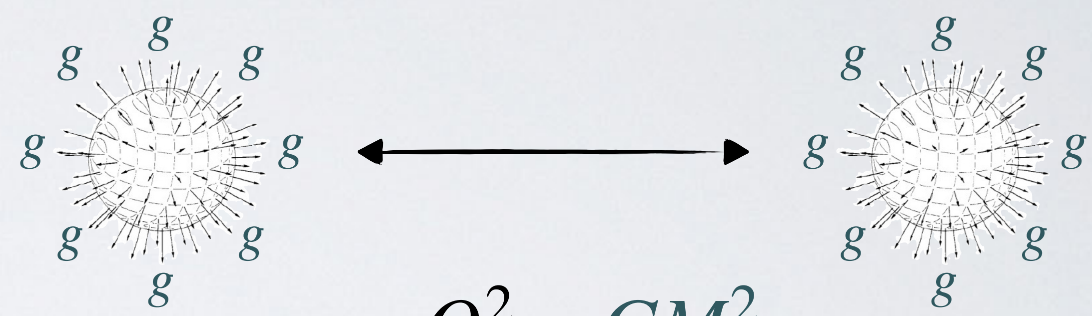




GRAVITY = FIELD DRESSING IN SPIN 2 FIELD

Coupling with gravity also introduce a long range attraction

$$\mathscr{L} = -\frac{\sqrt{-g}}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{\sqrt{-g}}{16\pi G}R$$



$$F \sim \frac{Q^2}{r^2} - \frac{GM^2}{r^2}$$

Depending on the strength of the 'dressing` either black hole or naked singularity is formed:

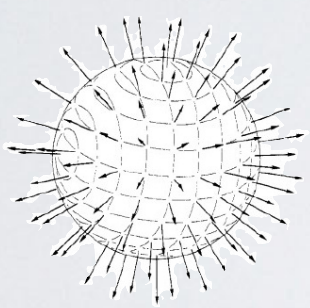
$$ds^2 = \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)dt^2 - \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1}dr^2 - r^2 d\Omega^2$$

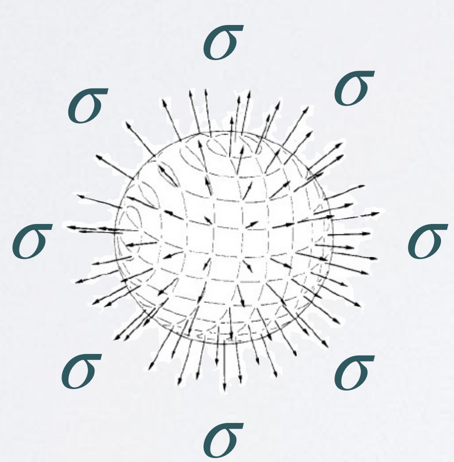
The balance is struck when $M \approx Q/\sqrt{G} = QM_P \sim 10^{19}\ \mathrm{GeV}$

This estimate is spectacularly wrong => gravity is not a fundamental `condensate'

CLOTHES MAKES THE MONOPOLE

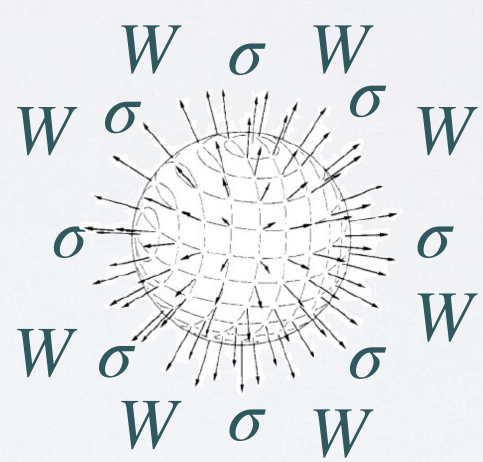
Dirac Monopole

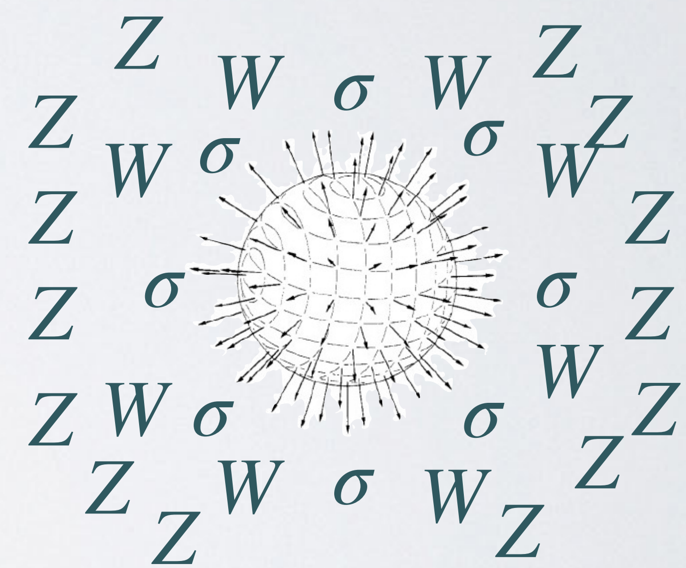




Dressed monopole

'’t Hooft-Polyakov monopole





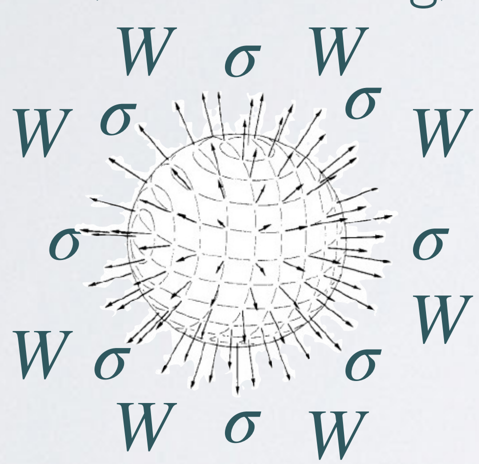
Cho-Maison monopole

DRESSING THE MONOPOLE IN VECTOR FIELDS:

Let's throw a Dirac monopole into a medium with a neutral scalar and charged vector field.

$$\mathscr{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}|D_\mu W_\nu - D_\nu W_\mu|^2 + \frac{i}{2e}(\bar{W}_\mu W_\nu - \bar{W}_\nu W_\mu)F^{\mu\nu}$$
$$+\frac{\lambda e^2}{4}\left(\bar{W}_\mu W_\nu - \bar{W}_\nu W_\mu\right)^2 + m^2(\sigma)|W_\mu|^2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)$$

K. Lee, E. Weinberg, 1994



If the value of a dipole moment of *W* is tuned just right and the self-coupling and mass is $\lambda = 1 \quad m(\sigma) = e\sigma$ the symmetry group magically enhances from U(1) into a spontaneously broken SU(2) gauge group with an adjoint triplet

$$\mathscr{L} = -\frac{1}{2e^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \mathrm{Tr}\left[D_\mu\Sigma D^\mu\Sigma\right] - V(\Sigma)$$

The Dirac monopole becomes a regular solution = a *topological soliton*!

$$A_i^{\mathrm{em}} = -\varepsilon_{ij3}\frac{x_j}{r(r+z)}$$

Singular gauge transformation ⟷

$$A_i = -\frac{1}{2}(1 - K(r))\varepsilon_{ijk}\frac{x_j\sigma_k}{r^2}$$

WEIGHING GUT MONOPOLES IN THE LANDSCAPE OF BPS MODELS

The strategy is now the same: find BPS limit and `weigh' the condensate:

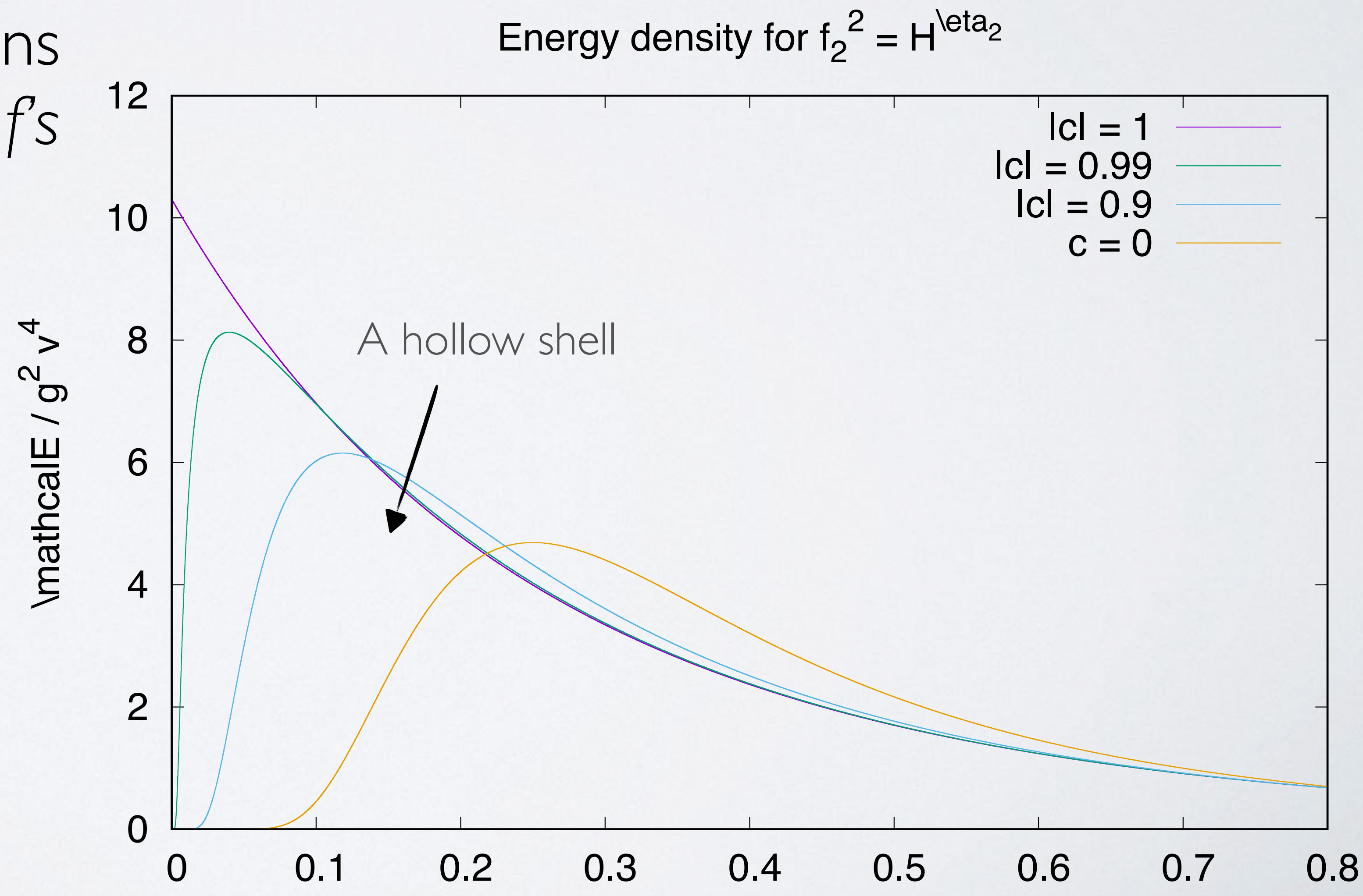
$$\mathscr{L} = -\frac{f_1(\sigma)}{2e^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - \frac{f_2(\sigma)}{2e^2}\mathrm{Tr}\left[F_{\mu\nu}\Sigma\right]^2 + f_3(\sigma)\mathrm{Tr}\left[D_\mu\Sigma D^\mu\Sigma\right] \qquad \sigma = \frac{1}{v^2}\mathrm{Tr}\left[\Sigma^2\right]$$

In the BPS limit, the mass is fixed entirely by topology: $M = \dfrac{4\pi v}{e}$

We have found novel solutions for specific choices of above f's for hollow shells

$$K(r) = ce^{-evr}$$

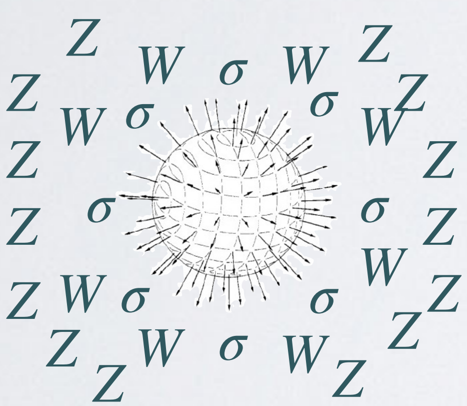
$$\sigma(r) = \exp\left(\frac{c^2e^{-2evr} - 1}{ver} + 2c^2\mathrm{Ei}(-2ver)\right)$$



EMBEDDING THE MONOPOLE IN THE STANDARD MODEL: CHO-MAISON MONOPOLE

Surprisingly, the Dirac monopole can be incorporated easily into SM by adding another neutral vector field (Z boson)

$$\mathscr{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2e^2}|D_\mu W_\nu - D_\nu W_\mu|^2 + \frac{i}{2e^2}(\bar{W}_\mu W_\nu - \bar{W}_\nu W_\mu)F^{\mu\nu}$$
$$+\frac{\lambda}{4}\left(\bar{W}_\mu W_\nu - \bar{W}_\nu W_\mu\right)^2 + m^2(\sigma)|W_\mu|^2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)$$
$$+\frac{1}{2e^2}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)^2 + \mathscr{L}_{\text{int}}\left(W_\mu, Z_\nu\right)$$



If we tune the values as $\lambda = \sin(\theta_W)^{-1}$ $\quad m(\sigma) = e\sigma/2$ the symmetry group is enhanced into a spontaneously broken SU(2)xU(1) gauge group = SM!

$$\mathscr{L} = -\frac{1}{2g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} + |DH_\mu|^2 - V(|H|)$$

The Dirac monopole becomes remains singular in the U(1) group but otherwise regular = *hybrid between Dirac and SU(2) monopole*!

THE TOPOLOGY OF THE CHO-MAISON MONOPOLE

The standard argument for why monopole cannot exist in SM was based on triviality of the second homotopy group $\pi_2(SU(2)xU(1)/U(1)) \sim \pi_1(SU(2)) = \{\}$

However, Cho and Maison (1997) found a topological solution based on the non-trivial second homotopy group of the normalized Higgs field, which is a CP1 coordinate: $\pi_2(\mathbb{C}P^1) \sim \pi_2(SU(2)) = \mathbb{Z}$

The spherical symmetric ansatz:

$$H = \frac{v}{\sqrt{2}}\rho(r)\xi \qquad \text{Higgs field}$$

$$\xi = i\begin{pmatrix}\sin\big(\theta/2\big)e^{-i\varphi} \\ -\cos\big(\theta/2\big)\end{pmatrix} \qquad \mathbb{C}P^1 \text{ coordinate}$$

$$A_i = \frac{1}{2}\varepsilon_{ijk}\frac{x_k\sigma_j}{r^2}\big(1 - K(r)\big) \qquad SU(2) \text{ gauge fields}$$

$$B_i = -\big(1 - \cos\theta\big)\partial_i\varphi \qquad U(1)_Y \text{ gauge fields}$$

$$HH^\dagger = \frac{v^2}{2}\rho^2(r)\xi\xi^\dagger = \frac{v^2}{4}\rho^2(r)\mathbf{1}_2 - \frac{v^2}{4}\rho^2(r)\frac{x_i\sigma_i}{r}$$

WEIGHING THE CHO-MAISON MONOPOLE

Cho and Maison realized that in the Electroweak theory the mass of the monopole is *divergent*!

This can be cured by considering theories beyond SM with *field dressing*

Later, the estimated was lowered by considering more appropriate function of the Higgs that take into account experimental bounds on H to 2 photon production

Studying the BPS Electroweak monopoles, we were able to obtain a lower bound:

Cho & Maison, Phys. Lett. B391 (1997)

$$M_{\rm mon} = \frac{2\pi}{g'^2} \int_0^\infty \frac{dr}{r^2} + \text{finite terms}$$

Cho, Kim & Yoon, Eur. Phys. J. C (2015)

$$-\frac{1}{4g^2} B_{\mu\nu}^2 \to -\frac{|H|^8}{4g^2} B_{\mu\nu}^2$$

$$M_{\rm mon} \approx 7.2 \text{ TeV}$$

Ellis, Mavromatos & You, *Phys. Lett. B (2016)*

$$f(|H|) = 5\left(\frac{H}{v}\right)^8 - 4\left(\frac{H}{v}\right)^{10}$$

$$M_{\rm mon} \approx 5.5 \text{ TeV}$$

Blaschke, Beneš, PTEP (2018)

$$M_{\rm mon} \geq 2.7 \text{ TeV}$$

THE BPS MASS

The BPS mass can be shown to lie within a range

$$\frac{2\pi v}{g} \leq \quad M = 4\pi v \left(\frac{1}{2g} + \frac{1}{g'} \int_0^1 dt\, f'(t) \right) \quad \leq \frac{2\pi v}{g} + \frac{4\pi v}{g'}$$

BPS 't Hooft-Polyakov term

BPS Dirac term

Thus, the mass of the BPS Cho-Maison monopole is bounded **both** from bellow and from above

BPS CHO-MAISON MONOPOLE

The most general BPS equation for Cho-Maison monopole:

$$D_i H = \eta \frac{\sqrt{2}}{g\rho} h(\rho) \left(M_i - \xi^\dagger M_i \xi \right) \xi + \eta \frac{1}{\sqrt{2}g} h'(\rho) \left(\xi^\dagger M_i \xi \right) \xi + \tilde{\eta} \frac{1}{\sqrt{2}g'} f'(\rho)\, G_i\, \xi$$

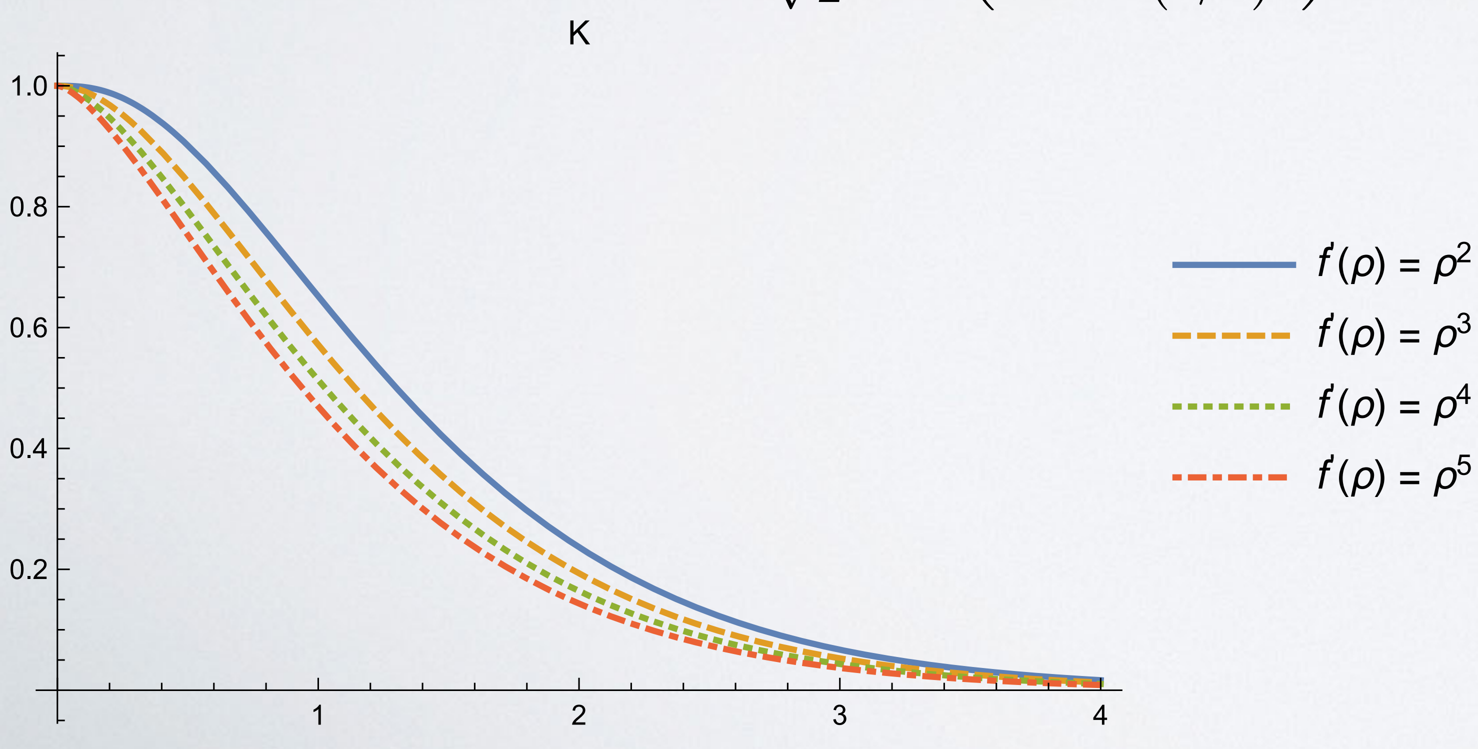
where $\quad M_i = \frac{1}{2}\varepsilon_{ijk} G_{jk} \quad G_i = \frac{1}{2}\varepsilon_{ijk} B_{jk}$

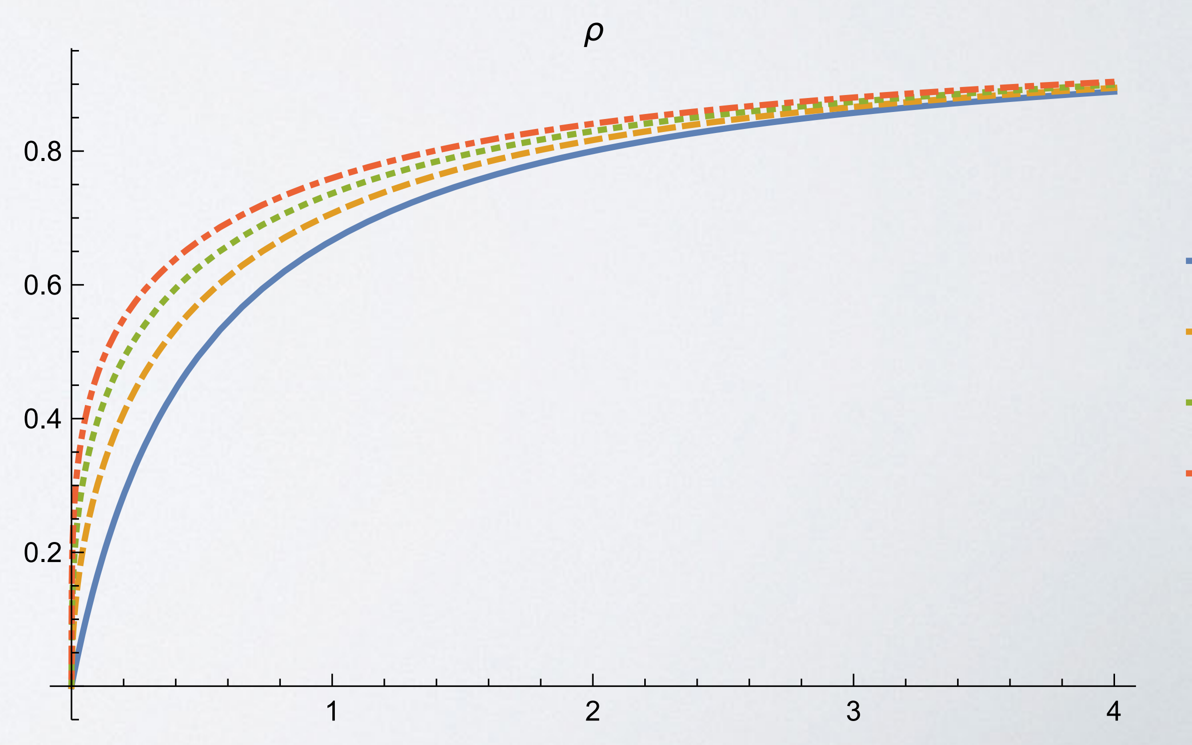
Some exact solutions:

$$K(r) = \exp\left\{ -\frac{g}{g'} \frac{\mu r}{2} \left(1 + \frac{n-1}{\mu r}\right)^{-\frac{2}{n-1}} \left[1 - \frac{2}{n+1}\, {}_2F_1\!\left(1, 1, \frac{2n}{n-1}; -\frac{\mu r}{n-1}\right) \right] \right\}$$

$$\rho(r) = \left(1 + \frac{n-1}{\mu r}\right)^{-\frac{1}{n-1}}$$

$$h(\rho) = 1 \quad f'(\rho) = \rho^n \quad H = \frac{v}{\sqrt{2}} \rho(r) \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix} \quad A_i = \frac{1}{2}\varepsilon_{ijk} \frac{x_k \sigma_j}{r^2} (1 - K(r)) \quad B_i = (1 - \cos\theta)\partial_i \varphi$$





BROADER LANDSCAPE AND DUALITY

Let us consider most general SU(2)xU(1) theory supporting a monopole

$$\mathscr{L} = -\frac{f_3(\rho)^2}{2g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - \frac{f_4(\rho)^2}{4g'^2}B_{\mu\nu}B^{\mu\nu} + f_1'^2(\rho)\,|\tilde{\xi}^\dagger D_\mu H|^2 + f_2'^2(\rho)\,|\xi^\dagger D_\mu H|^2$$

Asymptotically, we want to have the SM: $f'_{1,2}(1) = f_{3,4}(1) = 1$

If we further demand that the model is well-defined in the limit of large Higgs values, i.e. $f'_{1,2}(\infty) = 0 \quad f_{3,4}(\infty) < \infty$

$\tilde{\xi} = i\sigma_2\xi^*$ we can introduce a natural concept of **duality**

$$\begin{pmatrix}\rho\\ \xi\\ B_i\\ A_i\end{pmatrix} \underset{D}{\to} \begin{pmatrix}1/\rho\\ \tilde{\xi}\\ -B_i\\ A_i\end{pmatrix} \qquad \begin{pmatrix}f_1(\rho)\\ f_2(\rho)\\ f_3(\rho)\\ f_4(\rho)\end{pmatrix} \underset{D}{\to} \begin{pmatrix}f_1^D(\rho)\\ f_2^D(\rho)\\ f_3^D(\rho)\\ f_4^D(\rho)\end{pmatrix} \equiv \begin{pmatrix}f_1(1/\rho)\\ f_2(1/\rho)\\ f_3(1/\rho)\\ f_4(1/\rho)\end{pmatrix}$$

This duality allows to transform singular solutions (in the Higgs condensate) to regular ones! Hence, many solutions that have been deemed unworthy can be saved!

OUTLOOK

- We want to explore this duality further and look for *integrable* theories.
- Coupling to gravity = new source of `hairy' black hole solutions.
- Fermion condensates and Rubakov-Callan-like effects in EW theory.
- …

THANK YOU!