

SLEZSKÁ UNIVERZITA
FYZIKÁLNÍ ÚSTAV V OPAVĚ

Quantum spacetime effects on black hole ringdown

Based on: PLB 854 (2024) 138716; JHEP 06 (2024) 130; JHEP 05 (2025) 083; and ongoing work

With N. Herceg, T. Jurić, A. Samsarov, I. Smolić, and K. S. Gupta

---

A. Naveena Kumara

Rudjer Bošković Institute,
Zagreb, Croatia

Institut Ruđer Bošković

HrZZ

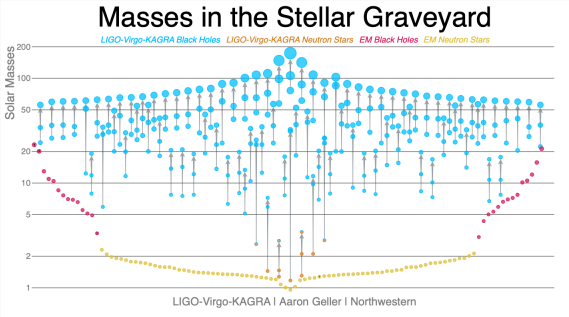
This research was supported by the Croatian Science Foundation
Project No. IP-2025-02-8625, *Quantum aspects of gravity*

Table of contents

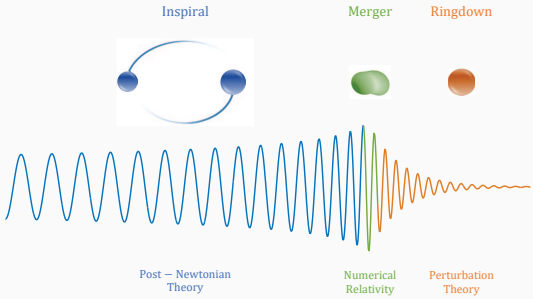
1. Black hole perturbations and ringdown
2. Noncommutative spacetime and geometry
3. *q*uantum gravitational perturbations
4. *q*uantum Kerr ringdown
5. Ringdown phenomenology
6. Conclusions

Black hole perturbations and ringdown

Motivation: gravitational waves and black hole spectroscopy



Reported gravitational-wave detections, as well as black holes and neutron stars previously studied through electromagnetic observations (up to the end of O3; GWTC-4/5 have since added many more). Credit: LIGO-Virgo / Aaron Geller / Northwestern University.
390 confirmed GW signals have been published by the LVK collaboration.



The temporal evolution of a binary system consists of three stages: the inspiral phase, during which the two objects orbit each other while gradually coming closer; the merger phase, in which they combine to form a single entity; and the ringdown phase (QNM), where the newly formed object settles into a stable equilibrium state. arXiv:1610.03567

Recent review article: arXiv:2505.23895

Black hole perturbations and quasinormal modes

1. Quasinormal modes

Ringdown $= \sum A_{\ell mn} e^{-i\omega_{\ell mn} t}$,
$\omega_{\ell mn} = \omega_R + i\,\omega_I$, $\omega_I < 0$;
$f = \omega_R/2\pi$, $\tau = 1/|\omega_I|$; ℓ, m angular, n overtone.

Schwarzschild $\ell = 2$:
$M\omega = 0.3737 - 0.0890\,i$; for
$M = 10 M_\odot$, $f \simeq 1.2\,\mathrm{kHz}$, $\tau \simeq 0.55\,\mathrm{ms}$.

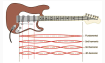
2. Black hole spectroscopy

For Kerr, every $\omega_{\ell mn}$ depends only on (M, χ): QNMs are the "footprints" of the black hole.

One mode $\Rightarrow M, \chi$; two or more $\Rightarrow$ a test of the Kerr hypothesis. Any extra dependence signals new physics.

CQG 21 (2004) 787; PRD 73 (2006) 064030

3. A string vs. a black hole



String: $f_n = nv/2L$, fixed by L and v.
Black hole: not self-adjoint, radiative boundary conditions $\Rightarrow$ complex ω, an incomplete set; a power-law tail follows the ringdown.

arXiv:2211.01103; PRD 5 (1972) 2419

4. Schwarzschild

Regge–Wheeler (axial) and Zerilli (polar) equations; Chandrasekhar transformations $\Rightarrow$ isospectrality.
Moncrief: gauge-invariant master functions.

PR 108 (1957) 1063; PRL 24 (1970) 737; Ann. Phys. 88 (1974) 323

5. Kerr: NP and Teukolsky

Kerr is Petrov type D, so the Weyl scalar ψ_4 decouples and separates:

$$\psi_4 \propto {}_{-2}R(r)\;{}_{-2}S_{\ell m}(\theta; a\omega)\; e^{-i\omega t + im\varphi}$$

and $\psi_4 \propto \ddot{h}_+ - i\ddot{h}_\times$ at infinity.

JMP 3 (1962) 566; PRL 29 (1972) 1114; ApJ 185 (1973) 635

6. QNM boundary conditions

$$\psi \sim \begin{cases} e^{-ik_H r_*}, & r_* \to -\infty \\ e^{+i\omega r_*}, & r_* \to +\infty \end{cases}$$

$k_H = \omega - m\Omega_H$. Solved by WKB or Leaver's continued fraction.

PRD 35 (1987) 3621; PRSLA 402 (1985) 285

Noncommutative spacetime and geometry

Why noncommutative spacetime?

1. Localization limit
Doplicher, Fredenhagen, and Robert:
Localizing an event within $\Delta x \sim \ell_P$ needs $E \sim \hbar c/\Delta x$: enough energy to form a horizon. Coordinates cannot be sharp simultaneously:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

PLB 331 (1994) 39; CMP 172 (1995) 187

2. A common thread
Strings, loop quantum gravity, asymptotic safety, causal dynamical triangulations, causal sets: the smooth continuum is expected to break down near the Planck scale.

NC geometry is a model-independent, effective way to encode this.

3. Twist deformation
Keep the manifold; deform the *product* of fields with a Drinfeld twist. Diffeomorphism covariance survives in twisted form.

Here: the Kerr background stays exact; only its perturbations feel the deformation.

4. The deformation parameter
q: dimension length^2; dimensionless $\hat{q} \equiv q/M^2$:

$$q = \hat{q}\left(\frac{GM}{c^2}\right)^2 \simeq 2.2 \times 10^6 \hat{q}\left(\frac{M}{M_\odot}\right)^2 \mathrm{m}^2$$

"$q$uantum" $\equiv q$-deformed.

5. Which scale?
Planck-scale $q \sim \ell_P^2$ gives $\hat{q} \sim 10^{-78}$–10^{-80} for stellar black holes: invisible.

Ringdown tests a mesoscopic scale: e.g. $\hat{q} = 0.2$ at $60M_\odot$ means $\sqrt{q} \approx 40$ km. Shifts are linear in $\hat{q}$, so any smaller scale follows by rescaling.

6. Why ringdown?
QNMs are set near the light ring and the horizon, where deviations from Kerr are expected to be largest.

Precision spectroscopy is improving fast (O4, then ET, CE, LISA).

Twisted geometry in a nutshell

1. Drinfeld twist
An invertible 2-cocycle $\mathcal{F} = f^{\alpha} \otimes f_{\alpha}$ deforms the Hopf algebra of diffeomorphisms $U\Xi$:

$$\Delta^{\mathcal{F}}(\xi) = \mathcal{F}\,\Delta(\xi)\,\mathcal{F}^{-1}$$

$$R = \mathcal{F}_{21}\mathcal{F}^{-1}, \qquad R^{-1} = \bar{R}^{\alpha} \otimes \bar{R}_{\alpha}$$

2. Abelian twist and $\star$-product

$$\mathcal{F} = e^{-\frac{i\eta}{2}(X_1 \otimes X_2 - X_2 \otimes X_1)}, \quad [X_1, X_2] = 0$$

$$f \star g = fg + \tfrac{i\eta}{2}\left[X_1 f X_2 g - X_2 f X_1 g\right]$$

R-symmetric: $f \star g = \bar{R}^{\alpha}(g) \star \bar{R}_{\alpha}(f)$.

3. $\star$-covariant derivative
$\mathbb{C}$-linear, with the twisted Leibniz rule

$$\nabla^{\star}_{V}(h \star z) = \pounds^{\star}_{V}(h) \star z + \bar{R}^{\alpha}(h) \star \nabla^{\star}_{\bar{R}_{\alpha}(V)} z$$

$\star$-torsion and $\star$-curvature follow (backup).

4. $\star$-inverse metric
$\widehat{g}^{\mu\rho} \star g_{\rho\nu} = \delta^{\mu}{}_{\nu}$; to $\mathcal{O}(\eta)$:

$$\widehat{g}^{\alpha\beta} = g^{\alpha\beta} - \tfrac{i\eta}{2}\Theta^{AB}(X_A g^{\alpha\sigma})(X_B g_{\sigma\gamma})\, g^{\gamma\beta}$$

5. Connection and curvature

$$\widehat{\Gamma}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\,\widehat{g}^{\rho\sigma} \star (\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu})$$

$$\widehat{R}^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\widehat{\Gamma}^{\rho}{}_{\nu\sigma} - \partial_{\nu}\widehat{\Gamma}^{\rho}{}_{\mu\sigma} + \widehat{\Gamma}^{\lambda}{}_{\nu\sigma} \star \widehat{\Gamma}^{\rho}{}_{\mu\lambda} - \widehat{\Gamma}^{\lambda}{}_{\mu\sigma} \star \widehat{\Gamma}^{\rho}{}_{\nu\lambda}$$

6. q-deformed vacuum
$\widehat{R}_{\mu\nu} = \widehat{R}^{\rho}{}_{\mu\rho\nu}$ is not symmetric $\Rightarrow$ use

$$\mathcal{R}_{\mu\nu} = \tfrac{1}{2}\left(\widehat{R}_{\mu\nu} + \widehat{R}_{\nu\mu}\right) = 0$$

(R-symmetrization for Abelian twists); $q \to 0$ gives $R_{\mu\nu} = 0$.

hep-th/0504183; hep-th/0510059; arXiv:0906.2774; arXiv:1210.1115

*q*uantum gravitational perturbations

q-deformed Einstein manifolds and the pseudo-semi-Killing twist

1. Field equation
Perturbations of $R_{\mu\nu} = 0$ become perturbations of $\mathcal{R}_{\mu\nu} = 0$, with the R-symmetrized Ricci tensor

$$\mathcal{R}_{\mu\nu} \equiv \tfrac{1}{2}\big\langle dx^\alpha,\, R^\star(\partial_\alpha, \partial_\mu, \partial_\nu) + R^\star(\partial_\alpha, \bar{R}^A(\partial_\nu), \bar{R}_A(\partial_\mu))\big\rangle_\star$$

2. Which twist?
Ideally fixed by experiment; for now, by the background symmetries.

Killing twist (both generators Killing): the background and modes adapted to its isometries stay undeformed at linear order; NC effects start only at $\mathcal{O}(h^2)$.

3. Pseudo-semi-Killing twist
One Killing generator K, and a commuting generator X that is *not* Killing:

$$\mathcal{F} = e^{-\frac{iq}{2}(K\otimes X - X\otimes K)}, \quad [K, X] = 0$$

The background stays an exact solution; NC corrections appear at **linear** order, $\mathcal{O}(qh)$.

4. Choice of vector fields
With Killing fields $\partial_t, \partial_\varphi$:

$$K = \alpha\,\partial_t + \beta\,\partial_\varphi, \qquad X = \partial_r$$

$$[t \overset{\star}{,} r] = iq\alpha, \qquad [\varphi \overset{\star}{,} r] = iq\beta$$

5. Action on the modes
For $h_{\mu\nu} \propto e^{-i\omega t + im\varphi}$:

$$\pounds_K h_{\mu\nu} = i\lambda\, h_{\mu\nu}, \qquad \lambda = -\alpha\omega + \beta m$$

The twist acts as an algebraic factor $q\lambda$ in the separated equations.

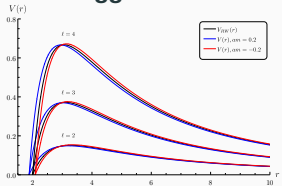
6. This work: the t–r twist
$\alpha = 1$, $\beta = 0 \Rightarrow \lambda = -\omega$; corrections enter as $q\omega$:

$$f \star g = fg + \tfrac{iq}{2}\left(\partial_t f\, \partial_r g - \partial_r f\, \partial_t g\right)$$

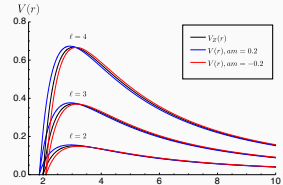
PLB 854 (2024) 138716; JHEP 06 (2024) 130; JHEP 05 (2025) 083

*q*uantum Schwarzschild potentials and isospectrality breaking

*q*uantum Regge–Wheeler



*q*uantum Zerilli

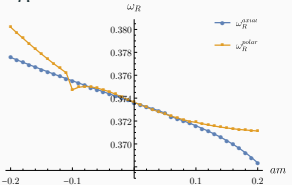


Isospectrality breaking

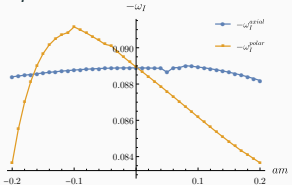
In GR, axial and polar modes share one spectrum. The deformation modifies the two potentials differently, so the spectra split:

$$\Delta\omega_{R,I} = 100 \times \frac{\omega_{R,I}^{\text{axial}} - \omega_{R,I}^{\text{polar}}}{\omega_{R,I}^{\text{polar}}}$$

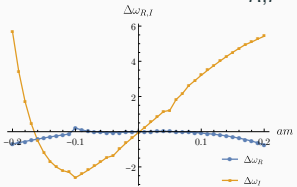
(a) ω_R vs. am



(b) ω_I vs. am



(c) Relative deviation $\Delta\omega_{R,I}$



$\ell = 2$, $n = 0$, $M = 1$ ($R = 2$); deformation enters as am; values at the optimal WKB order.

Improved QNM results in progress.

JHEP 05 (2025) 083

*q*uantum Kerr ringdown

Building Kerr ringdown on the Schwarzschild programme

2023	2023	2024	2024–25
*q*uantum Regge–Wheeler equation for axial modes	First *q*uantum QNM spectrum of Schwarzschild	*q*uantum Zerilli equation: axial–polar isospectrality broken	Near-horizon radial shift regularizes the NC boundary behaviour

Merger remnants spin, so the next step is Kerr

This work: the first systematic study of metric perturbations of a NC Kerr black hole.

Why slow rotation?

1. Teukolsky / Newman–Penrose decoupling is not established for the q-deformed equations.
2. Slow-rotation perturbation theory works directly with metric perturbations in tensor harmonics, expanding in χ.
3. At $\mathcal{O}(\chi)$, couplings to opposite-parity $\ell \pm 1$ modes do not shift the frequencies: the axial sector stands alone.

PLB 854 (2024) 138716; JHEP 06 (2024) 130; JHEP 05 (2025) 083

Kerr black hole and its symmetries

Kerr metric (Boyer–Lindquist), $\chi = J/M^2$

$$ds^2 = -\Big(1 - \frac{2Mr}{\Sigma}\Big)dt^2 - \frac{4M^2\chi\, r\sin^2\theta}{\Sigma}\, dt\, d\varphi + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2 + \Big(r^2 + \chi^2 M^2 + \frac{2\chi^2 M^3 r \sin^2\theta}{\Sigma}\Big)\sin^2\theta\, d\varphi^2$$

$$\Sigma = r^2 + \chi^2 M^2 \cos^2\theta, \qquad \Delta = r^2 - 2Mr + \chi^2 M^2$$

1. Horizons and limits

$\Delta = 0$: $r_\pm = M\big(1 \pm \sqrt{1-\chi^2}\big)$, event ($r_+$) and Cauchy ($r_-$) horizons; a black hole needs $|\chi| \le 1$.

$\Sigma = 0$: ring singularity at $r = 0$, $\theta = \pi/2$.

Vacuum ($R_{\mu\nu} = 0$), Petrov type D; $\chi \to 0$ gives Schwarzschild.

2. Killing symmetries

Stationary and axisymmetric: commuting Killing vectors ∂_t, ∂_φ, so any

$$K = \alpha\,\partial_t + \beta\,\partial_\varphi, \qquad \alpha, \beta \in \mathbb{R}$$

is Killing. The horizon is generated by

$$\xi_H = \partial_t + \Omega_H\,\partial_\varphi, \qquad \Omega_H = \frac{\chi}{2r_+}$$

3. Pseudo-semi-Killing twist

Pair K with the non-Killing ∂_r:

$$\mathcal{F} = e^{-\frac{iq}{2}(K\otimes\partial_r - \partial_r\otimes K)}$$

Kerr stays an exact solution. This work: $\alpha = 1$, $\beta = 0 \Rightarrow \lambda = -\omega$.

Expansion parameters: χ and $\hat{q} = q/M^2$.

Slow rotation to $\mathcal{O}(\chi)$

$$ds^2 = -F\,dt^2 + F^{-1}dr^2 - 2\varpi\sin^2\theta\, d\varphi\, dt + r^2 d\Omega^2, \qquad F = 1 - \frac{2M}{r}, \quad \varpi = \frac{2M^2\chi}{r}$$

PRL 11 (1963) 237; JMP 8 (1967) 265; PRD 86 (2012) 104017; IJMPA 28 (2013) 1340018

From field equations to a radial problem

1. Axial ansatz

Odd parity, Regge–Wheeler gauge ($h_2^\ell = 0$), $e^{-i\omega t}$, $A \in \{\theta, \varphi\}$:

$$\delta g_{tA} = h_0^\ell(r)\, S_A^\ell, \qquad \delta g_{rA} = h_1^\ell(r)\, S_A^\ell$$

$$S_\theta^\ell = -\frac{\partial_\varphi Y^\ell}{\sin\theta}, \qquad S_\varphi^\ell = \sin\theta\, \partial_\theta Y^\ell$$

on the $\mathcal{O}(\chi)$ Kerr background.

IJMPA 28 (2013) 1340018

2. q-deformed field equations

$$\mathcal{R}_{\mu\nu} = \tfrac{1}{2}\big(\widehat{R}_{\mu\nu} + \widehat{R}_{\nu\mu}\big) = 0$$

with the t–r star product

$$f \star g = fg + \tfrac{i\hat{q}}{2}\big(\partial_t f\, \partial_r g - \partial_r f\, \partial_t g\big)$$

Keep $\mathcal{O}(h)$, $\mathcal{O}(\chi h)$, $\mathcal{O}(\hat{q} h)$, $\mathcal{O}(\chi\hat{q} h)$; drop h^2, χ^2, $\hat{q}^2$.

3. Angular structure

Harmonic identities reduce the angular dependence to $\{Y, \partial_\theta Y, \partial_\varphi Y / \sin\theta, X^\ell, W^\ell\}$:

$$\mathcal{R}_{\theta\varphi}: \; f_\ell \sin\theta\, \partial_\theta Y + g_\ell\, \partial_\varphi Y + s_\ell \frac{X^\ell}{\sin\theta} + t_\ell\, W^\ell = 0$$

likewise $\mathcal{R}_{t\theta}$, $\mathcal{R}_{r\theta}$; coefficients split as GR $+\hat{q}M^2\times$NC.

4. Orthogonality and projection

$$\int d\Omega\, Y^*_{\ell m} Y_{\ell' m'} = \delta_{\ell\ell'}\delta_{mm'}$$

$$\int d\Omega\, \gamma^{ab} \big(S_a^{\ell m}\big)^* S_b^{\ell' m'} = \ell(\ell+1)\, \delta_{\ell\ell'}\delta_{mm'}$$

m decouples; $\ell \pm 1$ (opposite parity) couplings do not shift ω at $\mathcal{O}(\chi)$.

5. Radial equations

$$\ell(\ell+1)\,\beta_\ell^{(r)} + im\big[(\ell-1)(\ell+2)\,\zeta_\ell^{(r)} + \widetilde{\alpha}_\ell^{(r)} + \eta_\ell^{(r)}\big] = 0$$

$$\ell(\ell+1)\, t_\ell + im\, g_\ell = 0$$

(the $t\theta$ equation is not independent). Solve the first for h_0', insert into the second.

6. Towards Schrödinger form

$$A\, h_1'' + B\, h_1' + C\, h_1 = 0$$

with A, B, C each split as GR $+\hat{q}M^2\times$NC. Set $h_1 = \mathcal{P}(r)\,\psi$, $n = dr_*/dr$,

$$\frac{\mathcal{P}'}{\mathcal{P}} = -\frac{1}{2}\Big(\frac{B}{A} + \frac{n'}{n}\Big)$$

to remove $\psi' \Rightarrow$ the master equation.

The q-deformed axial master equation

Master equation

$$\frac{d^2\psi}{dr_*^2} + \left[\omega^2 - \frac{4m\chi M^2\omega}{r^3} - V_{\rm eff}(r,\omega)\right]\psi = 0, \qquad \frac{dr_*}{dr} = \frac{r}{r-2M} + \hat{q}\,\frac{M^3\omega}{(r-2M)^2}$$

$$V_{\rm eff} = V_{\rm axial} + \hat{q}M^2 V_q, \qquad V_{\rm axial} = F\left[\frac{L}{r^2} - \frac{6M}{r^3} + \frac{24m\chi M^2(3r-7M)}{L\,\omega\, r^6}\right], \qquad L = \ell(\ell+1)$$

$$V_q = \omega^2\frac{m\chi M^2}{r^4}\left(6 - \frac{3}{L}\right) + \frac{\omega}{2r^5}\left[2Lr^2 - 2(3L+5)Mr + 32M^2\right] + \frac{3m\chi M^2}{4Lr^9}\left[\dots\right]$$

Pure NC
The $\propto \omega$ term survives at $\chi = 0$. The tortoise coordinate has a double pole at $r = 2M$ (handled by a shifted coordinate in Leaver's method).

Mixed $\chi\hat{q}$
Rotation adds a new ω^2 dependence to the NC potential.

Limits
$\hat{q} \to 0$: slow Kerr; $\chi \to 0$: q-Schwarzschild; both: Regge–Wheeler.

The perturbative QNM spectrum

WKB, third order — independent check

- ω-dependent $Q(r,\omega)$: solve the complex peak $Q_1 = 0$ and the WKB condition together.
- Sector-by-sector tortoise derivatives: no spurious χ^2, $\hat{q}^2$ terms.

PRD 41 (1990) 374; PRD 113 (2026) 104052

Leaver continued fraction — primary benchmark

- Shift $\rho = r - \hat{q}M^2\omega/2$ removes the double pole; horizon wavenumber gets an NC term: $\kappa_H = w - 2j + \frac{3\hat{q}w}{8L}$ ($w = 2M\omega$, $j = m\chi/4$).
- 8-term recurrence $\to$ 3-term via 5 Gaussian eliminations; $N \approx 300$.

PRSLA 402 (1985) 285; JHEP 07 (2026) 075

$$M\omega_{\ell m} = \omega_{00}^{(\ell)} + m\chi A_\ell + \hat{q}B_\ell + m\chi\hat{q}\, C_\ell + \mathcal{O}(\chi^2, \hat{q}^2)$$

ℓ	$\omega_{00}^{(\ell)}$ (Schw.)	A_ℓ (rotation)	B_ℓ (pure NC)	C_ℓ (mixed)
2	$0.373672 - 0.088962i$	$0.062883 + 0.000998i$	$0.006834 - 0.001714i$	$-0.004048 + 0.000057i$
3	$0.599443 - 0.092703i$	$0.067355 + 0.000652i$	$0.007578 - 0.001467i$	$0.000142 - 0.000428i$
4	$0.809178 - 0.094164i$	$0.069814 + 0.000340i$	$0.007855 - 0.001183i$	$0.001071 - 0.000386i$

Fundamental axial modes (Leaver); A_ℓ, C_ℓ are m-independent (m factored out). WKB3 agrees to $|\Delta\omega|/|\omega| \lesssim 0.22\%$ (table in backup).

How far can first-order rotation go?

Working range: $\chi \leq 0.6$

Commutative slow-rotation Leaver spectrum vs full Kerr, $\ell = 2$–4, all m: maximum componentwise error below 10%.

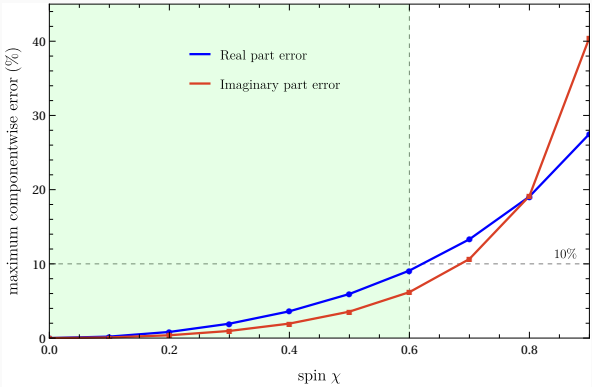
Maximum deviation from full Kerr

	Re	Im	
$\chi = 0.6$	9.1%	6.2%	within 10%
$\chi = 0.7$	13.3%	10.7%	outside

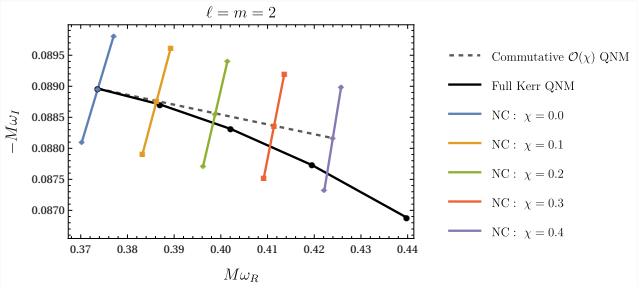
Sweet spot: $\chi \lesssim 0.2$

First-order rotation tracks Kerr closely.

This tests the commutative sector only; the accuracy of the NC terms beyond $\mathcal{O}(\chi)$ is not established by it.



Noncommutativity is not a change of spin



Reading the plot

Dashed: commutative $\mathcal{O}(\chi)$ spectrum; black dots: full Kerr; coloured segments: $-0.5 \leq \hat{q} \leq 0.5$ at fixed χ (co-rotating $\ell = m = 2$).

- Spin moves the mode mainly along the real axis.
- Noncommutativity moves it strongly in the damping direction.

q can mimic a spin change in one component, not in both.

Co-rotating: $m > 0$ for $\chi > 0$ (with $e^{-i\omega t + im\varphi}$), i.e. the mode travels with the black hole's rotation.

How $\hat{q}$ and spin shape the fundamental mode

Example: $\ell = m = 2$, $\chi = 0.1$

$\hat{q} = -0.2$: $0.385043 - 0.088422\,i$

$\hat{q} = +0.2$: $0.387453 - 0.089103\,i$

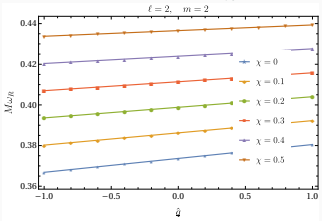
- $\hat{q} > 0$: higher frequency, stronger damping, shorter $\tau = 1/|\mathrm{Im}\,\omega|$.
- $\hat{q} < 0$: lower frequency, weaker damping, longer-lived modes.
- Stable: $\mathrm{Im}\,\omega < 0$ throughout the scanned range.

Slopes

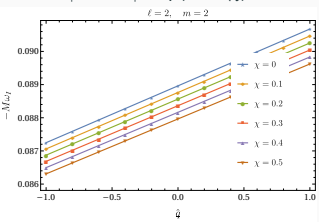
$$\left.\frac{\partial(M\omega)}{\partial\hat{q}}\right|_{\chi} = B_\ell + m\chi C_\ell, \quad \frac{\partial^2(M\omega)}{\partial\chi\,\partial\hat{q}} = mC_\ell$$

For $(2,2)$: $\partial_{\hat{q}}\mathrm{Re}(M\omega) \simeq 0.006834 - 0.008095\,\chi$: co-rotating spin weakens the shift; for $\ell = 3, 4$ it grows mildly.

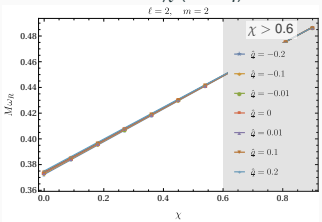
$\mathrm{Re}\,M\omega$ vs $\hat{q}$ (fixed χ)



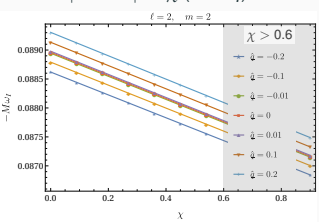
$|\mathrm{Im}\,M\omega|$ vs $\hat{q}$ (fixed χ)



$\mathrm{Re}\,M\omega$ vs χ (fixed $\hat{q}$)



$|\mathrm{Im}\,M\omega|$ vs χ (fixed $\hat{q}$)



$(\ell, m) = (2, 2)$. Shaded: $\chi > 0.6$, outside the slow-rotation range. Tables use $|\hat{q}| \leq 0.2$; wider $\hat{q}$ shown for the trend only.

Ringdown phenomenology

Kerr-anchored prescription for 11 LVK remnants

Slow-rotation error at the remnant spins

Event	χ	ϵ_R (%)	ϵ_I (%)
GWTC-3 · O3b			
GW191219_163120	0.14	+0.17	1.24
GW200210_092254	0.34	−2.88	0.71
GW200115_042309	0.42	−4.57	1.03
GW200105_162426	0.43	−4.80	1.10
GW191113_071753	0.45	−5.29	1.26
GWTC-5.0 · O4b			
GW240921_201835	0.52	−7.17	2.11
GW241110_124123	0.52	−7.17	2.11
GW240507_041632	0.55	−8.07	2.63
GW241127_061008	0.56	−8.38	2.83
GW240527_230910	0.58	−9.03	3.27
GW240930_035959	0.58	−9.03	3.27

ϵ: first-order slow rotation vs Kerr fit, $(2,2,0)$ mode. GW191219, GW200105, GW200115: NSBH candidates; GW200210 possibly NSBH; O4b events consistent with BBH.

Prescription

$$M\Delta\omega_q = M\left(\omega^{q,\mathrm{SR}} - \omega^{\mathrm{SR}}\right) = \hat{q}\,(B_2 + m\chi C_2)$$

$$\omega^{\mathrm{phen}}(\chi,\hat{q}) = \omega^{\mathrm{Kerr}}(\chi) + \Delta\omega_q(\chi,\hat{q})$$

$$\omega^{\mathrm{phen}}(\chi,0) = \omega^{\mathrm{Kerr}}(\chi)$$

$$f_{220} = \frac{M\omega_R}{2\pi M_f^{\mathrm{det}} t_\odot}, \qquad \tau_{220} = \frac{M_f^{\mathrm{det}} t_\odot}{|M\omega_I|}$$

$$t_\odot = GM_\odot/c^3 \simeq 4.925\,\mu\mathrm{s}$$

Why, and what it does

Why anchor? First-order rotation is off by up to 9% in frequency at $\chi = 0.58$ (left); that must not be mistaken for an NC shift.

Keeps: the full commutative spin dependence (Berti–Cardoso–Will fit) and the computed NC correction, unchanged.

Cannot fix: mixed terms from $\mathcal{O}(\hat{q}\chi^2)$ upward.

Predicted shifts and their signatures

$(2,2,0)$ shifts over the 11 remnants, $|\hat{q}| = 0.2$

Frequency: $|\delta f/f| \leq 0.29\%$ (0.087% – 0.29%)

Damping time: $|\delta\tau/\tau| \approx 0.39\%$ (0.38% – 0.39%)

Range over the remnants

$\hat{q}$	$\delta f/f$ ($\times 10^{-3}$)	$\delta\tau/\tau$ ($\times 10^{-3}$)
-0.2	$-2.92 \ldots -0.87$	$+3.83 \ldots +3.89$
-0.1	$-1.46 \ldots -0.44$	$+1.91 \ldots +1.94$
0	0	0
$+0.1$	$+0.44 \ldots +1.46$	$-1.93 \ldots -1.91$
$+0.2$	$+0.87 \ldots +2.92$	$-3.86 \ldots -3.80$

Well inside current LVK parameterized-ringdown uncertainties: an effect-size estimate, not a bound.

Signatures

- **Linear scaling:** $\delta f/f \simeq (0.004\text{–}0.015)\hat{q}$, $\delta\tau/\tau \simeq -0.019\hat{q}$. A 1% measurement probes $\hat{q} \sim 0.5\text{–}0.7$; $\hat{q} = 10^{-5}$ gives shifts $\sim 10^{-7}$ ($\sqrt{q} \approx 0.3$ km at $60M_\odot$).
- **Damping time is nearly universal**: $\delta\tau/\tau$ barely depends on spin ($\mathrm{Im}\, C_2 \approx 0$), while $\delta f/f$ does, through $\mathrm{Re}\, C_2$.
- **Sign pattern:** $\mathrm{sgn}\,\delta f = -\,\mathrm{sgn}\,\delta\tau$ for $\chi \lesssim 0.84$: a falsifiable correlation.
- **Fixed q vs fixed $\hat{q}$:** if q is universal, $\hat{q} \propto 1/M^2$: lighter remnants are more sensitive.

Illustrative mapping: GW250114

Inputs
Loudest event to date: network SNR ≈ 80, remnant spin $\chi = 0.68$, $M_f^{\rm det} = 68.1\, M_\odot$; reported central deviations $\delta f = 0.004$, $\delta\tau = 0.010$.

Kerr-anchored Leaver sensitivities at $\chi = 0.68$:

$$\frac{\delta f}{f} \simeq 0.002523\hat{q}, \qquad \frac{\delta\tau}{\tau} \simeq -0.019910\hat{q}$$

Formal mapping
From the frequency:
$q \approx 1.60 \times 10^{10}\,\mathrm{m}^2$ $\qquad \sqrt{|q|} \approx 127$ km

From the damping time:
$q \approx -0.51 \times 10^{10}\,\mathrm{m}^2$ $\qquad \sqrt{|q|} \approx 71$ km

Compare: Schwarzschild radius ≈ 201 km.

Sign-pattern check
Both central deviations are positive, but the model predicts opposite signs: no single $\hat{q}$ fits both, hence the opposite-sign mappings.

Illustrative only: catalog central values without posteriors; $\chi = 0.68$ exceeds the slow-rotation range; the implied $\hat{q} \approx 1.6$ and -0.5 are not small. Similar to the LVK mappings for EdGB/dCS, not a constraint.

Conclusions

Summary and outlook

Summary

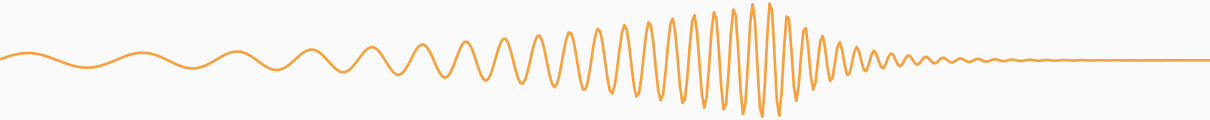
- **Framework:** twist-deformed geometry and $\mathcal{R}_{\mu\nu} = 0$; the pseudo-semi-Killing twist keeps Kerr exact and deforms perturbations at $\mathcal{O}(qh)$.
- **Schwarzschild:** $q$uantum Regge–Wheeler and Zerilli potentials; isospectrality broken.
- **Kerr (this work):** first metric perturbations of a $q$uantum Kerr black hole, to $\mathcal{O}(\chi, \hat{q}, \chi\hat{q})$, with a frequency-dependent potential and a new $\chi\hat{q}\,\omega^2$ term.
- **Spectrum:** Leaver and WKB3 agree; $\hat{q} > 0$ raises frequency and damping; C_ℓ makes the spin–NC interplay ℓ-dependent; not a change of spin; all modes stable.
- **Phenomenology:** $|\delta f/f| \lesssim 0.3\%$, $|\delta\tau/\tau| \approx 0.4\%$ at $|\hat{q}| = 0.2$ for 11 LVK remnants; linear in $\hat{q}$, with a spin-insensitive $\delta\tau$ and the sign pattern $\operatorname{sgn}\delta f = -\operatorname{sgn}\delta\tau$.

Outlook

- **Polar sector:** isospectrality for rotating q-deformed black holes.
- **Overtones, multipoles** and **mode ratios**: mass dependence if q is universal.
- **General twist:** $X_1 = \alpha\partial_t + \beta\partial_\varphi$, $X_2 = \partial_r$.
- **Rapid rotation:** beyond $\mathcal{O}(\chi)$, full numerical methods.
- **Bounds on q:** dedicated analysis with ringdown posteriors.
- **Greybody factors:** Hawking emission and evaporation.

Thank you

Questions and discussion



A. Naveena Kumara · nathith@irb.hr

Quantum Gravity Group
Division of Theoretical Physics
Rudjer Bošković Institute
Bijenička c.54, HR-10002 Zagreb, Croatia

Institut Ruđer Bošković