ISCO as a tool to alternate the Kerr metric with the charged stringy black hole metric.

Speaker: Bakhtiyor Narzilloev

PhD student at Fudan University, Shanghai, China.

Content

I. Electrically charged black hole

- Charged particle motion

II. Magnetically charged black hole

- Magnetically charged particle motion
- Magnetized particle motion

I. Electrically charged black hole

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2\left(1+\frac{2b}{r}\right)d\theta^2 + r^2\left(1+\frac{2b}{r}\right)\sin\theta d\phi^2$$

$$N(r) = \left[1-\frac{2(M-b)}{r}\right]\left(1+\frac{2b}{r}\right)^{-1}$$

$$b = Q^2/2M$$

$$r_h = 2(M-b)$$

$$b_{ext} = M \text{ or } Q_{ext} = \sqrt{2}M$$

Electric field around a black hole

$$A_\mu = \left\{ -\frac{Q}{r}\left(1+\frac{2b}{r}\right)^{-1},\ 0,\ 0,\ 0 \right\}$$

$$E^{\hat{r}} = \frac{M^2 Q}{\left(Mr + Q^2\right)^2}\ ,\ E^{\hat{\theta}} = E^{\hat{\phi}} = 0$$

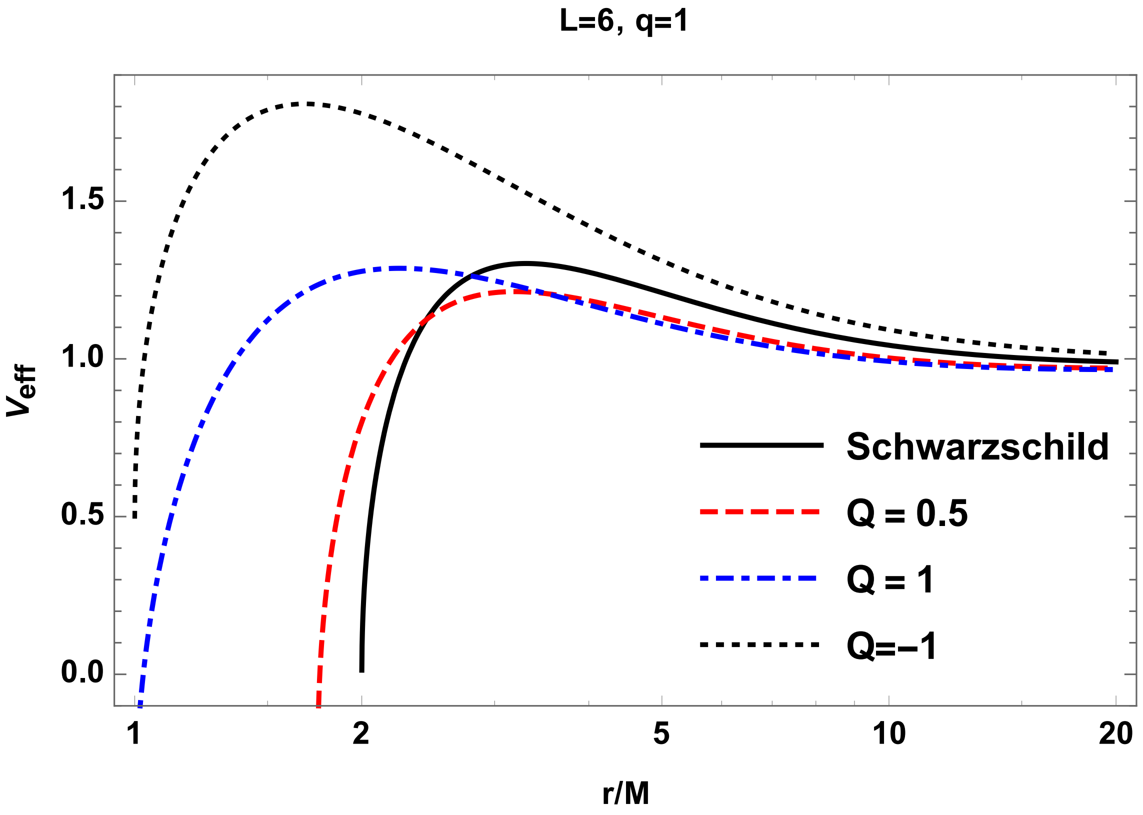
$$M/r \to 0$$

$$E^{\hat{r}} = \frac{Q}{r^2}$$

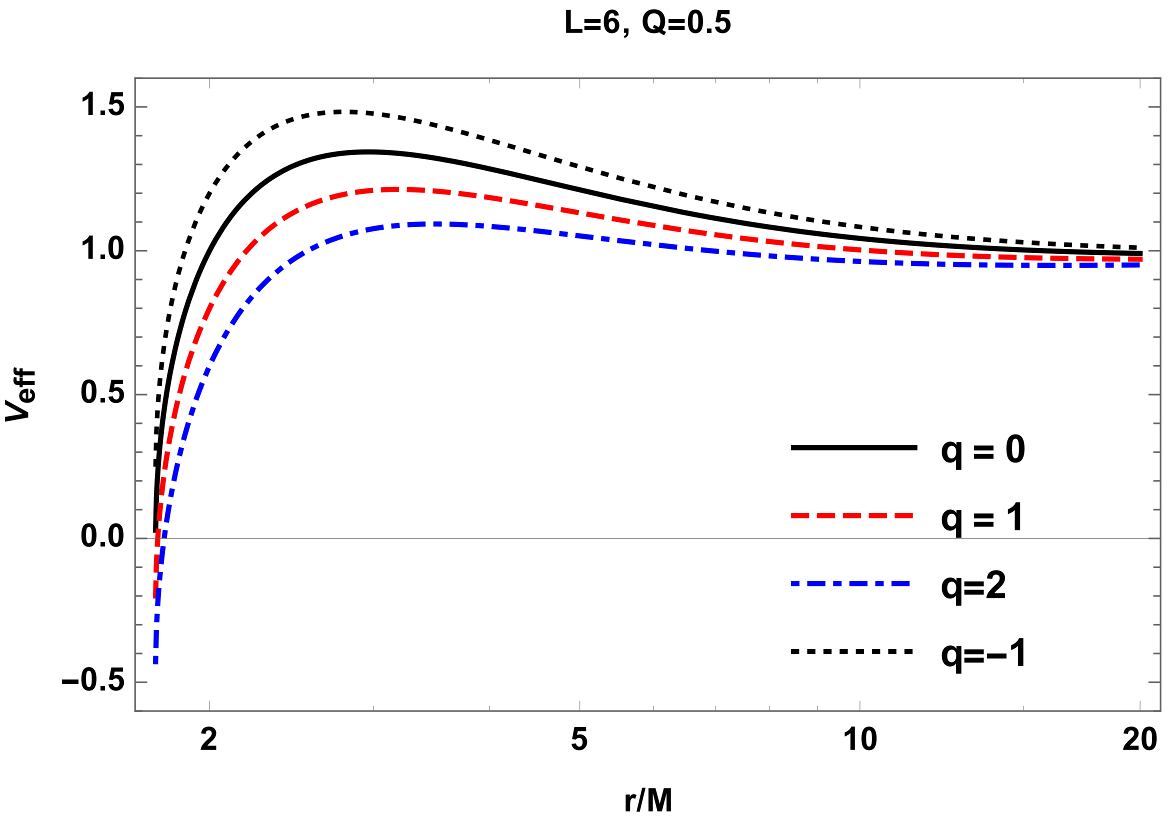
Charged particle motion

$$g^{\alpha\beta}\left(\frac{\partial S}{dx^{\alpha}}+qA_{\alpha}\right)\left(\frac{\partial S}{dx^{\beta}}+qA_{\beta}\right)=-m^2$$

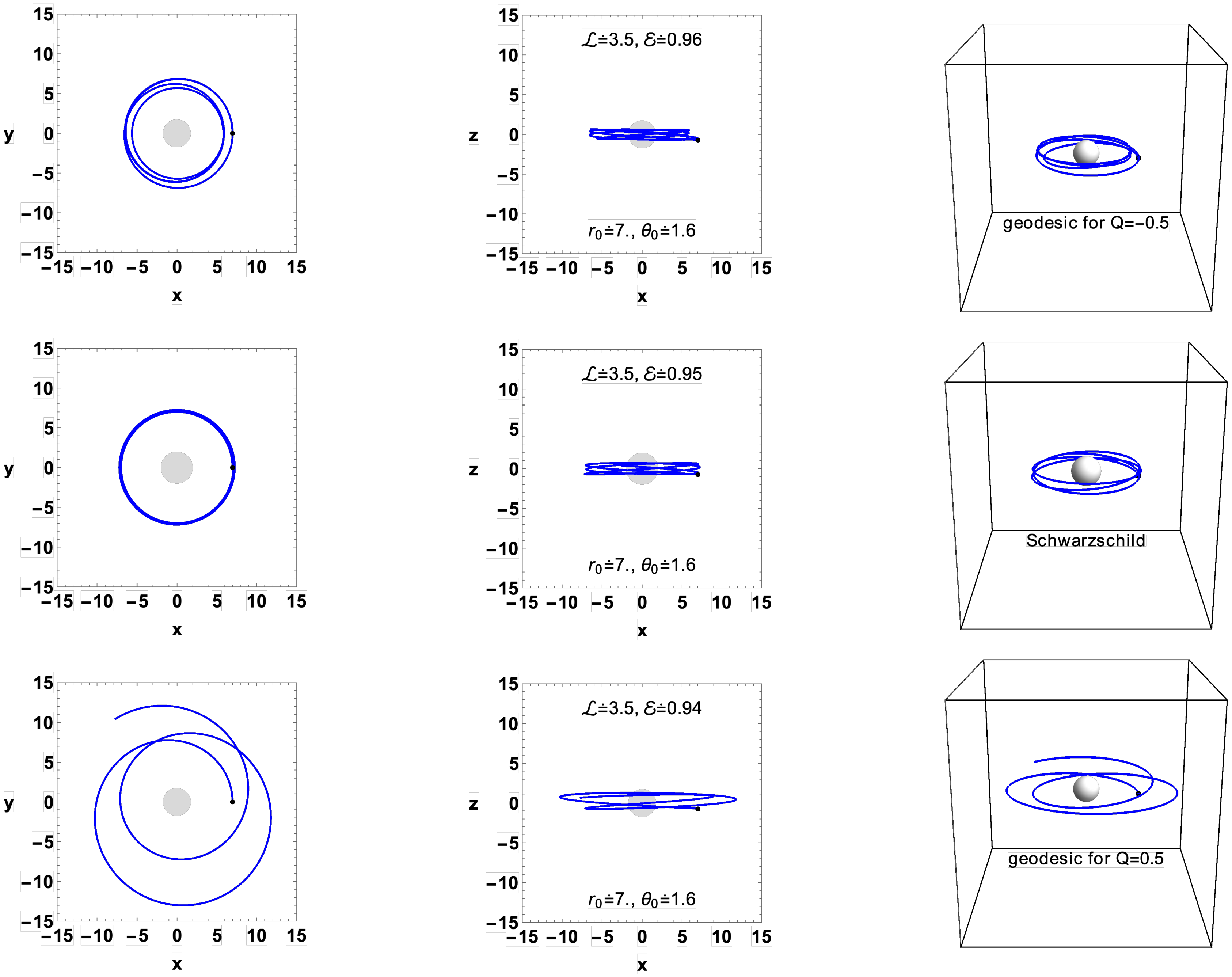
$$V_{\text{eff}}^{\pm}=\frac{qQ}{r}\left(1+\frac{Q^2}{M}\right)^{-1}\pm\sqrt{\frac{1-\frac{2M}{r}\left(1-\frac{Q^2}{2M^2}\right)}{1+\frac{Q^2}{Mr}}\left[1+\frac{\mathcal{L}^2}{r^2}\left(1+\frac{Q^2}{Mr}\right)^{-1}\right]}$$



L=6, q=1
V_{eff}
r/M
Schwarzschild
Q = 0.5
Q = 1
Q=−1

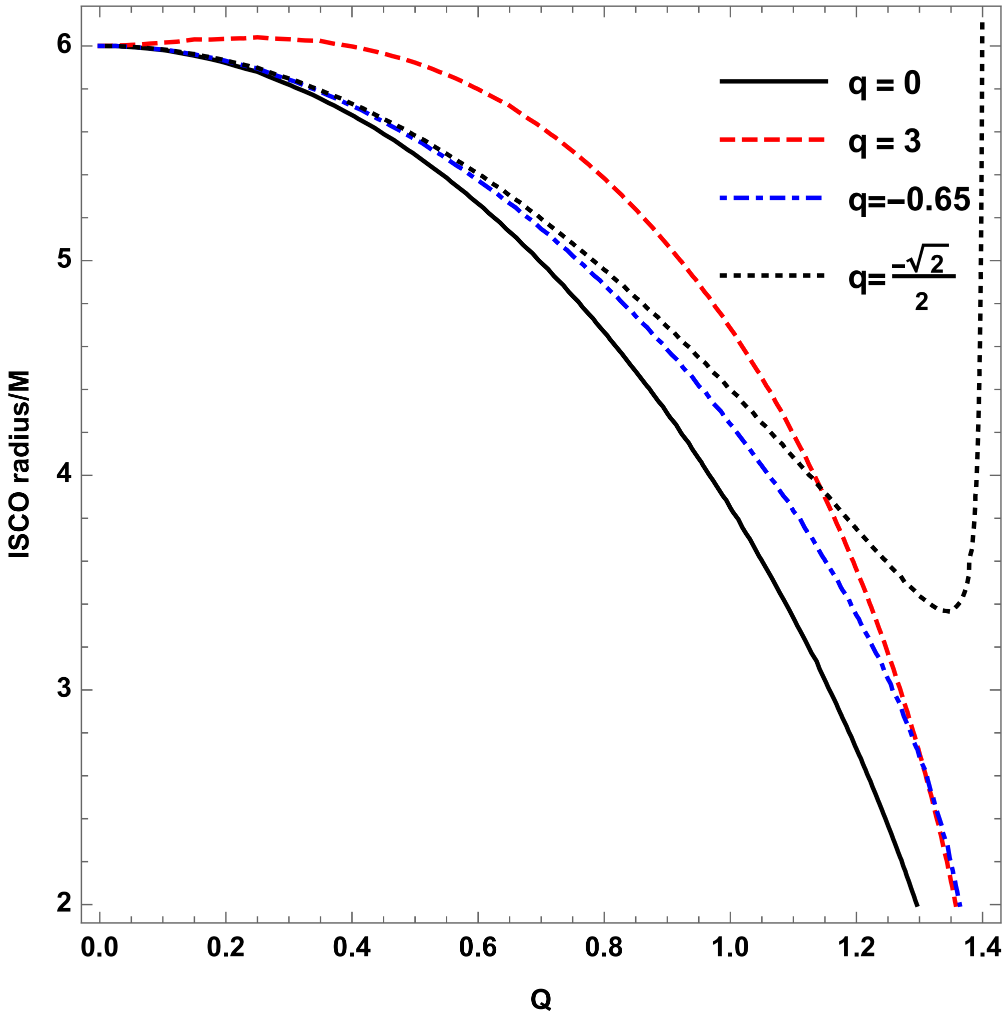


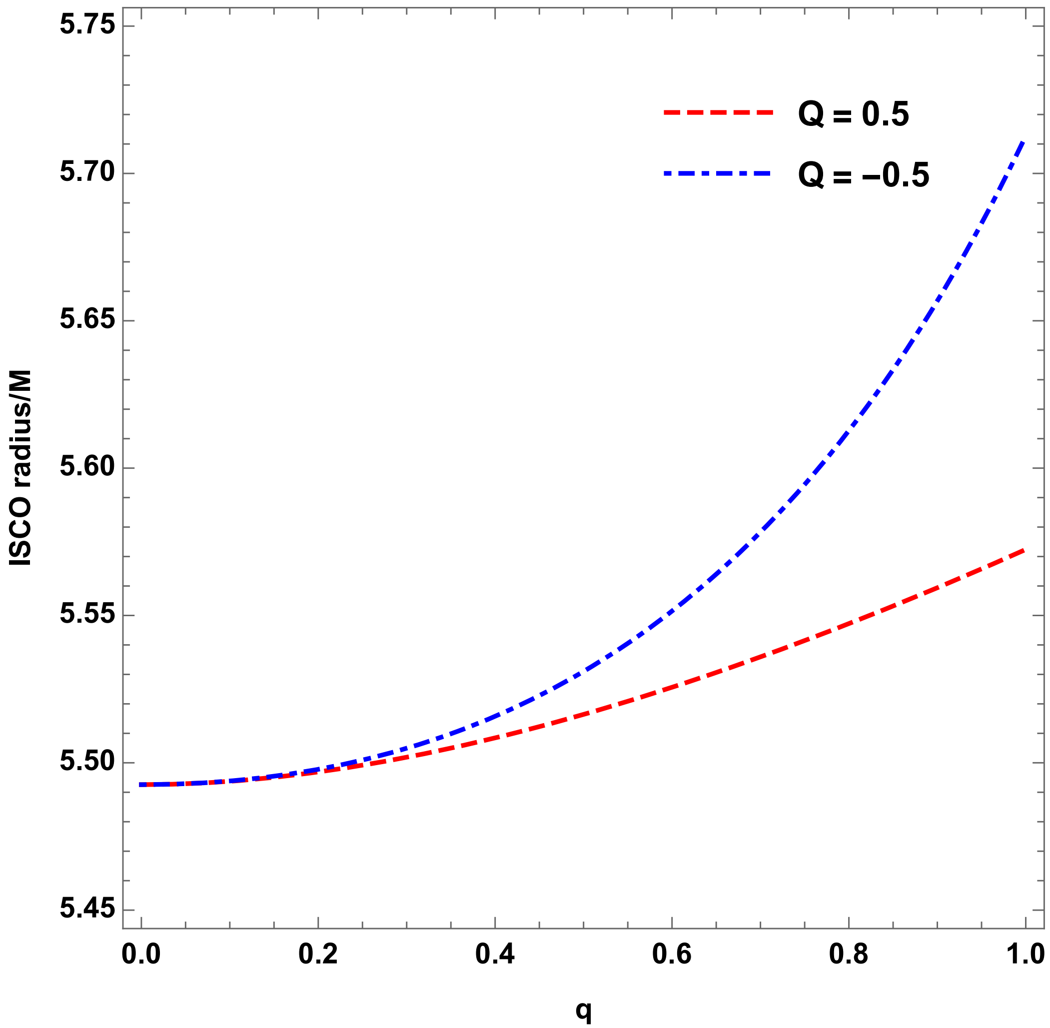
L=6, Q=0.5
V_{eff}
r/M
q = 0
q = 1
q=2
q=−1

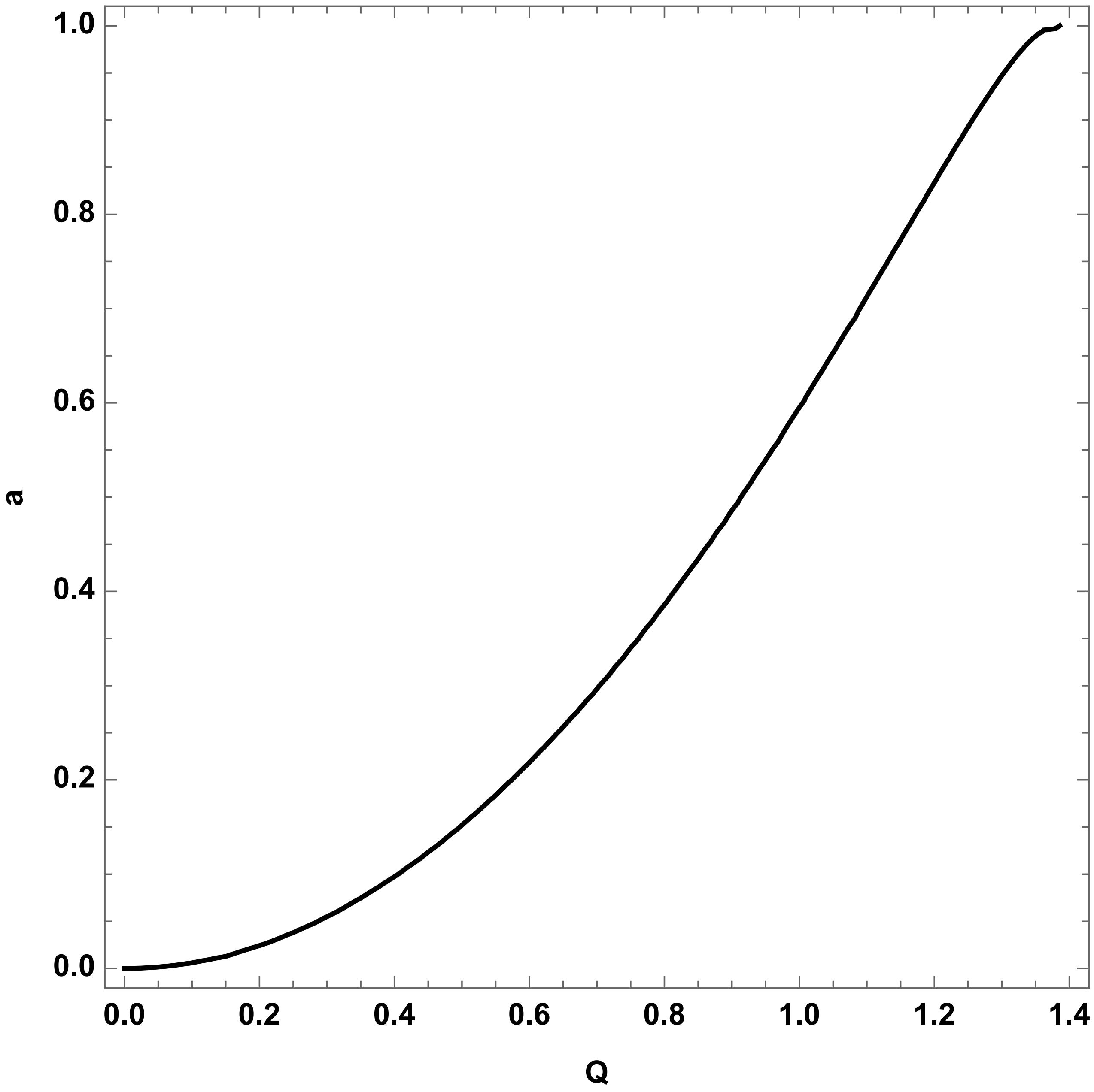


ℒ≐3.5, ℰ≐0.96
r₀≐7., θ₀≐1.6
geodesic for Q=−0.5
ℒ≐3.5, ℰ≐0.95
r₀≐7., θ₀≐1.6
Schwarzschild
ℒ≐3.5, ℰ≐0.94
r₀≐7., θ₀≐1.6
geodesic for Q=0.5

ISCO of the charged particle






a
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Q

II. Magnetically charged black hole

$$ds^2 = -\frac{f(r)}{h(r)}dt^2 + \frac{dr^2}{f(r)h(r)} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$f(r) = 1 - \frac{2\,M}{r}\,, \quad h(r) = 1 - \frac{Q_m^2}{M\,r}$$

Magnetically charged particle motion

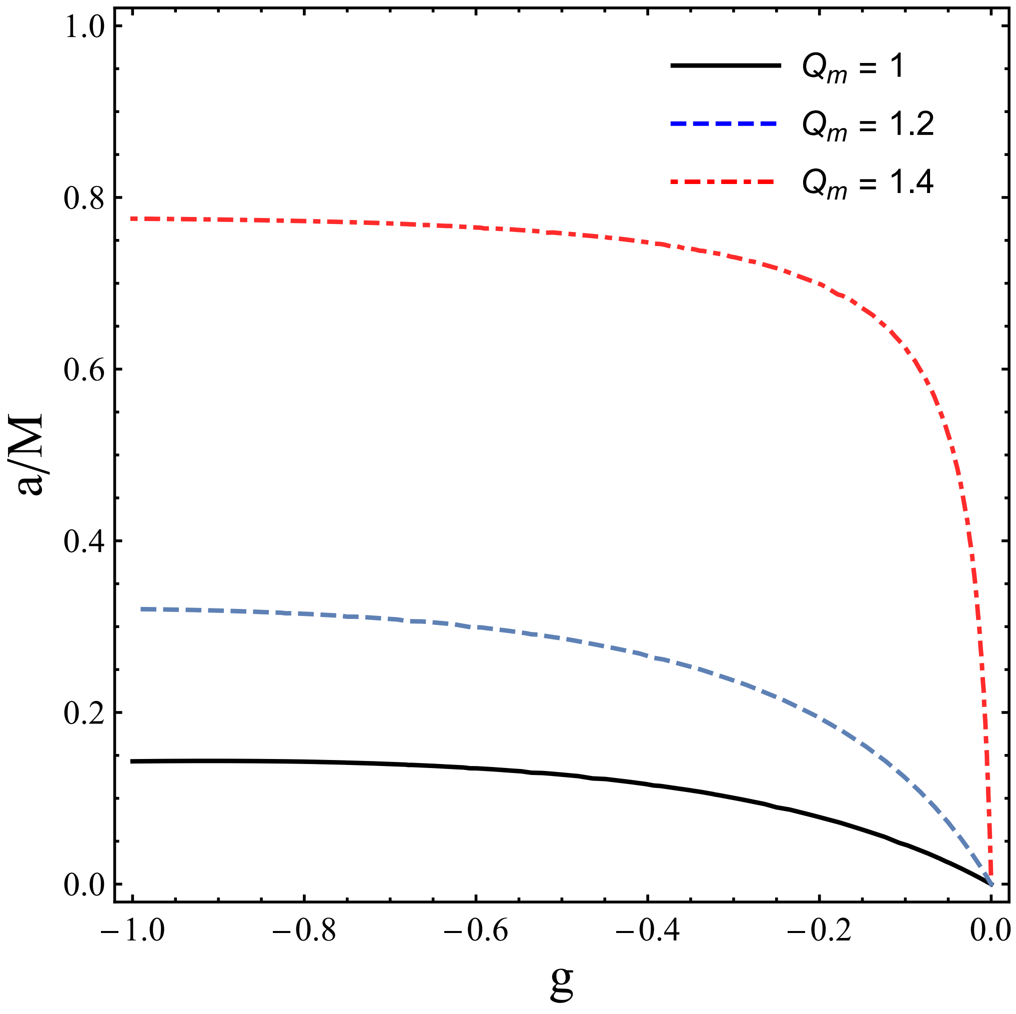
$$H \equiv \frac{1}{2} \; g^{\alpha\beta} \; \left(\frac{\partial \mathcal{S}}{\partial x^{\alpha}} - qA_{\alpha} + iq_m A^{\star}_{\alpha} \right) \left(\frac{\partial \mathcal{S}}{\partial x^{\beta}} - qA_{\beta} + iq_m A^{\star}_{\beta} \right)$$

$$A^{\star}_{t} = -\frac{iQ_m}{r} \quad \text{and} \quad A_{\phi} = -Q_m \cos\theta$$

$$V^{\pm}_{\text{eff}} = \frac{gQ_m}{r} \pm \sqrt{\frac{f(r)}{h(r)} \left(1 + \frac{\mathcal{L}^2}{r^2} \right)}$$

ISCO of the magnetically charged particle

Q_m	g = 0.01 / −0.01	0.05 / −0.05	0.10 / −0.1
0.1	6.00001 5.99999	6.00006 5.99997	6.00015 5.99996
0.2	6.00008 5.99992	6.00044 5.99967	6.00099 5.99944
0.5	6.00136 5.99868	6.00719 5.99379	6.01545 5.98850
0.8	6.00708 5.99313	6.03764 5.96765	6.08148 5.93985
1.0	6.01869 5.98203	6.10156 5.91674	6.22713 5.84776



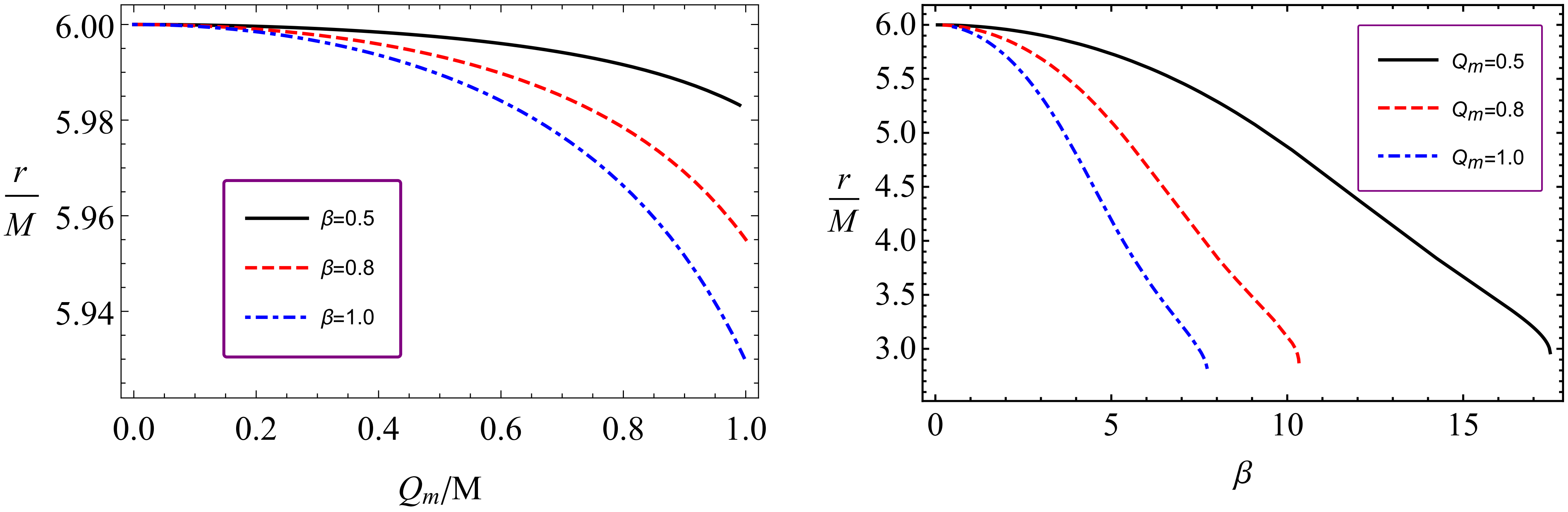
Magnetized particle motion

$$g^{\mu\nu}\frac{\partial\mathcal{S}}{\partial x^{\mu}}\frac{\partial\mathcal{S}}{\partial x^{\nu}} = -\left(m - \frac{1}{2}\mathcal{D}^{\mu\nu}F_{\mu\nu}\right)^2$$

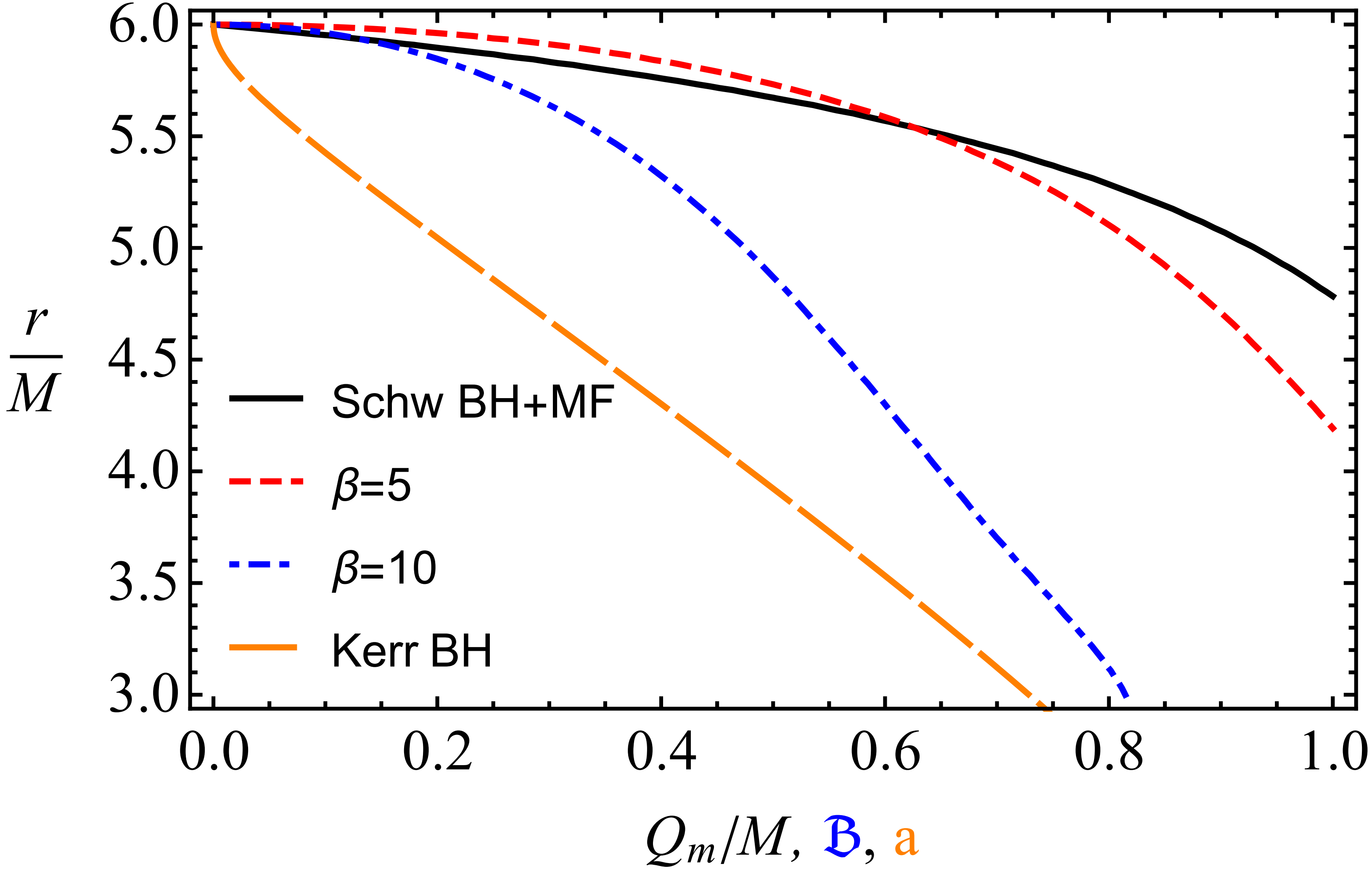
$$\mathcal{D}^{\alpha\beta} = \eta^{\alpha\beta\sigma\nu}u_{\sigma}\mu_{\nu}\ , \qquad \mathcal{D}^{\alpha\beta}u_{\beta} = 0$$

$$V_{\rm eff}(r;\mathcal{L},\mathcal{B}) = \frac{f(r)}{h(r)}\left[\left(1 - \frac{\mathcal{B}}{r^2}\right)^2 + \frac{\mathcal{L}^2}{r^2}\right] \qquad \mathcal{B} = \frac{\mu Q_m}{m}$$

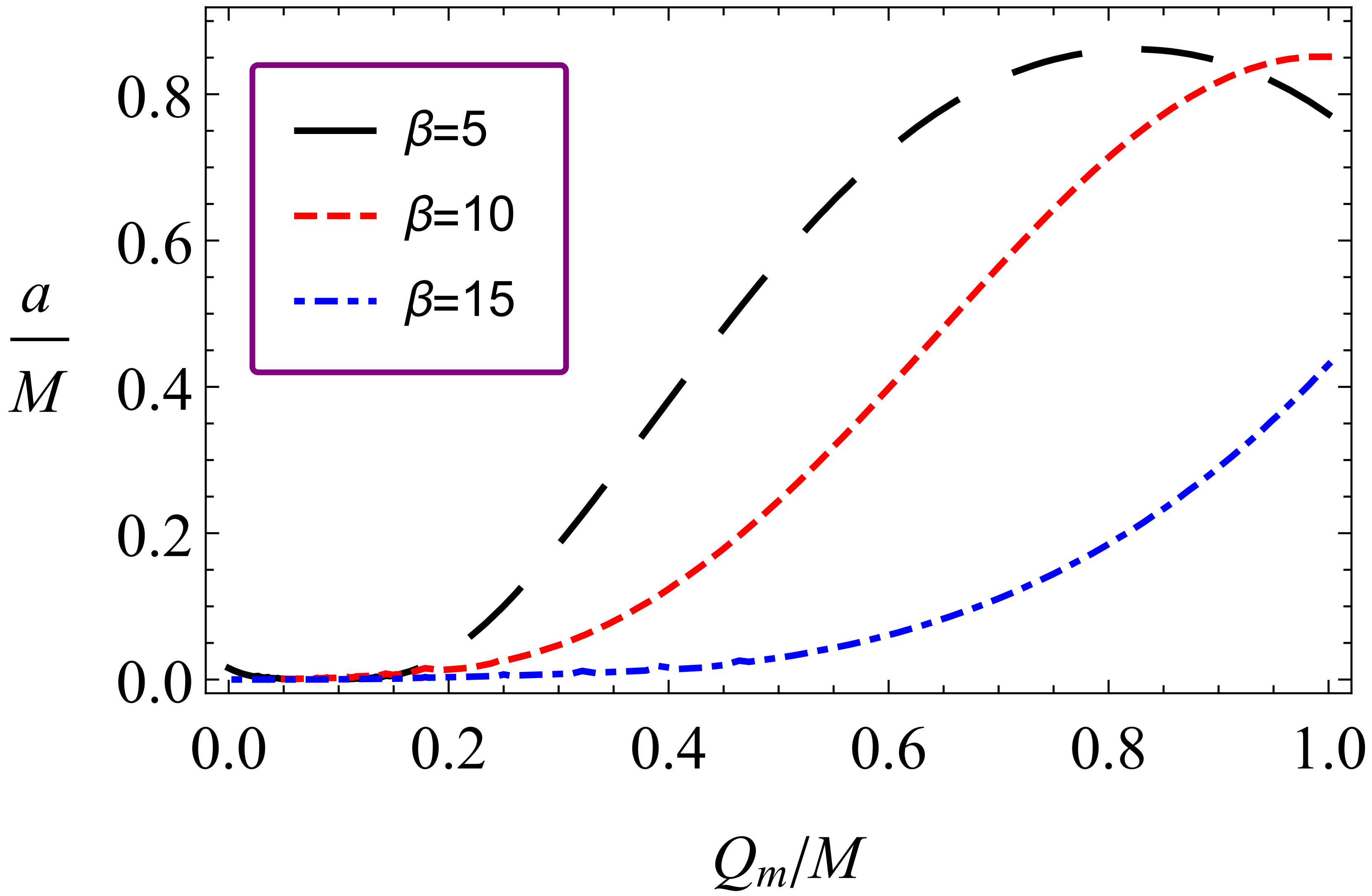
ISCO of the magnetized particle



$$\beta = \mu/(mM)$$



$\frac{r}{M}$
6.0
5.5
5.0
4.5
4.0
3.5
3.0
0.0
0.2
0.4
0.6
0.8
1.0
Q_m/M, $\mathfrak{B}$, a
Schw BH+MF
β=5
β=10
Kerr BH



β=5
β=10
β=15
0.0
0.2
0.4
0.6
0.8
1.0
a/M
Q_m/M

Conclusion

- The electric charge of a static stringy black hole can completely mimic the rotation parameter of a Kerr black hole in the case of charged particle motion.

- The magnetic charge parameter of the magnetically charged particle can mimic the black hole spin parameter up to a $\sim$ 0.8.

- The magnetic charge of the stingy black hole can mimic the spin parameter of Kerr black hole up to a$\sim$0.85.

Thank you for your attention!