Innermost stable circular orbits around charged, rotating BHs

RAGTIME 23

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» Black Holes with Charge?

* selective accretion restricts BH charge to very small values ($Q \propto 10^{-18} M$) (M. Zajacek, A. Tursunov, A. Eckart, and S. Britzen, 2018.)
* particle-pair creation restricts the BH charge even in a vacuum environment to $Q \propto 10^{-5} M$
* if magnetic fields are present BH charges more likely (Wald, 1974)
* high charges are posible on short timescales
* "graviational" charges for BH solutions in MOG not restricted by selective accretion (Moffat, 2015)

$G = c = 1$

» Whats interesting about the ISCO?

Purely relativistic phenomenon! → **Characteristic property of a BH spacetime**

- Good first estimation for an inner edge of an accretion disc (this discs)
- limiting case of bound solutios in the polish doughnut model
- limit for slow inspiral

» ISCOs in literature

1. Circular motion, charged particles in Reissner-Nordström spacetime
   D. Pugliese+, Phys. Rev. D 83, 104052 (2011).
2. Charged particle motion in Kerr-Newman spacetime
   J. Bicak+, Bulletin Astron. Institutes of Czechoslovakia 40, 65/133 (1989),
   E. Hackmann and H. Xu, Phys. Rev. D 87, 124030 (2013),
   K. Schroven+, Phys. Rev. D 96, 063015 (2017).
3. Charged BHs immersed in electromagnetic background,
   J. P. Hackstein and E. Hackmann, Gen. Rel. Grav. 52, 22 (2020),
   B. Narzilloev+, Phys. Rev. D 99, 104009 (2019).
4. ISCO in Reissner-Nordström, Kerr-Newman and Kerr-Sen spacetime,
   Kris Schroven and Saskia Grunau, Phys. Rev. D 103, 024016 (2021).

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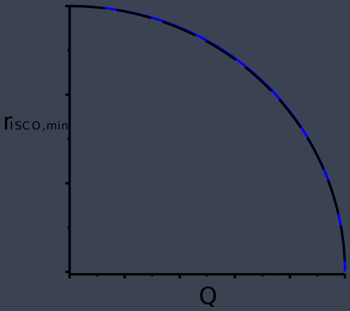
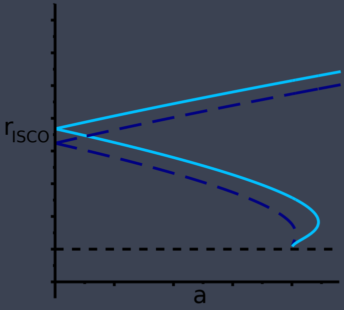
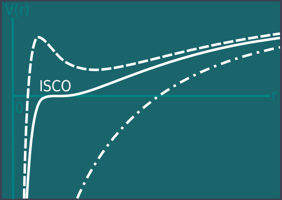
How is the ISCO effected by charge?



How is the ISCO affected by BH spin and charge?



» Charged BH vs classic Schwarzschild



BH charge →

Schwarzschild limit:

$$Q, q \to 0: \quad r_{\mathrm{ISCO}} = 6M$$

↖ particle charge

Innermost stable circular orbit – equations:

$$\frac{m}{2}\left(\frac{dr}{d\tau}\right)^2 = R(r) = \mathcal{E}^2 - mc^2 - V_{eff}(r), \qquad \begin{bmatrix} R(r) \\ R'(r) \\ R''(r) \end{bmatrix} = 0$$

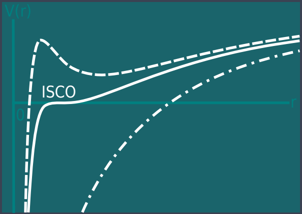
(spherical symmetry: $\theta = \pi/2 \to \frac{d\theta}{d\tau} = \frac{d^2\theta}{d\tau^2} = \frac{d^3\theta}{d\tau^3} = 0$)

classic: $$V_{\mathrm{SS}}(r) = -\frac{1}{r} + \frac{L^2}{2r^2} - \frac{L^2}{r^3}$$

charged: $$V_{\mathrm{charged}}(r) = -\frac{1}{r} + \frac{\hat{L}^2}{r^2} - \left(1 - \frac{Q^2}{2r}\right)\left(\hat{L}^2 + \frac{\bar{q}^2 - Q^2}{1 - E\bar{q}}\right)\frac{1}{r^3}$$

redefined $\hat{L}(E, L, \bar{q})$ — charge product qQ

» The charged, non-spinning BH



Limit for bound orbits ($r > r_+$):

$$E^2 < 1,\ \bar{q} < 1,\ L^2 > 1 - Q^2$$

Limit ISCOs:

$$r_{\mathrm{ISCO}} \to \infty : \begin{cases} E^2 = 1,\ \bar{q} = 1\ L^2 = 1 - Q^2 \\ E^2 \to 0,\ \bar{q} \to -\infty \end{cases}$$

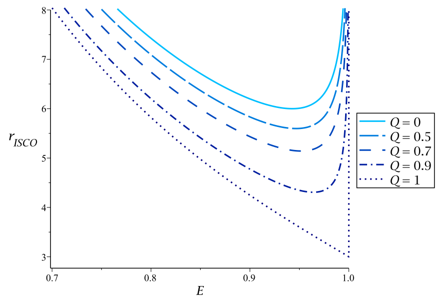
classic: $$V_{\mathrm{SS}}(r) = -\frac{1}{r} + \frac{L^2}{2r^2} - \frac{L^2}{r^3}$$

charged: $$V_{\mathrm{charged}}(r) = -\frac{1}{r} + \frac{\hat{L}^2}{r^2} - \left(1 - \frac{Q^2}{2r}\right)\left(\hat{L}^2 + \frac{\bar{q}^2 - Q^2}{1 - E\bar{q}}\right)\frac{1}{r^3}$$

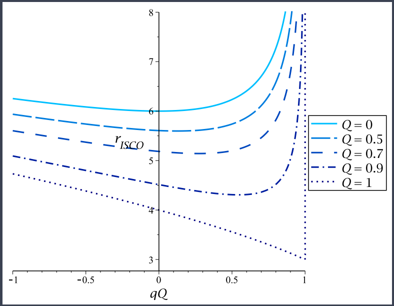
Occurance of a minimal ISCO

» Minimal ISCOs occur for charged BHs

r_{ISCO} over energy E for different BH charges Q



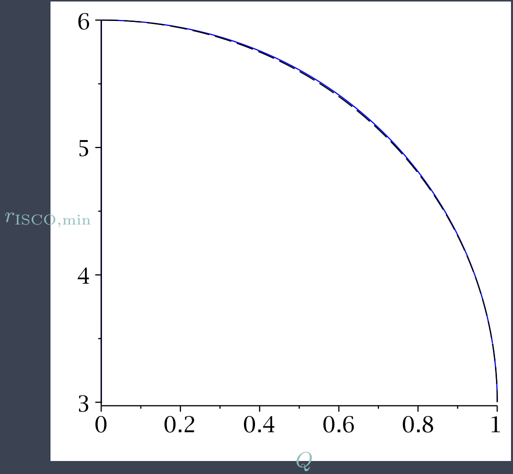
r_{ISCO} over particle charge product $\bar{q}$ for different Q



Comparable effects seen for a charged BH in a homogenious background field (Hackstein, Hackmann, 2020)

» Minimal ISCOs occur for charged BHs

Minimal ISCO radius over BH charge Q:



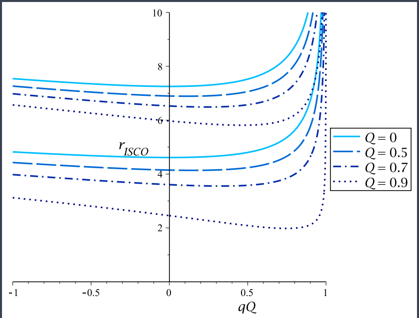
minimal ISCO:

$$qQ = 2\sqrt{\frac{-5Q^2 + 9\left(1 - \sqrt{1 - Q^2}\right)}{-9 + 25Q^2}}\,Q$$

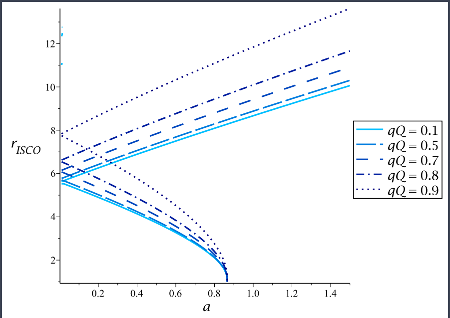
- course nearly circular
- How to explain the minimal ISCO?

» ISCO affected by BH spin and charge: Kerr-Newman BH

r_{ISCO} over charge product $\bar{q}$ for BH spin $a = 0.4$



r_{ISCO} over spin a for BH charge $Q = 0.5$

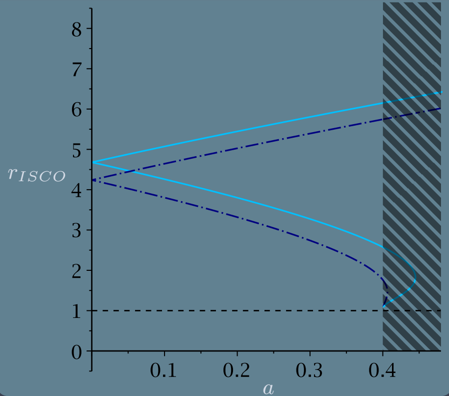


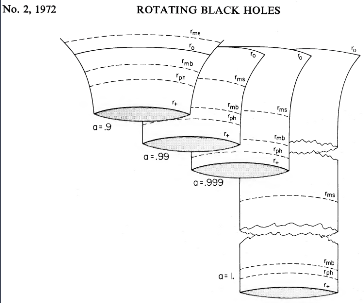
* minimal ISCO occures in the uncharged case, if BH charge has negligible effect on spacetime (Q sufficiently small)
* Corotating orbits reaches $r = 1$ for extremal BHs, if $\bar{q}_{\mathrm{L}}(a) < qQ < 1$

» Be aware of your coordinates!

* Corotating orbits reach $r = 1$ for extremal BHs, if $\frac{1-2a^2}{\sqrt{1-a^2}} < qQ < 1$

r_{ISCO} over BH spin a for $Q = \sqrt{1-0.4^2}$ and $\bar{q} = 0.4$ ($\bar{q} = 0.4 < 0.74 \approx \frac{1-2a^2}{\sqrt{1-a^2}}$)





Bardeen, Press, Teukolsky (1972).

With $a = 1 - \delta$, the proper distance between the outer horizon r_+ and r_{ISCO}:

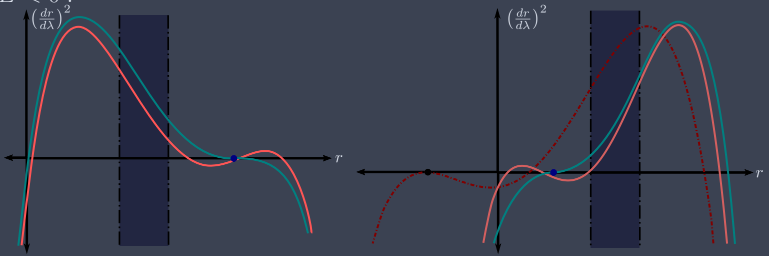
$$\lim_{\delta\to 0} r_{\text{ISCO}} - r_+ =$$

$$const + \lim_{\delta\to 0} \frac{\ln 2^{\frac{7}{6}}(\sqrt{2}-1)}{\delta^{1/6}}$$

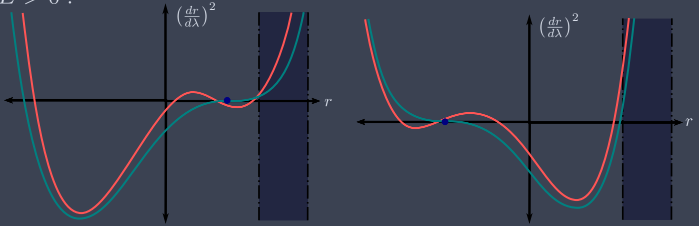
» Outermost stable circular orbit byd. the horizon

$$\dot{r} = R(r) = \frac{1}{r^4} P_4(E, L, \bar{q}, r) = \frac{1}{r^4} P_4(-E, -L, -\bar{q}, r)$$

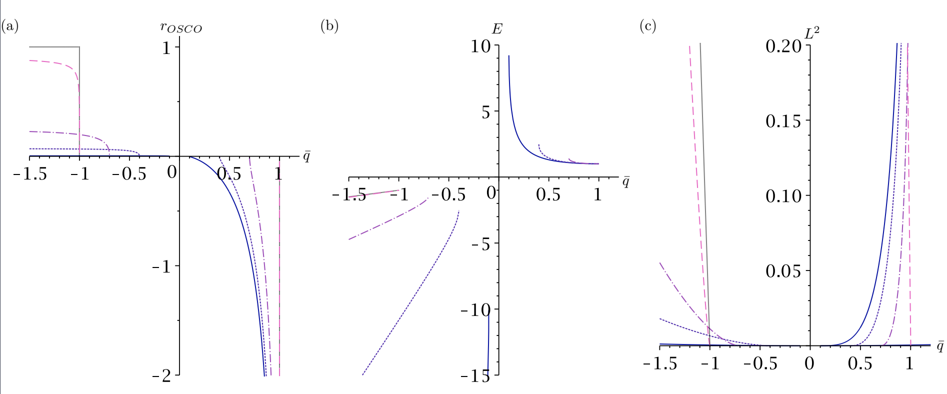
$E < 0$:



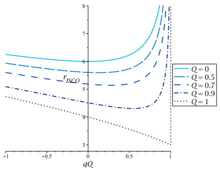
$E > 0$:

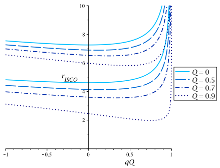


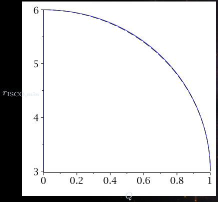
» OSCOs byond the horizon

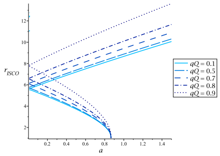


$Q = 0.1$ (blue), $Q = 0.4$ (dark violet), $Q = 0.7$ (violet), $Q = 0.999$ (bright violet)



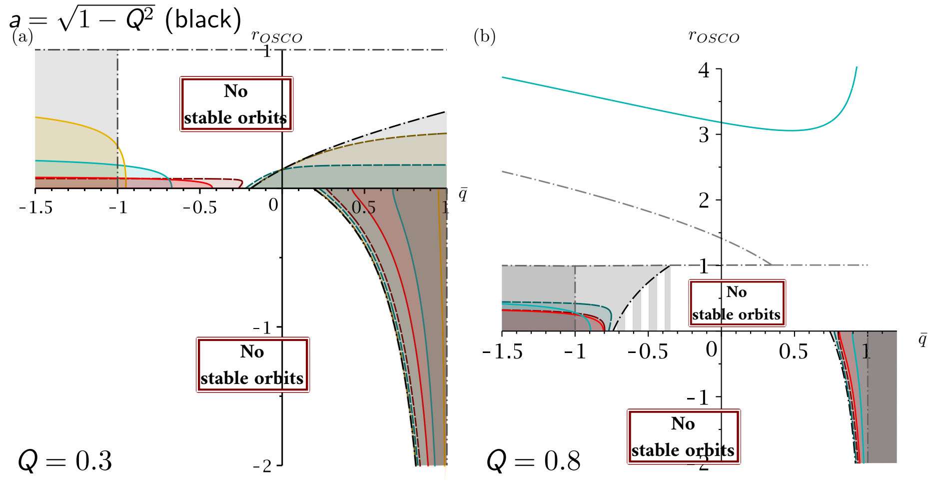






* Existence of a minimal ISCO
* Minimum drops below estimated ISCO for uncharged BH
* Outermost stable orbits are found beyond the horizons

» OSCOs in Kerr-Newman spacetime byd. the horizon



$a = 0.3$ (red), $a = 0.6$ (blue), $a = 0.9$ (yellow) $a = 0.1$ (red), $a = 0.4$ (blue)

$$\frac{d^k}{d\,r^k} R(r) = 0, \quad k = 0..3 \qquad \Rightarrow \qquad r = 0, \quad \bar{q} = a^2 + Q^2, \quad E = \frac{1}{\bar{q}}, \quad K_{\mathrm{eq}} = 0$$