Strong gravitational lensing around Kehagias-Sfetsos black hole surrounded by plasma

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Overview

Motivation

- To constrain the value of the “Hořava” parameter.
- Investigating the effects of plasma on strong lensing.

Kehagias-Sfetsos metric

[Kehagias et. al, Phys. Lett. B, 678, 123-126, 2009]

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

where, $A(r) = f(r) = B^{-1}(r)$, $C(r) = r^2$,

$f(r) = 1 + r^2\omega\left(1 - \sqrt{1 + \frac{4M}{r^3\omega}}\right)$,

M is the mass of the compact object and ω is the "Hořava" parameter.

The metric is singular if $f(r) = 0$. The roots of this equation are $r_{\pm} = M \pm \sqrt{M^2 - \frac{1}{2\omega}}$. If,

$\omega \geq \omega_h = \frac{1}{2M^2} \implies$ black hole
$\omega < \omega_h = \frac{1}{2M^2} \implies$ naked singularity

Effective geometry due to plasma

[Synge, 1960]
The modification in metric is as follows,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \left(1 - \frac{1}{n^2}\right) V_\mu V_\nu \ , \tag{2}$$

where V_μ is the 4-velocity of the plasma and n is the refractive index of plasma.
The 4-velocity of plasma reads

$$V_\mu = (\sqrt{-g_{tt}}, 0, 0, 0) \ . \tag{3}$$

Only the temporal component of the metric modifies as,

$$\tilde{g}_{tt} = \frac{f(r)}{n^2} \ . \tag{4}$$

Models of plasma

$$n^2 = 1 - \frac{\omega_e^2}{\omega_{ph}^2(x^i)} \ , \quad \omega_e^2 = \frac{4\pi e^2 N}{m} = K_e N \ . \tag{5}$$

Here ω_e is the electron plasma frequency, ω_{ph} is the frequency of photon, N is the number density of electron, m is the mass of electron.

Model A

We consider this model without plasma.

Model B

$$N = \frac{N_0 r_0}{r} \ , \quad n^2 = 1 - \frac{K_e N_0 r_0}{\omega_{ph}^2(x^i) r} = 1 - \frac{k}{r} \tag{6}$$

where $k = K_e N_0 r_0 / \omega_{ph}^2(x^i)$.

Schematic diagram of lensing

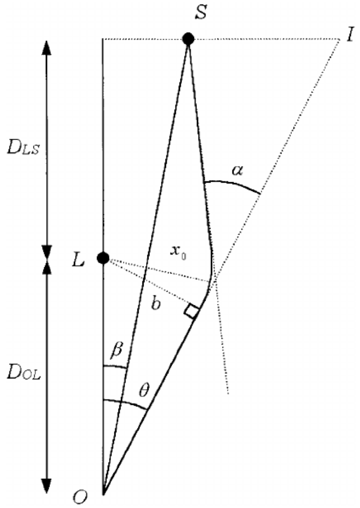

Figure: Schematic diagram of gravitational lensing. (credit : Bozza, 2001)

Deflection angle

[V. Bozza, Phys. Rev. D, 66, 2002]

The deflection angle α reads

$$\alpha(\theta) = -\bar{a} \log \left[\frac{\theta \, D_{OL}}{l_\gamma} - 1 \right] + \bar{b}. \tag{7}$$

where θ is image angular position, D_{OL} is distance between observer and lens, l_γ is the impact parameter of the photon orbit and

$$\bar{a} = \frac{R(0, r_\gamma)}{2\sqrt{\beta_\gamma}}, \tag{8}$$

$$\bar{b} = -\pi + b_R + \bar{a} \log \left(\frac{2\beta_\gamma}{A_\gamma} \right) \,, \tag{9}$$

$$R(0, r_\gamma) = \frac{2\sqrt{B_0 A_0}}{C_0 \, A'_0} (1 - A_\gamma) \sqrt{C_\gamma}, \tag{10}$$

$$\tilde{\beta}_\gamma = \frac{C_\gamma (1 - A_\gamma)^2 \left[C''_\gamma A_\gamma - C_\gamma A''_\gamma \right]}{2A_\gamma^2 \, C'^2_\gamma} \,. \tag{11}$$

The parameter b_R is calculated by numerical integration

$$b_R = \int_0^1 g(z, r_\gamma) dz \ , \tag{12}$$

where function g is given by expression

$$g(z, r_\gamma) \equiv R(z, r_\gamma) f(z, r_\gamma) - R(0, r\gamma) f_0(z, r_\gamma) \ , \tag{13}$$

with following functions

$$f(z, r_\gamma) = \left(A_\gamma - [(1 - A_\gamma)z + A_\gamma] \frac{C_\gamma}{C} \right)^{-1/2}, \tag{14}$$

$$f_0(z, r_\gamma) = \left(\tilde{\alpha} z + \tilde{\beta} z^2 \right)^{-1/2} \ , \tag{15}$$

and

$$\tilde{\alpha} = \frac{1 - A_\gamma}{C_\gamma A'_\gamma}\left(C'_\gamma A_\gamma - C_\gamma A'_\gamma\right), \quad (16)$$

$$\tilde{\beta} = \frac{(1 - A_\gamma)^2}{2C_\gamma^2 A'^3_\gamma}\left[2C_\gamma C'_\gamma A'^2_\gamma + (C_\gamma C''_\gamma - 2C'^2_\gamma)A_\gamma A'_\gamma - C_\gamma C'_\gamma A_\gamma A''_\gamma\right], \quad (17)$$

Index '0' $\implies$ evaluated at turning point r_0.
Index 'γ' $\implies$ evaluated at photon orbit, i.e. at r_γ.

Variables z and r are mutually connected with transformation

$$z \equiv \frac{A - A_\gamma}{1 - A_\gamma}. \quad (18)$$

Image position and magnification

The lens equation for case of strong deflection limit also by Bozza and it reads

$$\beta = \theta - \frac{D_{LS}}{D_{OS}} \Delta\alpha_n \tag{19}$$

parameter $\Delta\alpha_n$ is the offset of deflection angle given by formula

$$\Delta\alpha_n \equiv \alpha(\theta) - 2\pi n. \tag{20}$$

Angular position of the *n*th order image is

$$\theta_n = \theta_n^0 + \frac{I_\gamma \, H_n \left(\beta - \theta_n^0\right) \, D_{OS}}{\bar{a} \, D_{LS} \, D_{OL}} \ , \tag{21}$$

where

$$\alpha(\theta_0^n) = 2n\pi \ , \quad H_n \equiv \exp\left(\frac{\bar{a} - 2n\pi}{\bar{b}}\right). \tag{22}$$

The image magnification is associated with the ratio between image angular position θ and angular position of the source β, it reads

$$\mu_n = \left(\frac{\beta}{\theta} \frac{\partial \beta}{\partial \theta} \right)^{-1}_{\theta_n^0}. \tag{23}$$

Applying strong lensing lens equation, the resulting magnification formula is given by equation

$$\mu_n = H_n \frac{I_\gamma^2 \left(1 + H_n\right) D_{OS}}{\bar{a}\beta \, D_{OL}^2 \, D_{LS}}. \tag{24}$$

Observable quantities

The angular separation parameter S is

$$S \equiv \theta_1 - \theta_\infty, \tag{25}$$

It is reasonable to define R as quantity relating magnification of the first order image with the sum of magnification contributions of second and higher order images, i.e.

$$R \equiv \frac{\mu_1}{\mu_{2+}} \tag{26}$$

where is

$$\mu_{2+} \equiv \sum_{i=1}^{\infty} \mu_i \tag{27}$$

The observable parameters read

$$S = \theta_\infty \exp\left[\left(\bar{b} - 2\pi\right)/\bar{a}\right] \tag{28}$$

and

$$R = \exp\left(2\pi/\bar{a}\right). \tag{29}$$

Plot of $\bar{a}$ and $\bar{b}$.

$$\alpha(\theta) = -\bar{a} \log \left[\frac{\theta D_{OL}}{I_\gamma} - 1 \right] + \bar{b}. \tag{30}$$

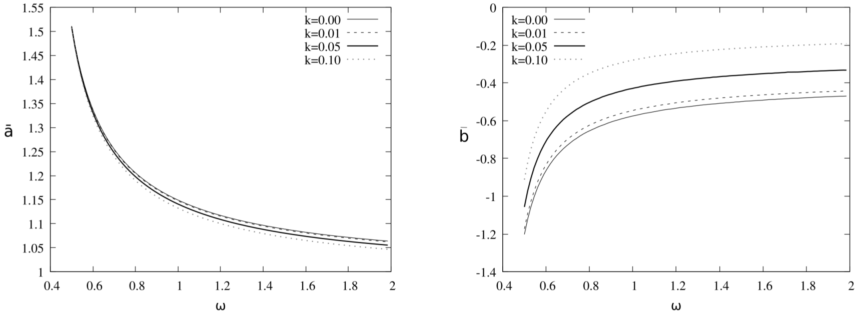

Figure: Plots of $\bar{a}$ (left) and $\bar{b}$ (right) as functions of KS parameter $\omega \in [0.5, 2.0]$ prepared for four representative values of plasma parameter $k = 0.0$, 0.01, 0.05, and 0.1 and plasma model B. Note: the parameters $\bar{a}$ and $\bar{b}$ are in units of *rad*.

Plot of angular size of image

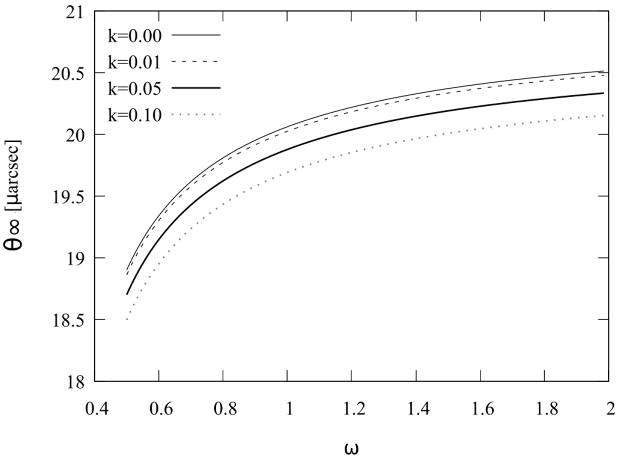

Figure: The black hole shadow angular size θ_∞ calculated for black-hole mass $M = 6.5 \times 10^9\, \mathrm{M}_\odot$ and distance $D_{OL} = 16.0\mathrm{Mpc}$ (distance between lens and observer).

Plot of angular separation of image

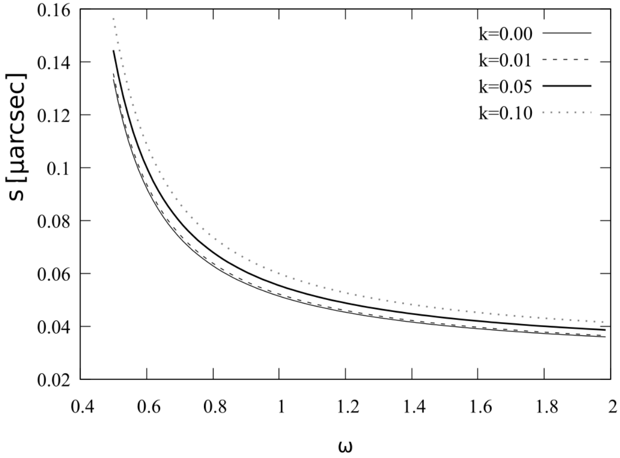

Figure: S is plotted as function of KS parameter ω for plasma parameter $k = 0$,0.01, 0.05, and 0.1. The central lens mass is $M = 6.5 \times 10^9\,\mathrm{M}_\odot$ and distance $D_{OL} = 16.0\mathrm{Mpc}$ (distance between lens and observer).

Plot of relative magnification of image

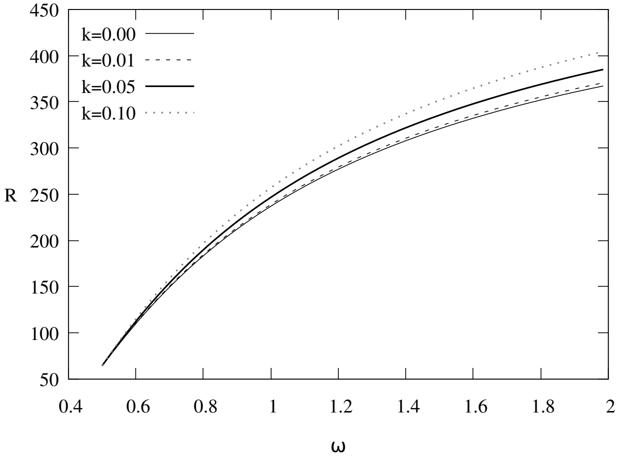

Figure: R is plotted as function of KS parameter ω for plasma parameter $k = 0,0.01$, 0.05, and 0.1. The central lens mass is $M = 6.5 \times 10^9\,\mathrm{M}_\odot$ and distance $D_{OL} = 16.0\mathrm{Mpc}$ (distance between lens and observer).

Conclusions

- The deflection angle increases due to the effect of plasma.
- Plasma reduces the angular size of black hole shadow.
- Angular separation of first order and higher order images increases because of plasma.
- For higher value of "Hořava" parameter, relative magnification of images increase due to plasma.

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