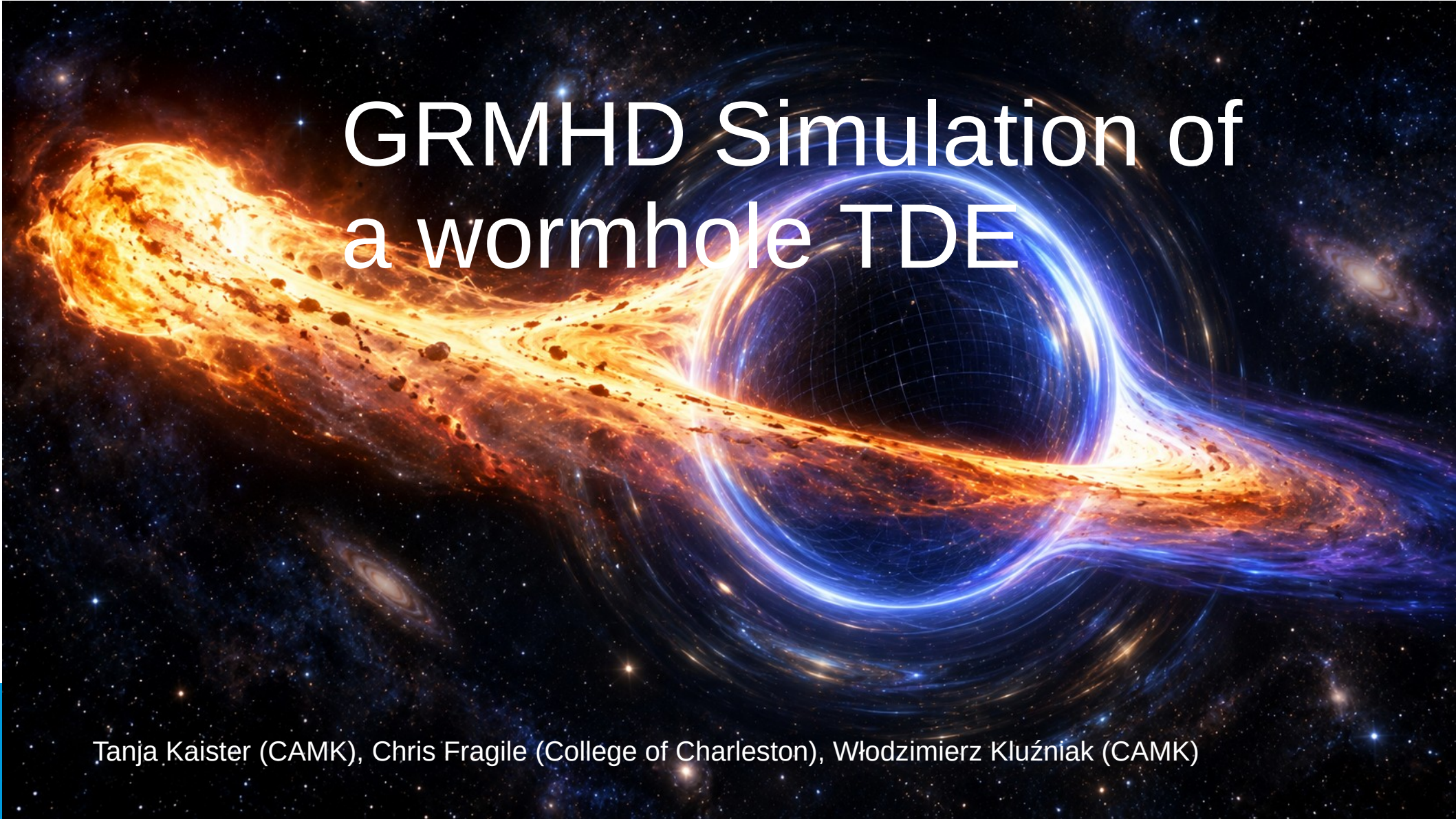


GRMHD Simulation of a wormhole TDE

Tanja Kaister (CAMK), Chris Fragile (College of Charleston), Włodzimierz Kluźniak (CAMK)

Motivation

- Distinguish wormholes from black holes
  - Fallback curve has been measured to be different [1]
    - What else is different?
- Stellar mass compact objects constrained to be black holes by gravitational waves by mergers
  - Supermassive compact objects unconstrained
- Need bright events to see distant sources
  - TDE is a good candidate

[1] Relativistic Tidal Disruption in Black Hole and Wormhole Backgrounds, P. Banerjee et al.

Cosmos++ Code [3]

- General Relativistic magnetohydrodynamics (GRMHD)
- Adaptive mesh refinement
  - Octree

[3] Cosmos++ : Relativistic Magnetohydrodynamics on Unstructured Grids with Local Adaptive Refinement, P. Anninos et a.

Cosmos++ Code: Governing equations [3]

$$\partial_t D + \partial_i \left(D V^i \right) = 0$$ Continuity

$$\partial_t E + \partial_i \left(E V^i \right) = -\left(P + k_{\dot{W}} \left\| Q \right\| \right) \partial_t W - \left(\delta_i^j - Q_i^j \right) \partial_j \left(W V^i \right)$$ Energy

$$P_B = \frac{B^\mu B^\nu g_{\mu\nu}}{8\pi}$$

$$\partial_t S_j + \partial_i \left(S_j V^i \right) = \frac{1}{4\pi} \partial_t \left(\sqrt{-g} B_j B^0 \right) + \frac{1}{4\pi} \partial_i \left(\sqrt{-g} B_j B^i \right)$$

$$+ \left(\frac{S^\mu S^\nu}{2 S^0} - \frac{\sqrt{-g}}{8\pi} B^\mu B^\nu \right) \partial_j g_{\mu\nu} - \sqrt{-g} \partial_i \left(\left(P + P_B \right) \delta_j^i + Q_j^i \right)$$ Momentum

$$\mathscr{B}^\mu = W \left(B^\mu - B^0 V^\mu \right)$$

$$\partial_t \mathscr{B}^j + \partial_i \left(\mathscr{B}^j V^i \right) = \mathscr{B}^i \partial_i V^j + \eta \partial^j \left(\partial_i \mathscr{B}^i \right)$$ Induction

Wormhole metric: Formula

Kerr-like Wormhole metric: [2]

$$ds^2 = -\left(1 - 2\frac{Mr}{\Sigma}\right)dt^2 - \frac{4\,M\,r\,\Sigma\sin^2\theta}{\Sigma}\,dt\,d\phi + \frac{\Sigma}{\hat{\Delta}}\,dr^2 + \Sigma\,d\theta^2 + \left(r^2 + a^2 + \frac{2\,Ma^2\,r\sin^2\theta}{\Sigma}\right)\sin^2\theta\,d\phi^2$$

$$\hat{\Delta} = r^2 - 2M(1+\lambda^2)r + a^2 \qquad \Sigma = r^2 + a^2\cos^2\theta$$

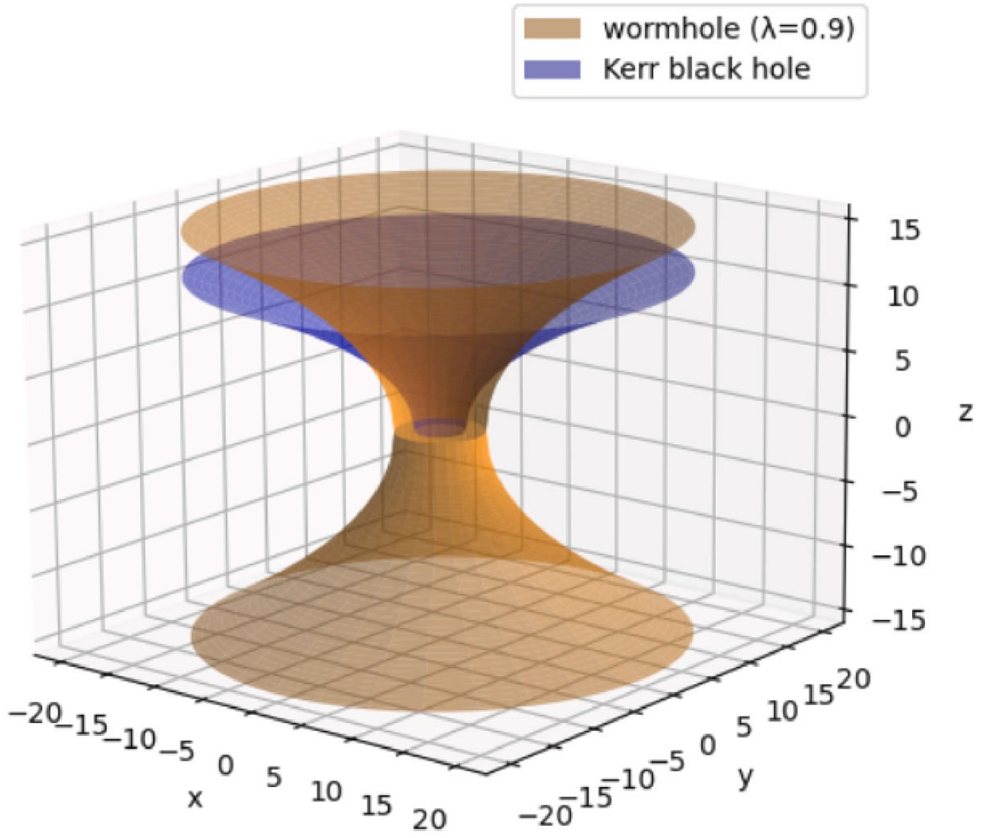
Root of $\hat{\Delta}$ leads to throat surface:

$$r_+ = (1+\lambda^2)M + \sqrt{M^2(1+\lambda^2)^2 - a^2}$$

Rescaling coordinates to $r = r_+ + \frac{\rho^2}{M}$ removes the coordinate singularity and leads to two connected regions → Wormhole!

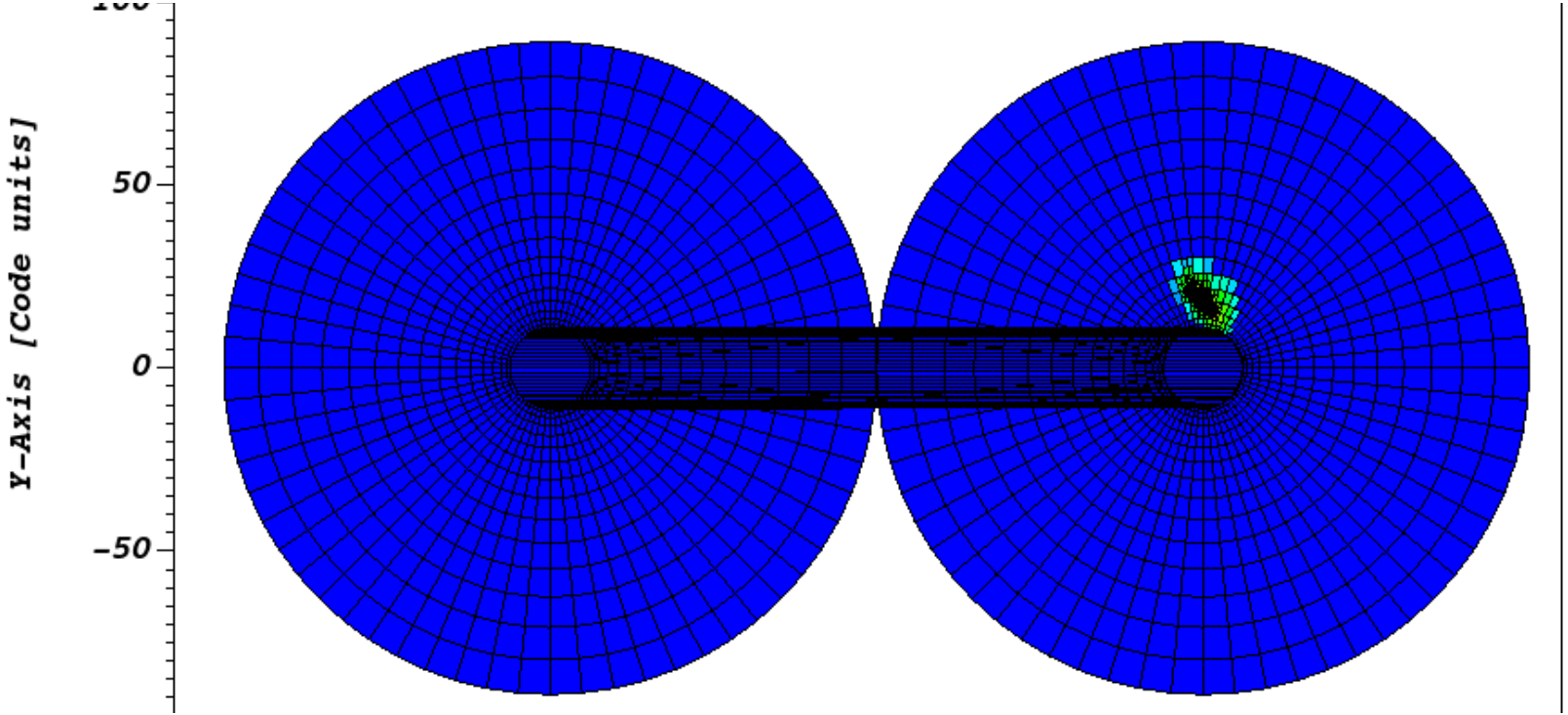
[2] Echoes of Kerr-like wormholes, Bueno et al.

Wormhole metric: Diagram

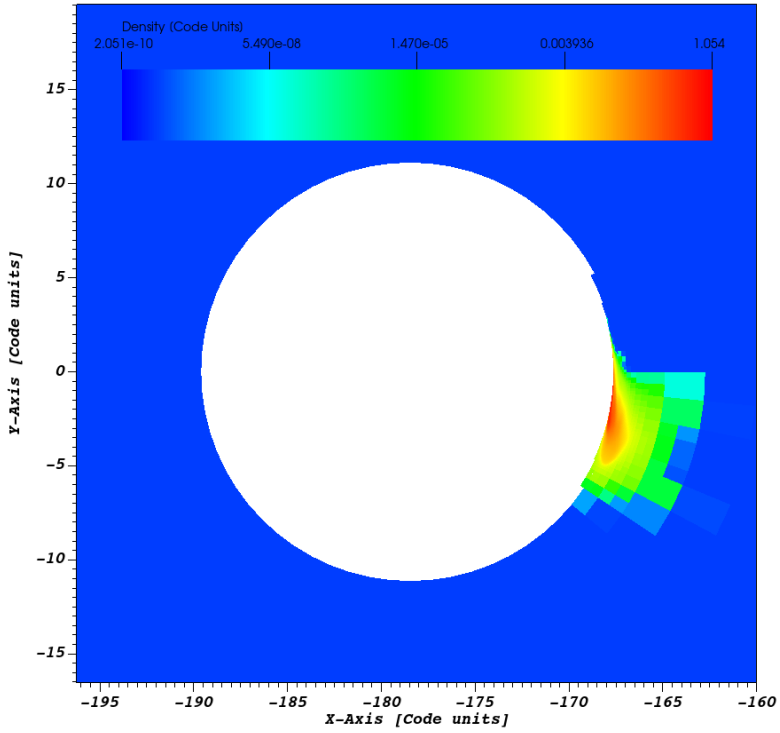


[4] Image from signature of iron line profile from a Kerr-like wormhole, Liu et al.

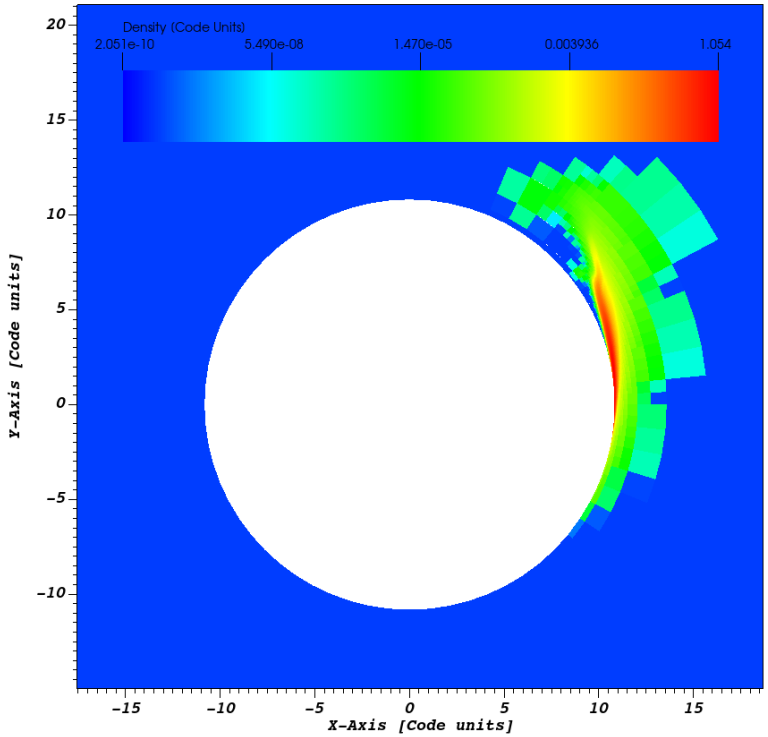
Wormhole metric: Overview



Wormhole metric: Passthrough



Left mouth



Right mouth

Star model: MESA

- Use a B type main sequence star to mimic the S-star cluster
- To build the model: Start with 8 solar mass pre main sequence star and evolve until main sequence
- Used so that the star has a realistic internal structure
  - Important for disruption dynamics

[5] Modules for Experiments in Stellar Astrophysics (MESA), Paxton et al.

Equation of motion

To simplify, we assume radial infall from a position of rest:

$\theta = 0,\ \sin\theta = 0,\ \cos\theta = 1,\ \Sigma = r^2 + a^2$

The metric simplifies to:

$$ds^2 = -\left(1 - \frac{2Mr}{r^2+a^2}\right)dt^2 + \frac{r^2+a^2}{r^2 - 2M\left(1+\lambda^2\right)r + a^2}dr^2 + \left(r^2+a^2\right)d\theta^2$$

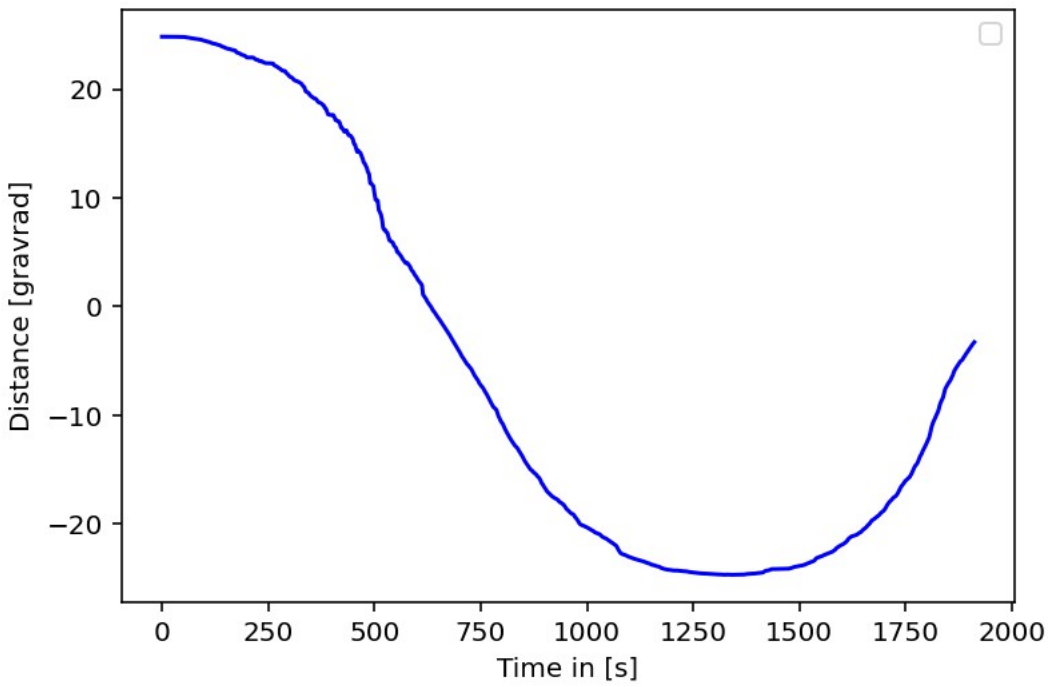
From the Lagrangian we can derive the radial equation of motion $\dot{r}^2 = \frac{\hat{\Delta}}{r^2+a^2}\left(\frac{1 - \frac{2Mr_0}{r_0^2+a^2}}{1 - \frac{2Mr}{r^2+a^2}} - 1\right) = F(r)$

From this, we can compute the half period, which can be tested by comparing it to the simulation result

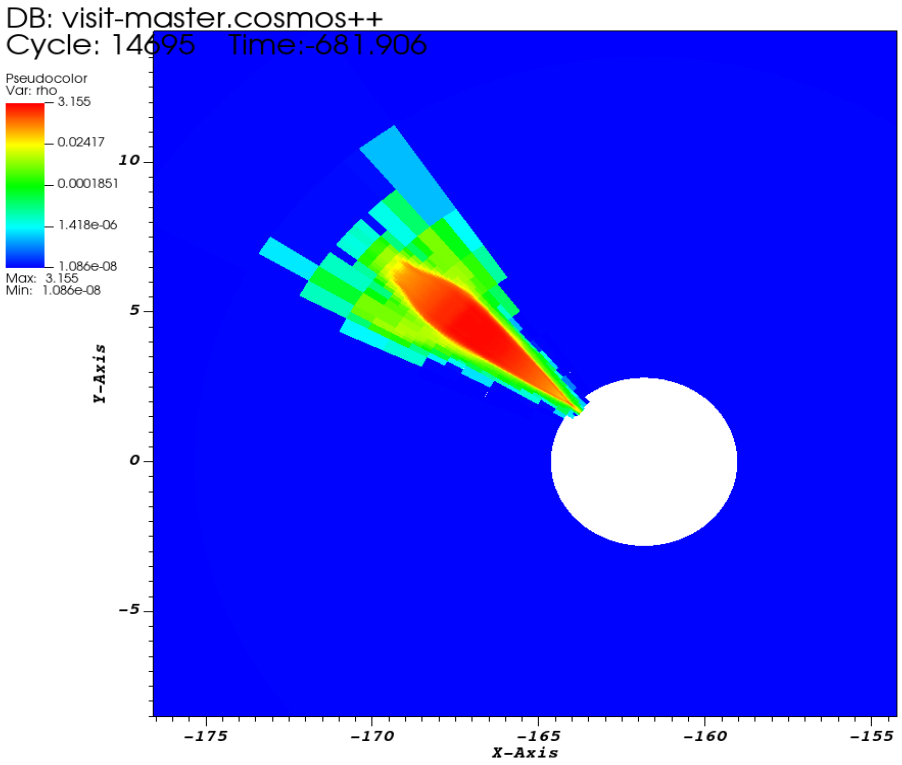
$$T/2 = 2\int_{r_+}^{r_0} \frac{dr}{\sqrt{F(r)}}$$

Oscillation result

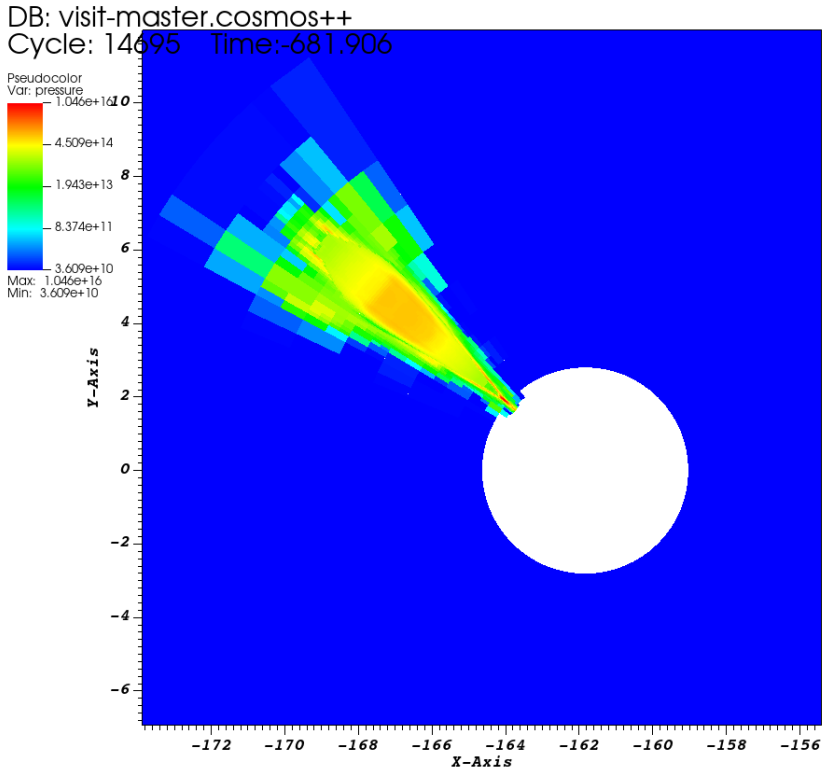
- Theoretical prediction: 1560s
- Measured: 1371s
- Agreement ~12%
- It behaves as expected!



Shock search: After passthrough

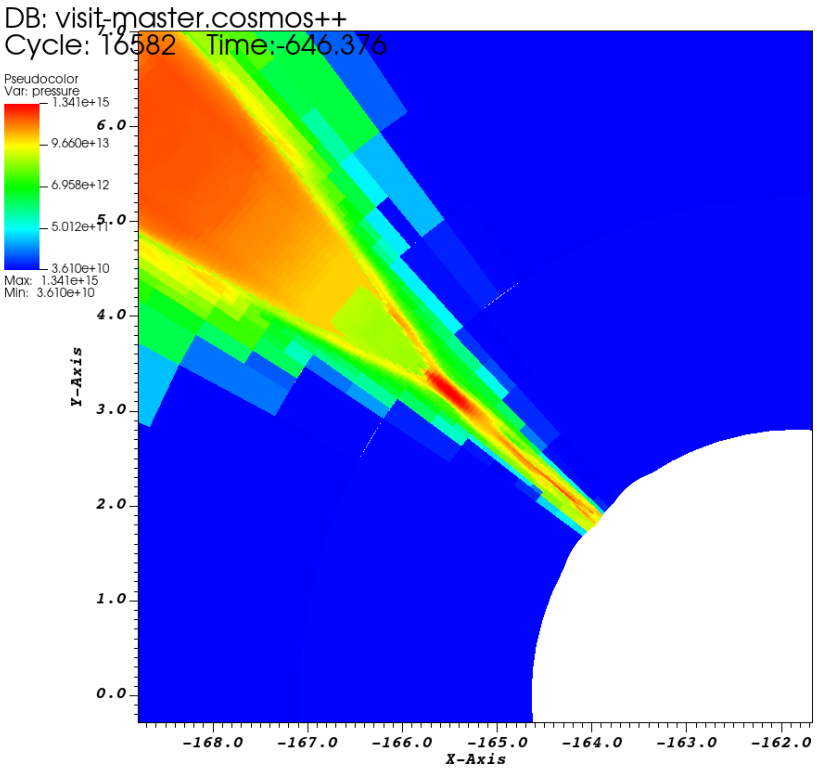


Density

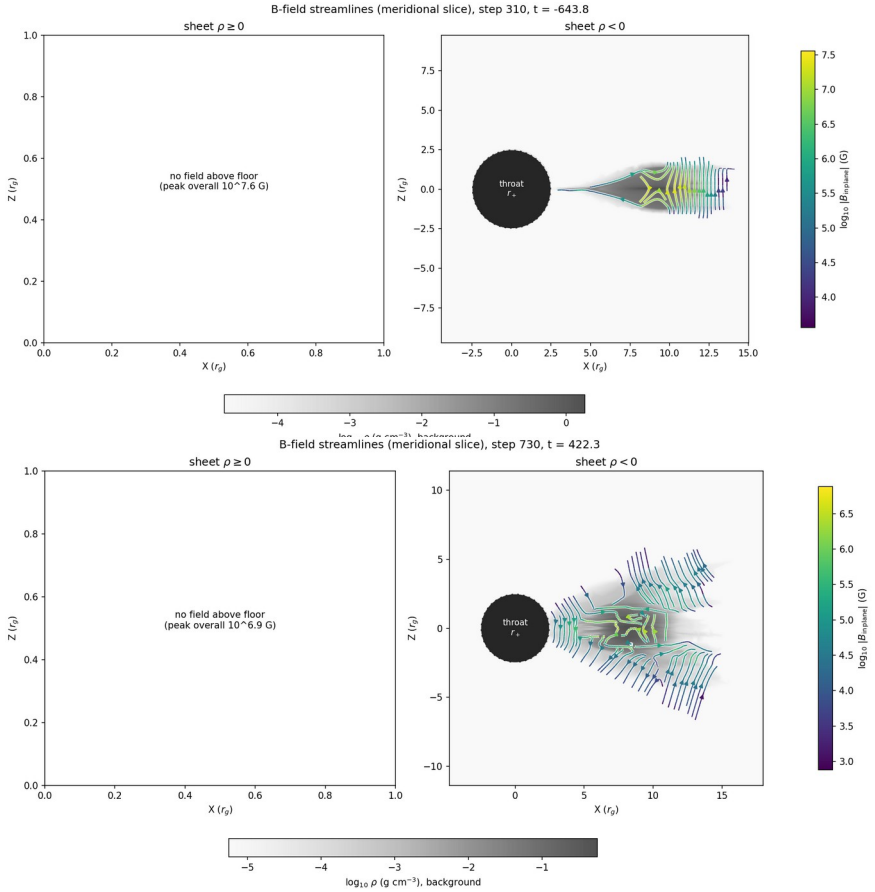


Pressure

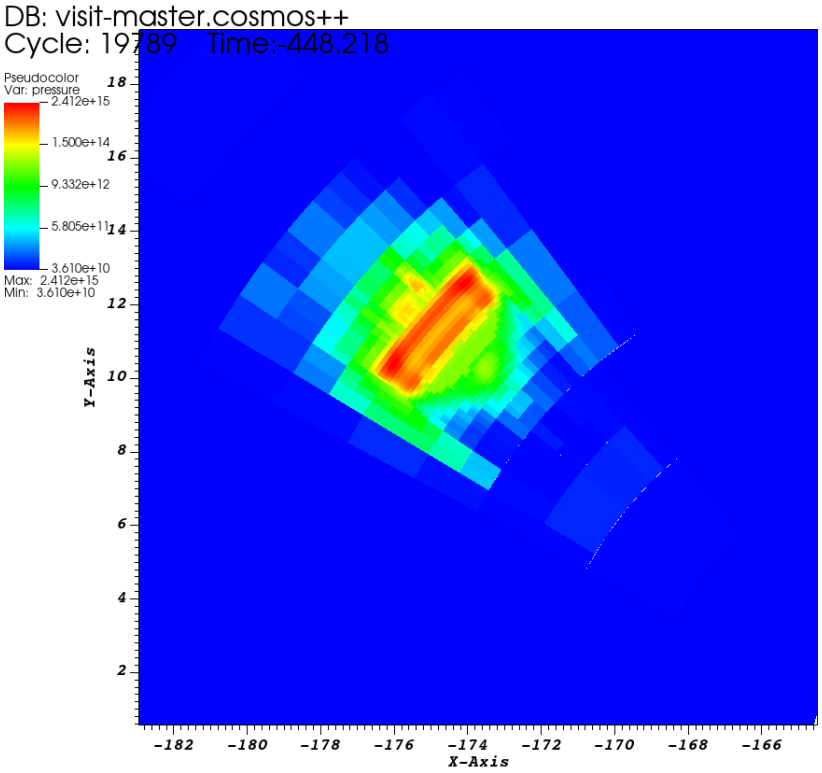
After passthrough: Magnetic field



Pressure

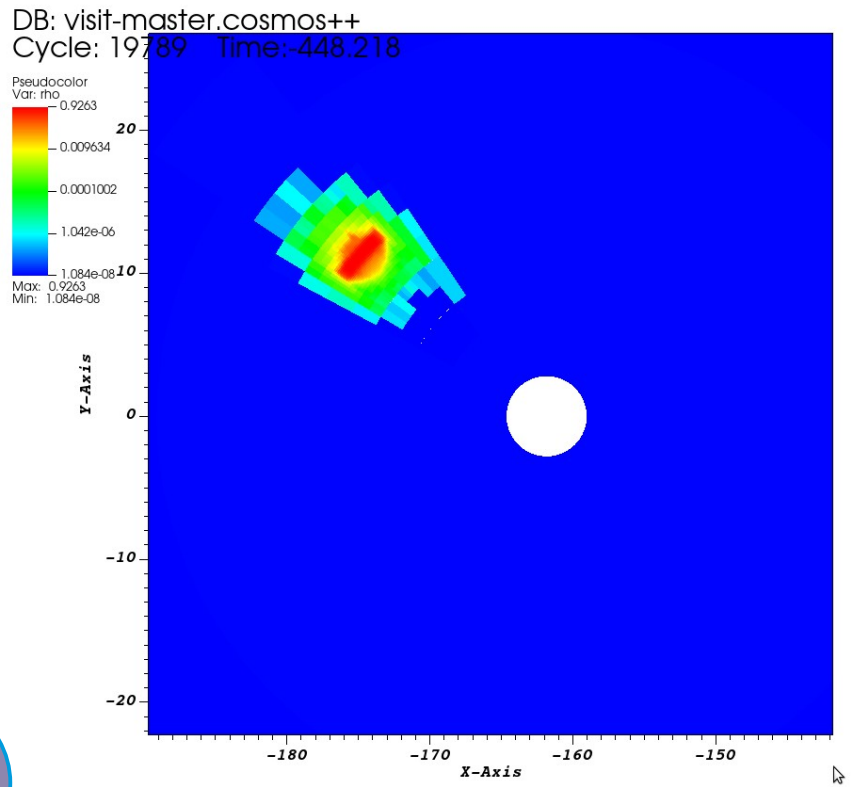


Shock search: Maximum extension



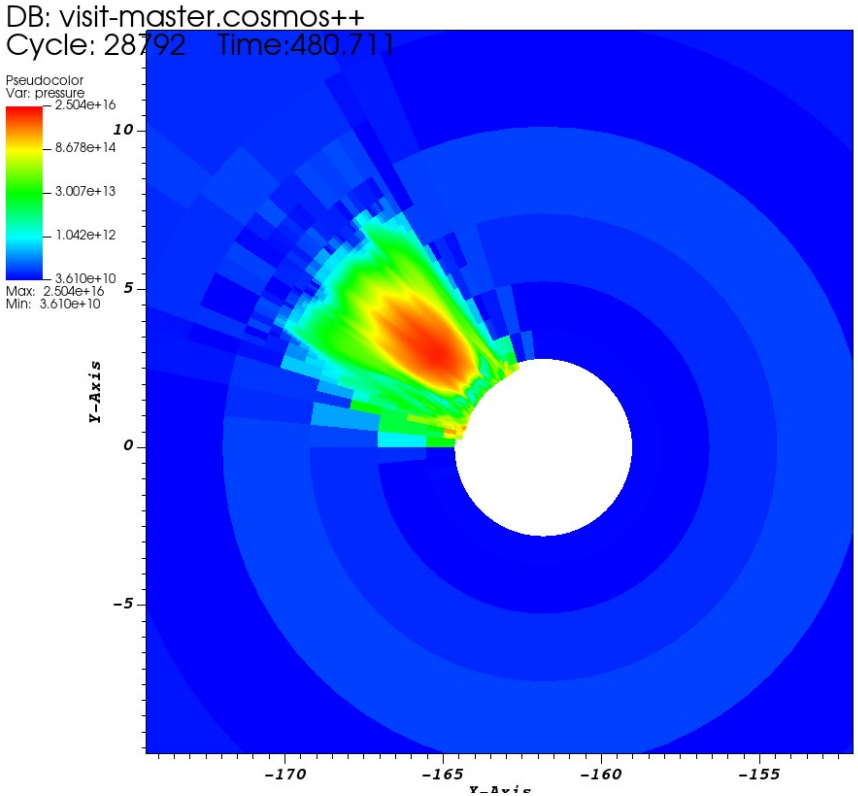
Pressure

WH

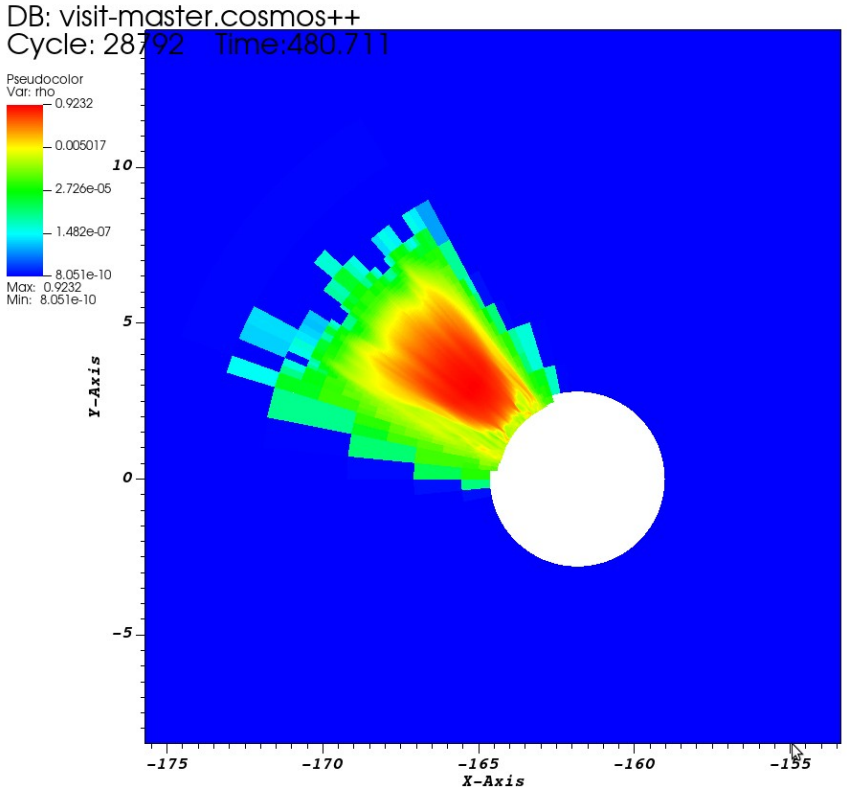


Density

Shock search: Second throat passthrough

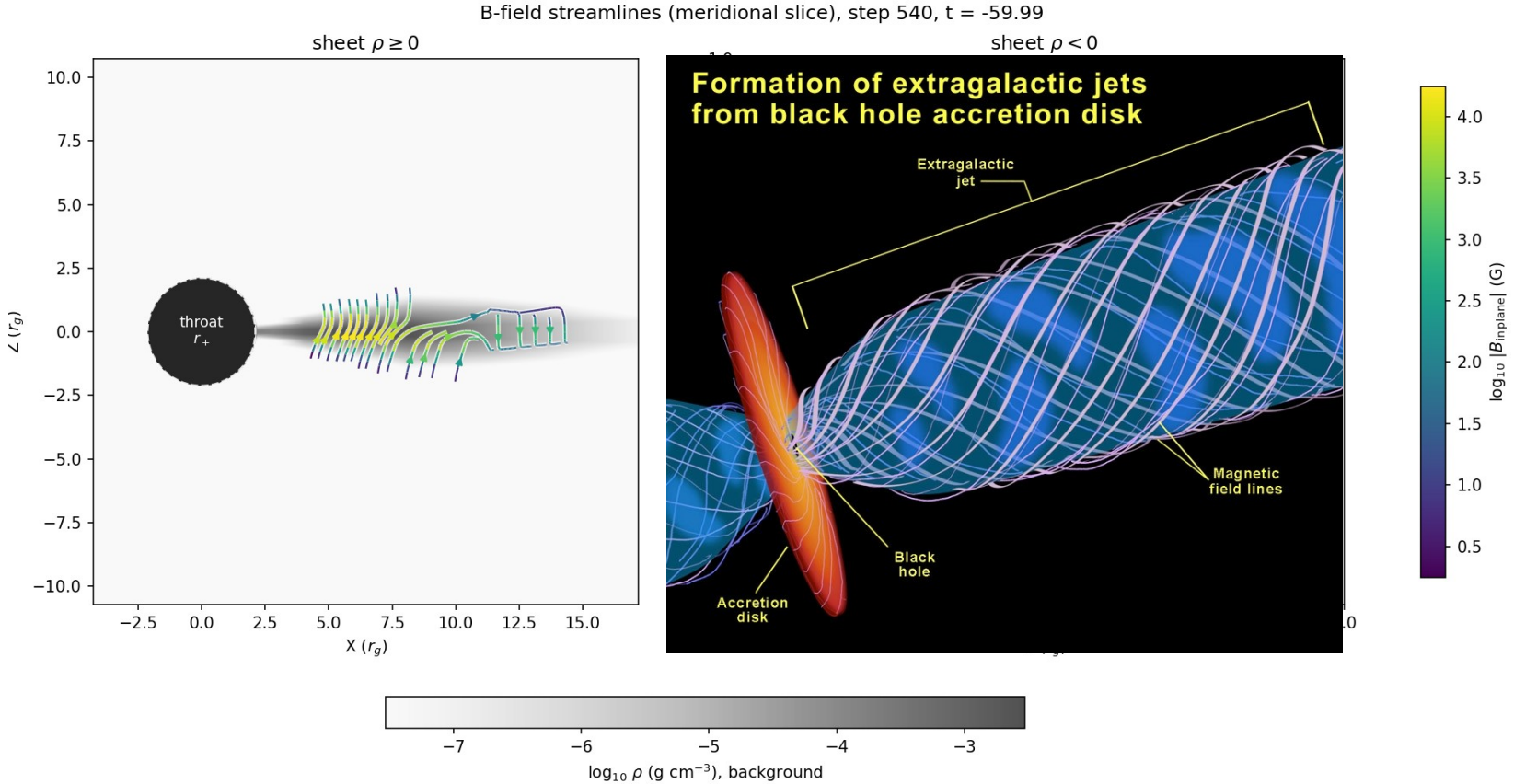


Pressure



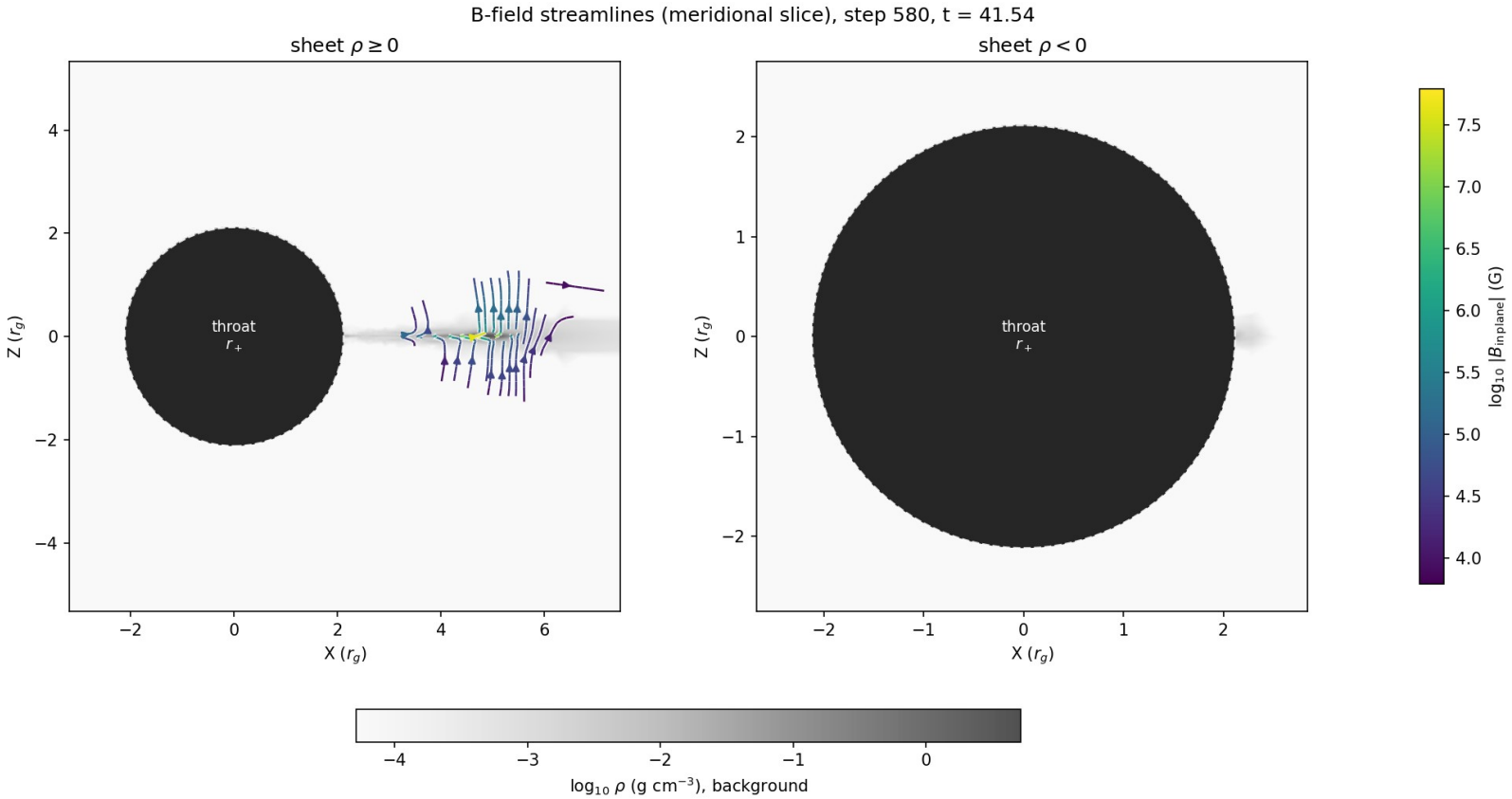
Density

Preview: Spin 0.9 run



[5] Image source: NASA

Preview: Spin 0.9 run



Takeaways

- Star will pass through throat and oscillate as predicted
- Star can not retrain its shape
- Some shocks found but for radiation calculation ray tracing is needed
- Rotating wormhole acts like a funnel but no jets (yet?)