Extreme mass ratio inspirals: accessing generic orbits

Ondřej Zelenka, Michal Stratený, Petr Kotlařík, Georgios Lukes-Gerakopoulos

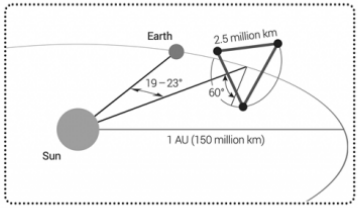
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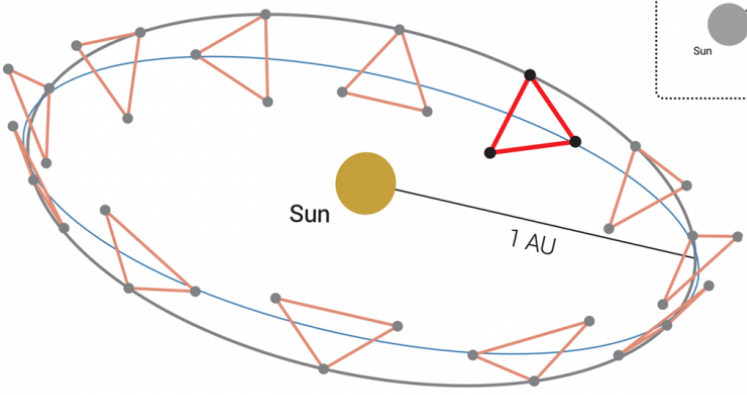
4 November 2026
RAGtime 28, Opava

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LISA mission



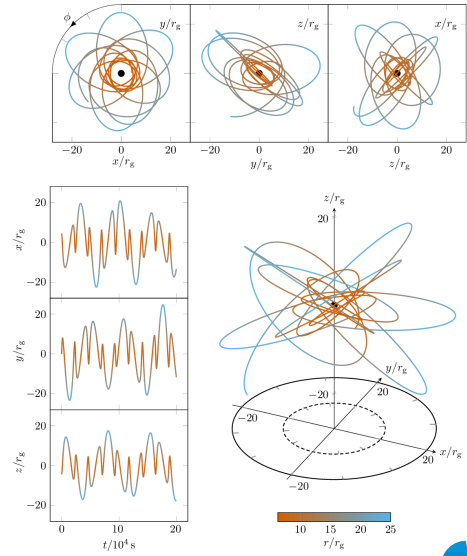


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Extreme Mass Ratio Inspirals (EMRIs)

- compact binary
  - primary: supermassive black hole
  - secondary: stellar mass black hole/neutron star
- $\mu/M \lesssim 10^{-5}$
- complex dynamics, adiabatic models
  - test particle limit
  - averaged radiation reaction (Teukolsky)
  - slow dissipation



Berry et al. (2019), arXiv:1903.03686

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Teukolsky equation

separates in the frequency domain

$$\psi_4 = (r - ia\cos\theta)^{-4} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \int \mathrm{d}\omega R_{lm}(r,\omega)\, S_{lm}(\theta,\omega)\, e^{-i\omega t + im\phi}$$

radial equation

$$\left[\Delta^2 \frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{-1}\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{K^2 + 4i(r-M)K}{\Delta} - 8i\omega r - \lambda_{lm}(\omega)\right] R_{lm}(\omega) = T_{lm}(\omega)$$

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Source: particle on a geodesic

Also separates with appropriate parameterization

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 = V_r(r) \ , \quad \left(\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}\right)^2 = V_\theta(\theta)$$

Fourier expansion:

$$r(\lambda) = \sum_{n=-\infty}^{\infty} r_n e^{-in\Upsilon_r \lambda} \ , \quad \theta(\lambda) = \sum_{k=-\infty}^{\infty} \theta_k e^{-ik\Upsilon_\theta \lambda}$$

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Flux computation

- fast: e.g. FEW
- precomputed $Z^{H,\infty}_{lmkn}(a, p, e, x = \cos I)$
- densely cover 8-dimensional grid
- lot of computations!

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle^{\infty} = \sum_{l,m,k,n} \frac{1}{4\pi\omega^2_{mkn}} \left| Z^{H}_{lmkn} \right|^2$$

$$\left\langle \frac{\mathrm{d}J_z}{\mathrm{d}t} \right\rangle^{\infty} = \sum_{l,m,k,n} \frac{1}{4\pi\omega^3_{mkn}} \left| Z^{H}_{lmkn} \right|^2$$

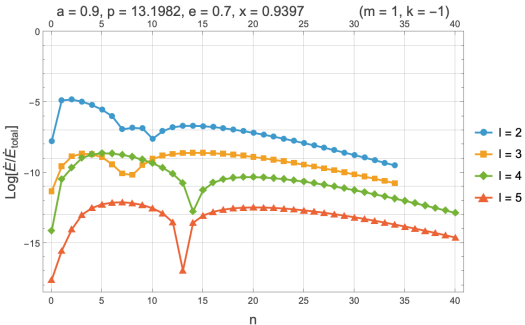
$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle^{H} = \sum_{l,m,k,n} \frac{1}{4\pi\omega^2_{mkn}} |Z^{\infty}_{lmkn}|^2$$

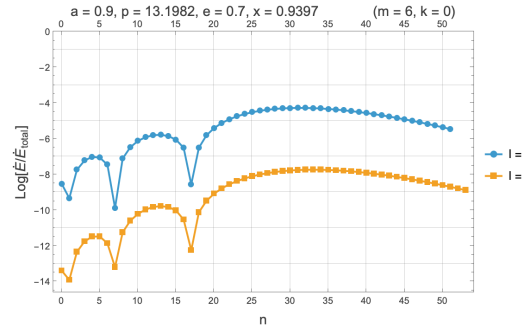
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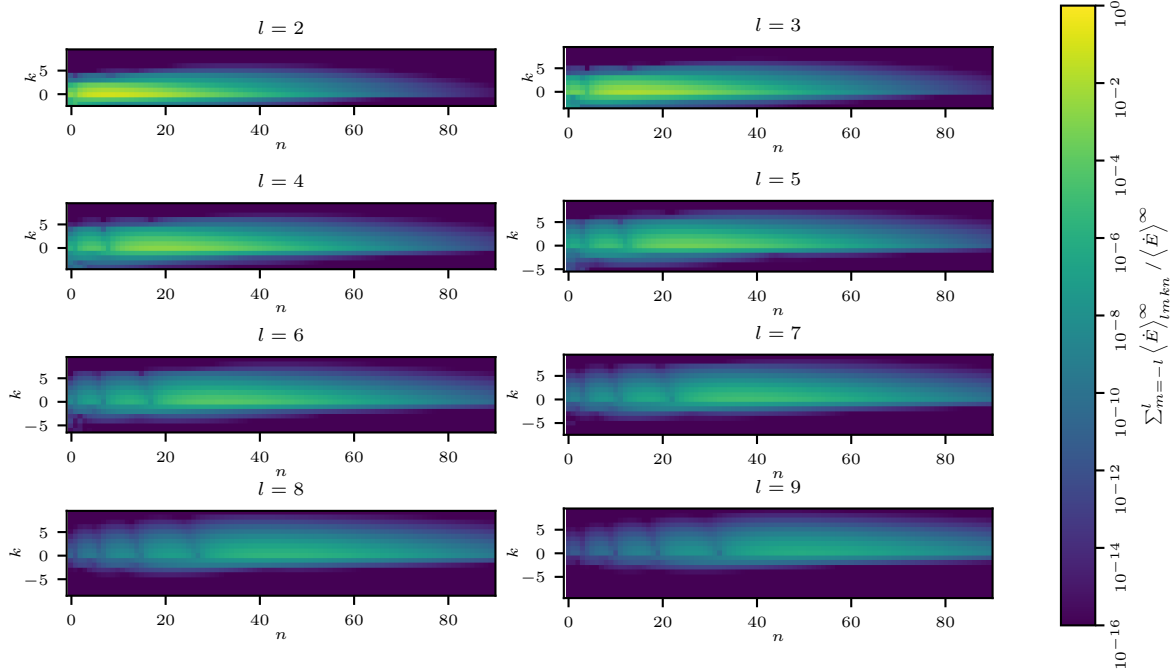
Examples

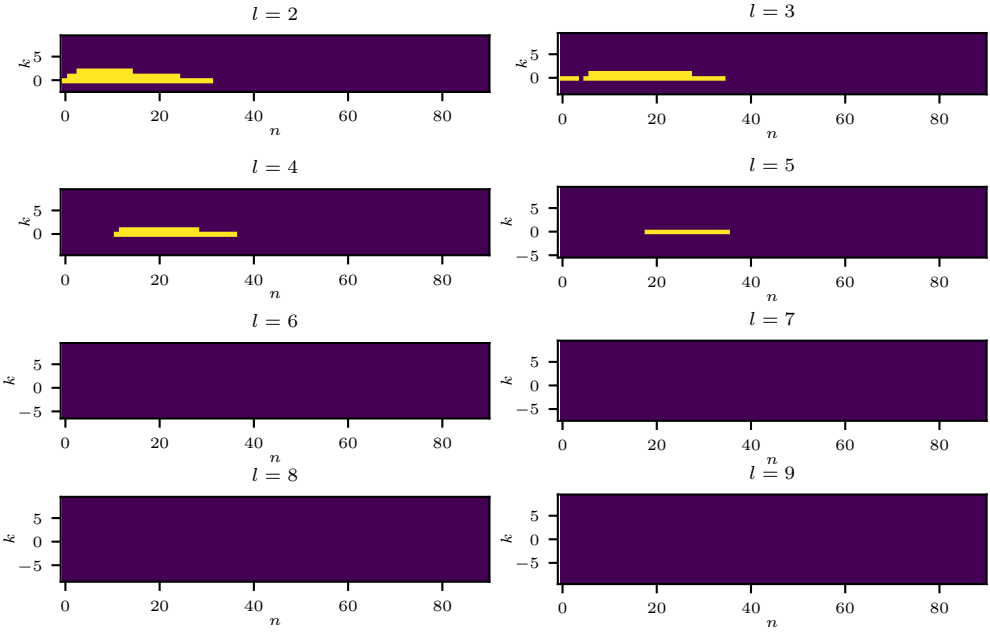




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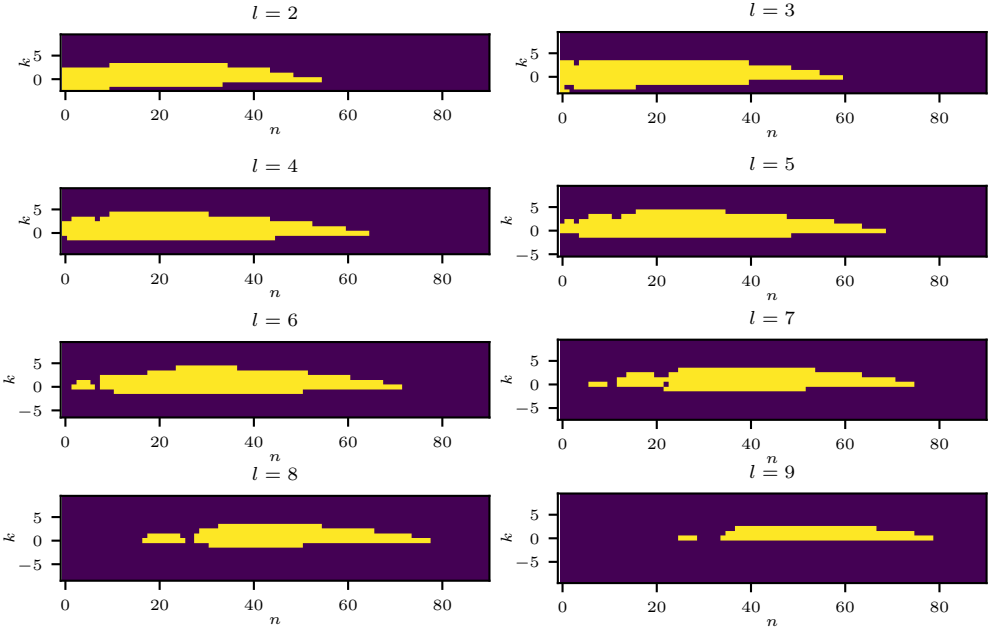
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$l = 2$
$l = 3$
$l = 4$
$l = 5$
$l = 6$
$l = 7$
$l = 8$
$l = 9$
k
n
$\sum_{m=-l}^{l} \left\langle \dot{E} \right\rangle_{lmkn}^{\infty} / \left\langle \dot{E} \right\rangle^{\infty}$



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Neural Networks

layers of artificial neurons

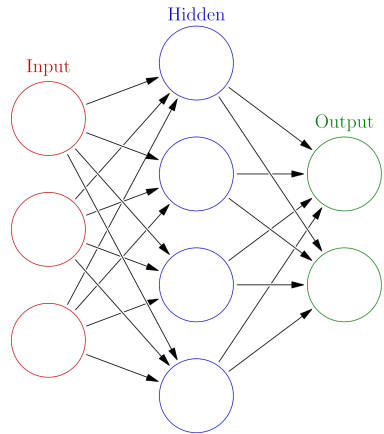
$$f(\mathbf{x}) = \sigma\left(\sum_j w_j x_j + b\right)$$

training:

- dataset
- loss function

$$\mathrm{MSE}\left(\bar{\mathbf{Y}}, \mathbf{Y}\right) = \frac{1}{N}\sum_{i=1}^{N}\left(\bar{Y}_i - Y_i\right)^2$$

- GD-based optimizer



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Can we exclude some modes?

- typically dominant $l = \pm m$
- can we find dominant n for a given l, m, k?
- consider $n_{\max}$ produces the maximal energy flux
- simple model (193 parameters):

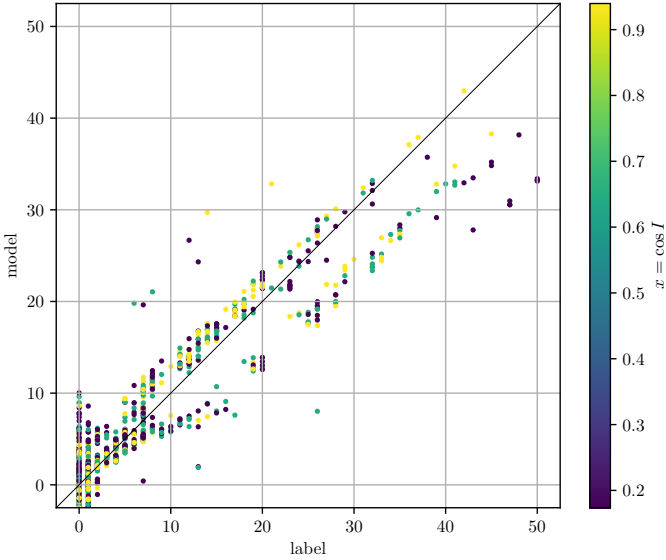
$$\alpha, \beta, \gamma = \mathrm{NN}\left(a, p, e, x\right)$$
$$n_{\max} = \alpha l + \beta m + \gamma k$$

- $4 \to 10 \to 10 \to 3$

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Simple model



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Slightly more complex

- non-linear

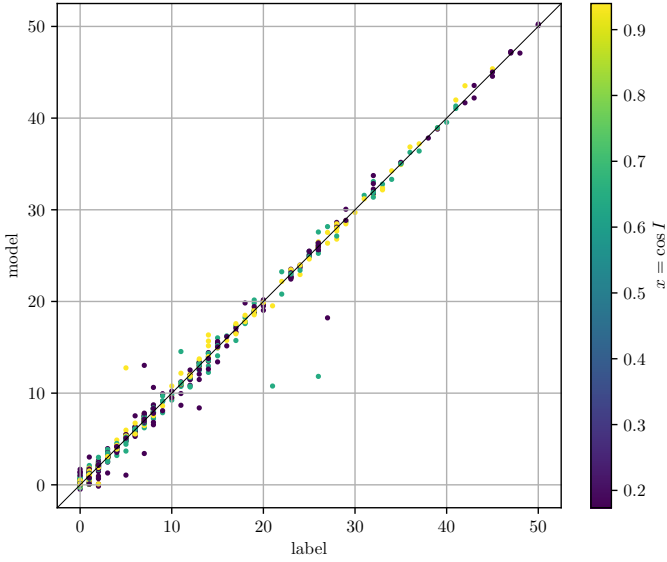
$$n_{\max} = \mathrm{NN}\left(a, p, e, x, l, m, k\right)$$

- $7 \to 20 \to 50 \to 20 \to 1$
- 2251 parameters

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Slightly more complex



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Conclusion/outlook

- if reliable - significant computational savings
- optimization + more data!
- high eccentricity
- not linear in l, m, k

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