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BH SHADOWS AND PHOTON PROPAGATION IN A DYNAMICAL LTB SPACETIME

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Outline

- Lemaître–Tolman–Bondi (LTB) spacetime
- Gullstrand–Painlevé (GP) coordinates and photon dynamics
- Photon trajectory from the observer to the black hole (BH)
- Shadow edge in different cases
- Shadow evolution
- Separation of kinematic, spatial, and dynamical effects
- Conclusions

LTB spacetime

Metric in comoving coordinates [1]

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + r_R^2\,\mathrm{d}R^2 + r^2(\tau, R)\mathrm{d}\sigma^2 \tag{1}$$

$$r(\tau, R) = \left(\frac{3}{2}\sqrt{r_\mathrm{g}}\sqrt{1 + AR^3}(\tau - \tau_0(R))\right)^{2/3} \tag{2}$$

$$r_\tau = \frac{\partial r}{\partial \tau}, \quad r_R = \frac{\partial r}{\partial R}, \quad r_\tau(\tau, R) = \sqrt{\frac{r_\mathrm{g}(1 + AR^3)}{r}}, \quad \epsilon(R) = AR^3$$

R – a radial coordinate labeling spherical shells of dust
τ – proper time measured by a comoving observer
$\tau_0(R)$ – the Big Bang time at each shell R
$A = a_0/r_\mathrm{g}$ – dimensionless parameter shows how the expansion rate changes, where a_0 is the radius of the observable universe

[1] M. Korkina and E. Kopteva. The mass function method for obtaining exact solutions in general relativity. Space, Time and Fundamental Interactions, 2012

LTB to GP coordinates

$$t = \tau, \qquad r = r(\tau, R), \qquad R = R(t, r) \tag{3}$$

$$\mathrm{d}s^2 = -F(t,r)\,\mathrm{d}t^2 - 2v(t,r)\,\mathrm{d}t\,\mathrm{d}r + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{4}$$

where

$$F(t,r) = 1 - v^2(t,r)$$

$$v(t,r) \equiv r_\tau(t, R(t,r)) = \sqrt{\frac{r_\mathrm{g}(1+\epsilon(t,r))}{r}}$$

$$\epsilon(t,r) = AR(t,r)^3 \tag{5}$$

- r is now the observable areal radius
- The cosmological correction enters through $\epsilon(t,r)$
- Therefore, the effective metric is explicitly time-dependent

Photon dynamics

The Hamiltonian is

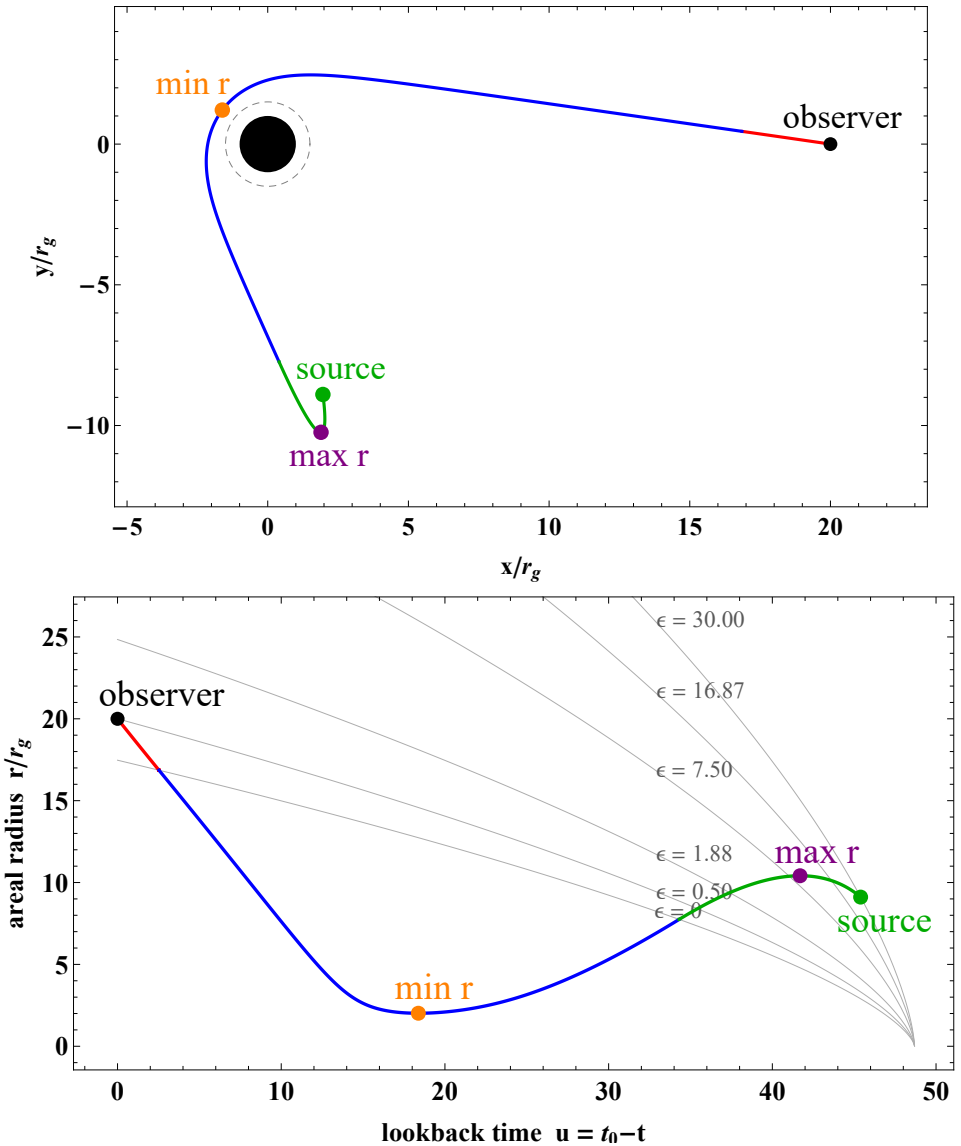
$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0 \tag{6}$$

and the canonical equations are

$$\dot{x}^\mu = \frac{\partial H}{\partial p_\mu}, \qquad \dot{p}_\mu = -\frac{\partial H}{\partial x^\mu} \tag{7}$$

- null photon trajectory
- direct integration in GP coordinates
- shadow obtained by tracing rays backward

Photon ray at $r_o = 20 r_g$

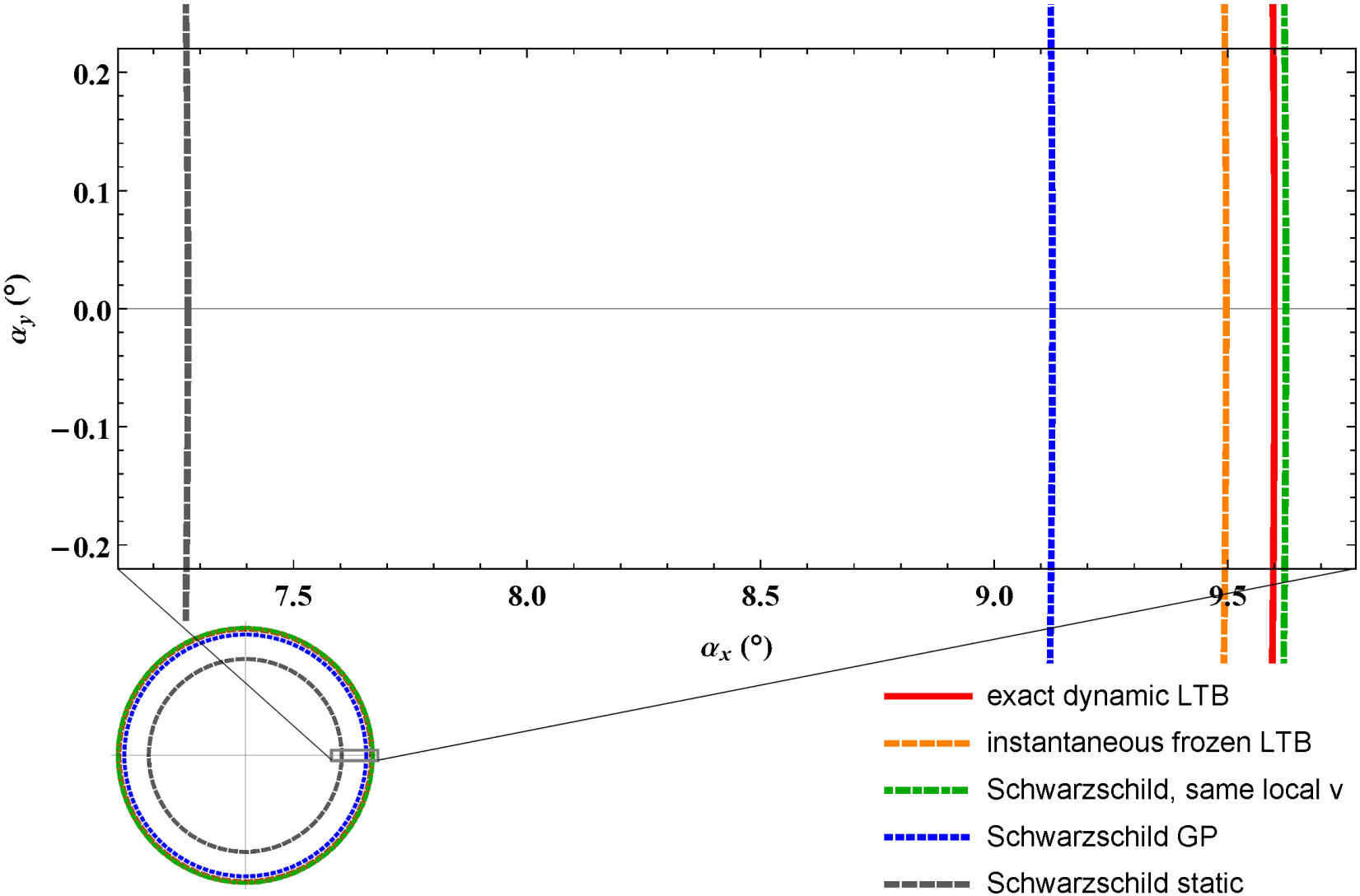


- Red: exact dynamic LTB
- Blue: static Schwarzschild
- Green: exact dynamic LTB
- Extrema of r

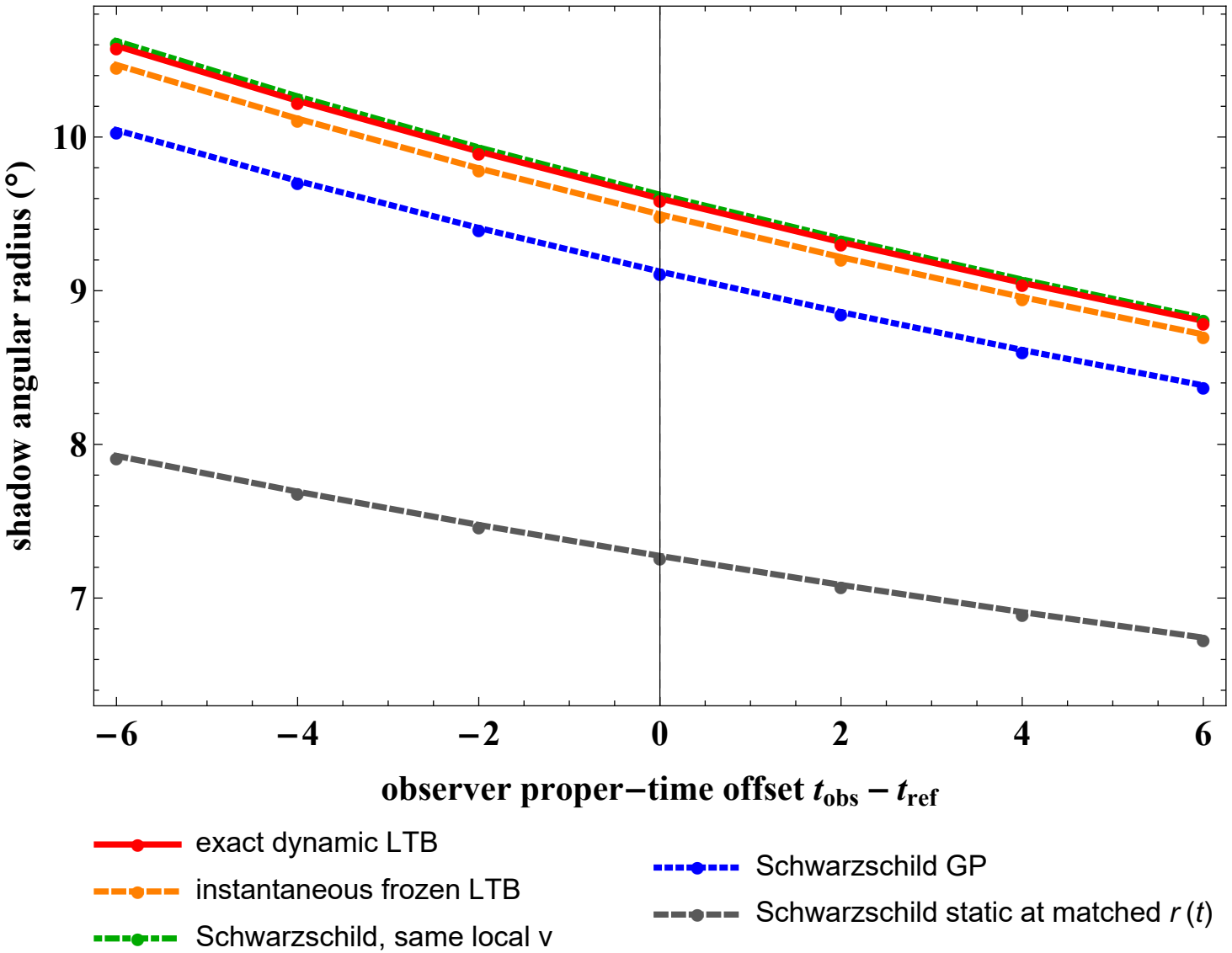
$$\frac{\mathrm{d}r}{\mathrm{d}\lambda} = 0$$

Zoom of the shadow edge, $r_o = 20 r_g$

static → obs. kinematics → matched v → frozen LTB → exact LTB



Shadow evolution, $r_o = 20 r_g$



Angle effects, decomposition, $r_o = 20 r_g$

$$\boxed{\Delta_{\textbf{total}} = \alpha_{\text{exact,LTB}} - \alpha_{\text{Sch,stat}} = \Delta\alpha_{\textbf{kinematic}} + \Delta\alpha_{\textbf{spatial}} + \Delta\alpha_{\textbf{dynamic}}}$$

$$\Delta\alpha_{\textbf{kinematic}} = \alpha_{\text{Sch,}(v_{\text{LTB}})} - \alpha_{\text{Sch,stat}} = 2.536^\circ$$

$$\left.\begin{aligned} \Delta\alpha_{\textbf{spatial}} &= \alpha_{\text{frozen,LTB}} - \alpha_{\text{Sch,}(v_{\text{LTB}})} = -0.182^\circ \\ \Delta\alpha_{\textbf{dynamic}} &= \alpha_{\text{exact,LTB}} - \alpha_{\text{frozen,LTB}} = 0.149^\circ \end{aligned}\right\} = -0.033^\circ (1.3\%)$$

$$\Delta_{\textbf{total}} = 2.503^\circ$$

Conclusions

- We studied a BH embedded in an expanding LTB spacetime
- We followed the photon from the observer through the strong-field region and through the evolving LTB spacetime
- As ϵ grows, the expansion affects the photon trajectory
- The difference in the shadow angle contains kinematic, spatial, and dynamical contributions
- In this example, the spatial and dynamical LTB effects partly cancel each other

Main physical message: the LTB correction acts through the full photon path, not only through the local metric at the observer

Thank you!

Backup, $\tau_0(R)$

$$r(\tau, R) = \left(\frac{3}{2}\sqrt{r_g}\sqrt{1 + AR^3}(\tau - \tau_0(R))\right)^{2/3} \tag{8}$$

The Big Bang singularity, $r(\tau, R) = 0$, occurs at $\tau = \tau_0(R)$
A shell-crossing singularity, $r_R = 0$, arises when neighboring shells begin their evolution at different times. To avoid the shell-crossing:

$$\tau_{\rm sc}(R) < \tau_0(R) < \tau_{\rm hor}(R) \quad \text{with} \quad \tau_0'(R) < 0 \tag{9}$$

The arbitrary function $\tau_0(R)$ must be strictly decreasing.
Therefore, we choose it in the form

$$\tau_0(R) = (1 + R^{3/2})^{-1} \tag{10}$$

The function $\tau_0'(R)$ describes shell desynchronization

$$\tau_0'(R) = -\frac{3}{2}\frac{\sqrt{R}}{(1 + R^{3/2})^2} \tag{11}$$

Backup, LTB to GP

We rewrite the LTB metric

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + r_R^2\,\mathrm{d}R^2 + r^2\mathrm{d}\sigma^2 \tag{12}$$

in the general GP form

$$\mathrm{d}s^2 = -g_{tt}\,\mathrm{d}t^2 + 2g_{tr}\,\mathrm{d}r\,\mathrm{d}t + g_{rr}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{13}$$

Using

$$\mathrm{d}t = t_\tau \mathrm{d}\tau + t_R \mathrm{d}R, \quad \mathrm{d}r = r_\tau \mathrm{d}\tau + r_R \mathrm{d}R, \tag{14}$$

$$r_\tau = \frac{\partial r}{\partial \tau}, \qquad r_R = \frac{\partial r}{\partial R}, \qquad t_\tau = \frac{\partial t}{\partial \tau}, \quad t_R = \frac{\partial t}{\partial R}, \tag{15}$$

and comparing coefficients of $\mathrm{d}\tau^2$, $\mathrm{d}R^2$, and $\mathrm{d}\tau\mathrm{d}R$, one obtains

$$-g_{tt}t_\tau^2 + 2g_{tr}t_\tau r_\tau + g_{rr}r_\tau^2 = -1, \tag{16a}$$

$$-g_{tt}t_R^2 + 2g_{tr}t_R r_R + g_{rr}r_R^2 = r_R^2, \tag{16b}$$

$$-g_{tt}t_\tau t_R + g_{tr}\left(t_\tau r_R + t_R r_\tau\right) + g_{rr}r_\tau r_R = 0. \tag{16c}$$

Backup, LTB to GP

A particularly natural gauge choice is

$$t_\tau = 1, \qquad t_R = 0, \qquad \longrightarrow \qquad t = \tau. \tag{17}$$

This means that the GP time is chosen to coincide with the synchronous comoving proper time of the dust; $t_R = 0$ – all shells are assigned the same time. The metric coefficients reduce to

$$g_{tt} = 1 - r_\tau^2, \qquad g_{tr} = -r_\tau, \qquad g_{rr} = 1. \tag{18}$$

With the branch choice $r_\tau > 0$ used in the main text, the mixed metric component is negative, $g_{tr} = -r_\tau < 0$. This term represents the radial flow of the LTB shells in the GP-type representation. The metric (13) takes the form

$$\mathrm{d}s^2 = -(1 - r_\tau^2)\,\mathrm{d}t^2 - 2r_\tau\,\mathrm{d}t\,\mathrm{d}r + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{19}$$

Thus the same quantity r_τ which in LTB represents the shell velocity becomes the radial flow velocity in the GP representation.

Backup, Hamiltonian formalism

$$H = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0, \tag{20}$$

$$\dot{x}^\mu = \frac{\partial H}{\partial p_\mu}, \qquad \dot{p}_\mu = -\frac{\partial H}{\partial x^\mu}. \tag{21}$$

$$\dot{t} = -p_t - v p_r, \qquad \dot{r} = -v p_t + F p_r, \qquad \dot{\varphi} = \frac{L}{r^2}, \tag{22a}$$

$$\dot{p}_t = v_t p_r (p_t + v p_r), \quad \dot{p}_r = v_r p_r (p_t + v p_r) + \frac{L^2}{r^3}. \tag{22b}$$