Modeling Dense Nuclear Matter with a Modified Excluded Volume Equation of State

Based on A. Kumar et al. (doi:10.1088/1361-6382/aebd5d) (2026)

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Outline

Introduction
- Nuclear matter EOS for Supernova simulations
- Phase transition to quark matter and State of the art
- EOS uncertainties at high density

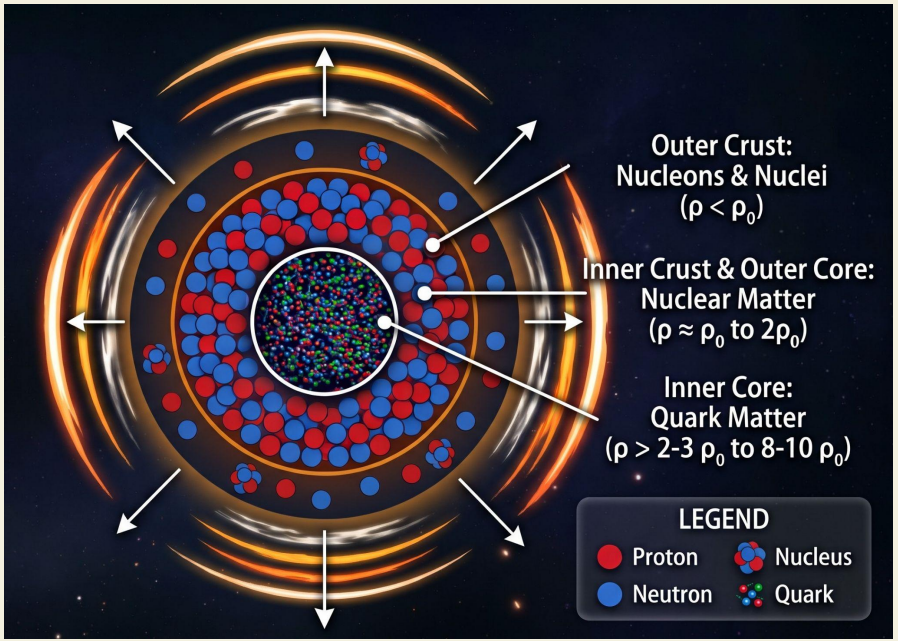
My approach
- Concept of excluded volume
- EOS with modified excluded volume

Supernova simulations
- Impacts on shock revival
- Impacts on neutrino ejecta
- Asteroseismology

State of matter in Supernova

- A huge shock is generated to collision of infalling matter on core
- Density of core increases with time
- Nucleons exist in outer region along with nuclei
- At higher density nuclei decompose into uniform nuclear matter
- Quark matter may exist in most dense region of core or inner core

Tobias' talk

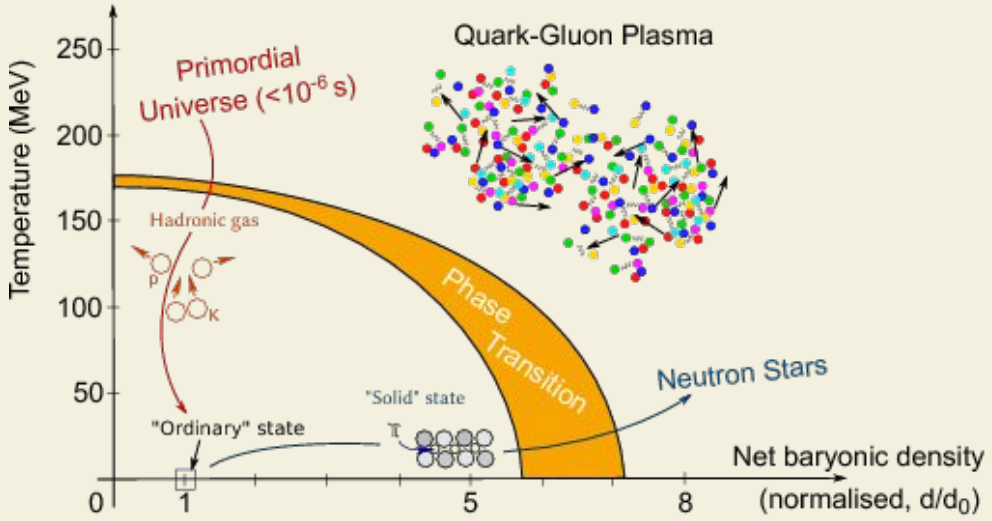


Source: AI generated image

QCD phase transition and State of the Art

- Phase Transition: nuclear matter to quark matter
- Nuclear matter: RMF models for description
- Quark matter is modeled with BAG models and NJL models
- RDF models use a string flip model for QM

RDF models: Noshad (Andia) talk



Source:https://cds.cern.ch/record/2025215

Uncertainties at high density

- **BAG model does not explain chiral symmetry breaking**
- **NJL model lacks of confinement at low density**
- **RDF model density dependence need improvements**

$$U_{\text{eff}} = U(n_s, n_v) - \frac{\partial U}{\partial n_s} n_s - \frac{\partial U}{\partial n_v} n_v$$

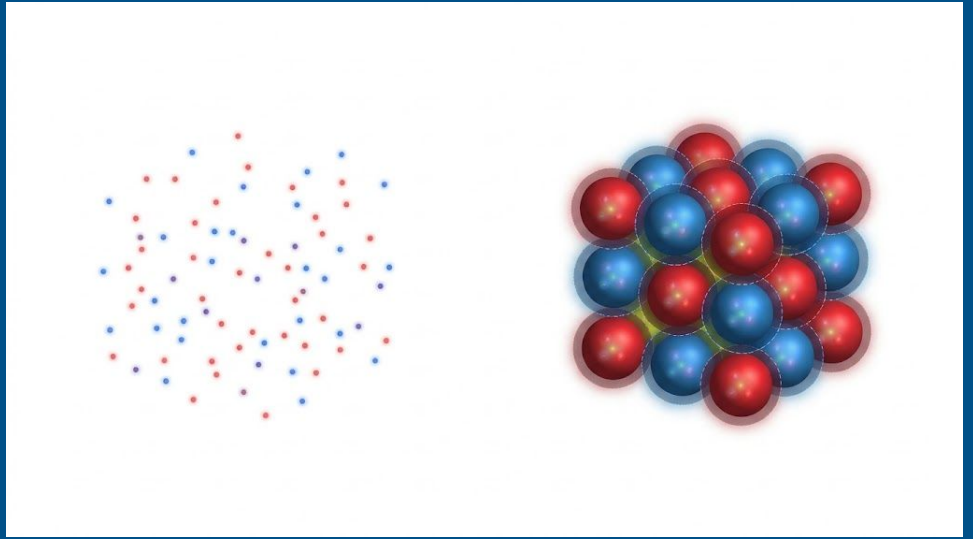
$$U(n_s, n_v, n_{vi}) = C n_s^{4/3} + D(n_v) n_s^{2/3} + \omega_4 n_v^2 + \frac{\omega_8 n_v^4}{1 + \omega_8' n_v^2} + \rho_4 n_{vi}^2$$

U. Shukla and P.M. Lo JSPC 3, 100058 (2025)

Source: ***N.F. Bastian*** PRD 103,023001 (2021)

Excluded Volume

$$\Phi = V'/V$$



Source: AI generated image

EOS with Modified Excluded volume

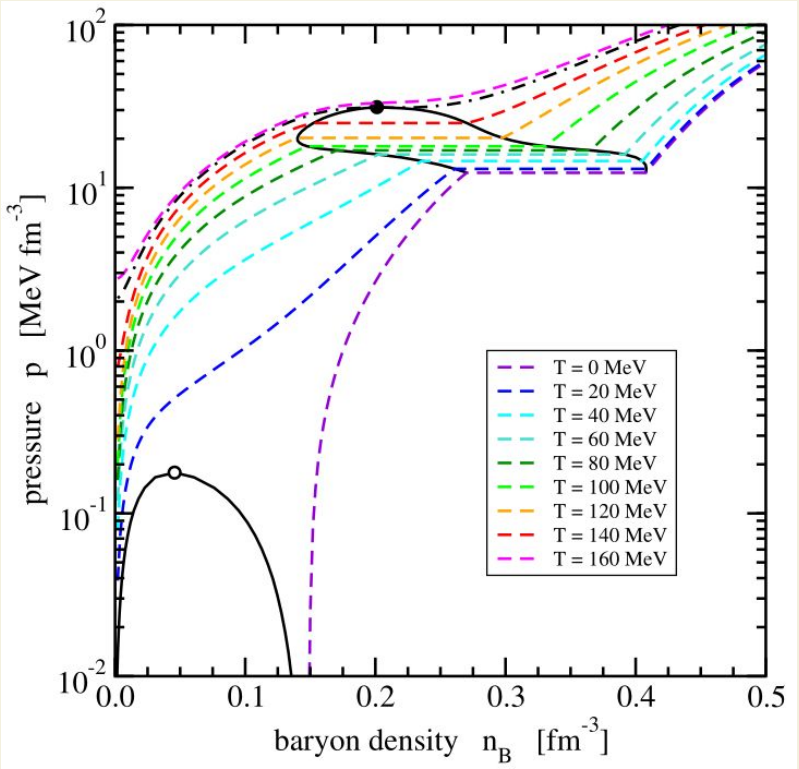
$$\Phi_N(T,x) = 1 + \mathcal{S}\, g_1(T)\, \Theta(x)\, \exp\left(-\frac{1}{2x^2}\right)$$

$$x = v\,(n_p^{(v)} + n_n^{(v)} - g_8(T)\, n_{cut})$$

$$g_t(T) = \Theta(T_0 - T)\, \exp\left[-\frac{t}{2}\left(\frac{T}{T_0 - T}\right)^2\right]$$

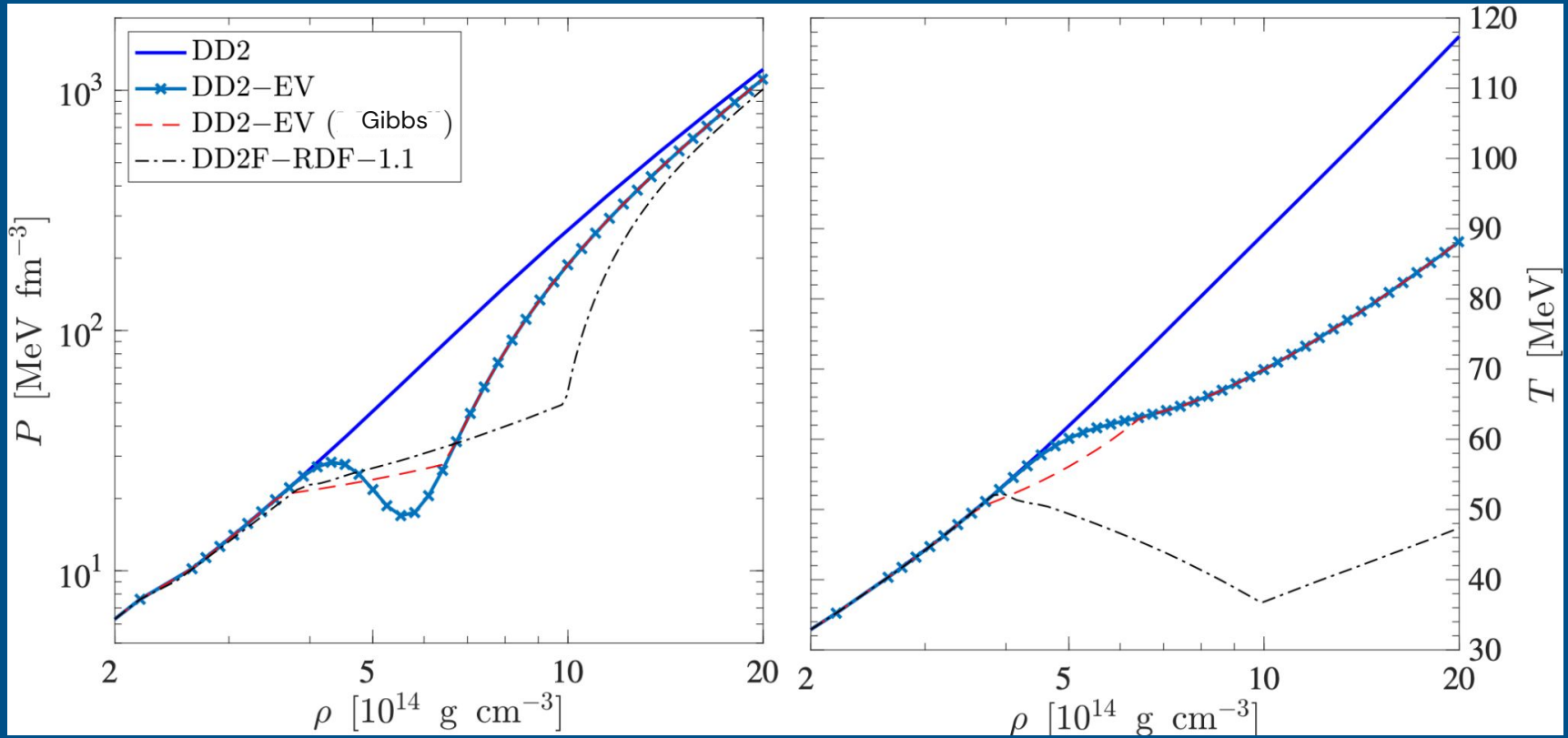
Gibbs Equilibrium conditions

$$T_1 = T_2$$
$$P_1 = P_2$$
$$\mu_1 = \mu_2$$

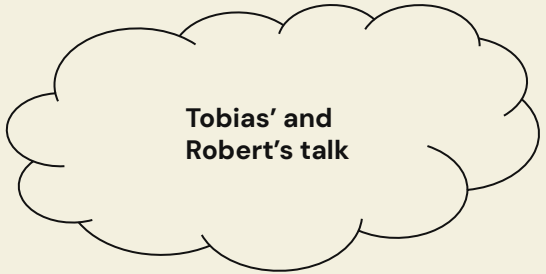


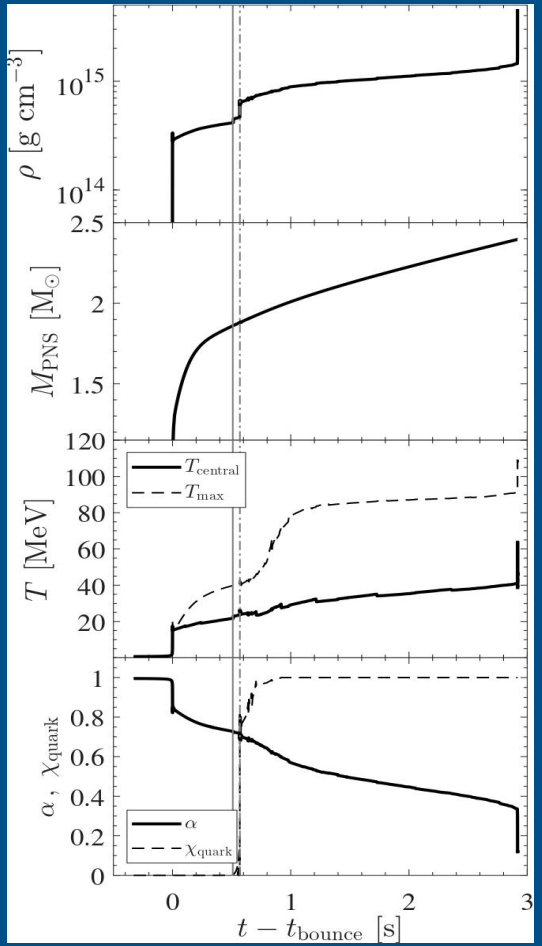
S. Typel et al Universe 2018, 4(2), 32

(s = 3 kB, Yp = 0.3)

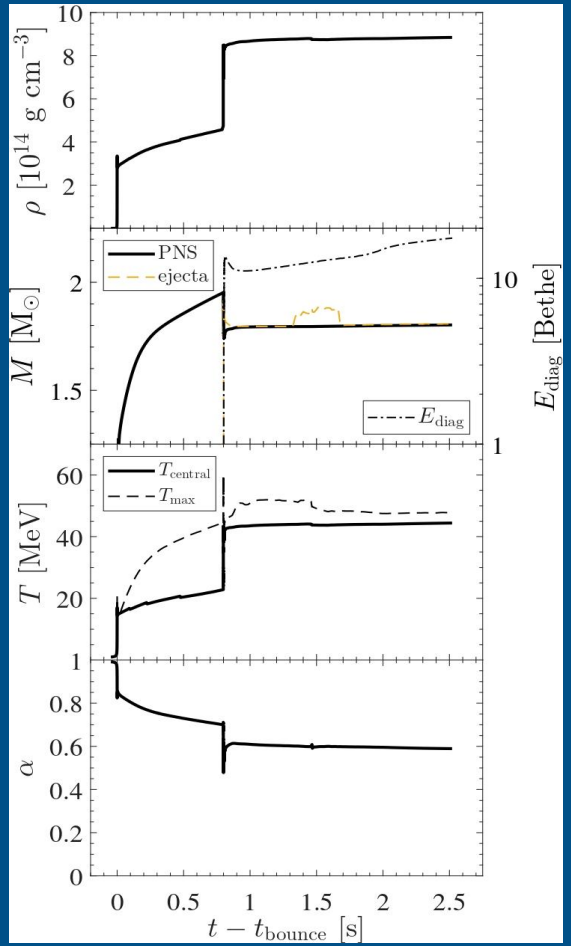

DD2
DD2−EV
DD2−EV (Gibbs)
DD2F−RDF−1.1
P [MeV fm^{-3}]
10^3
10^2
10^1
2
5
10
20
ρ [10^{14} g cm^{-3}]
T [MeV]
120
110
100
90
80
70
60
50
40
30

GRMHD

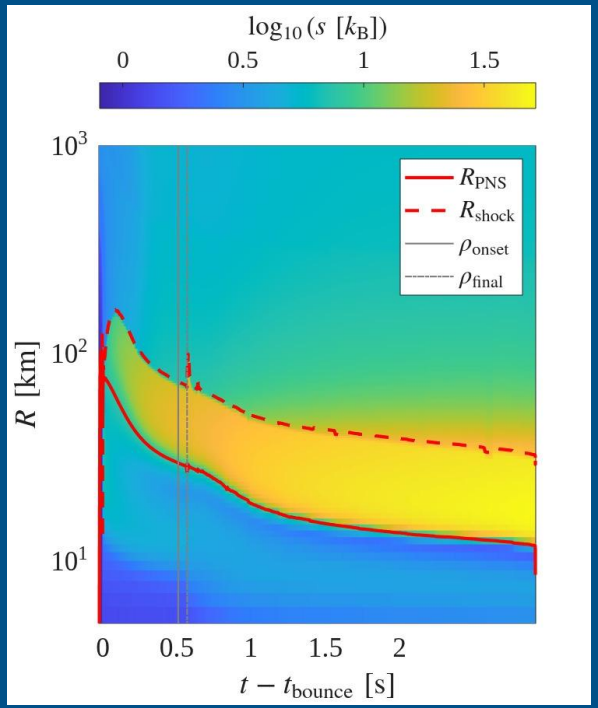




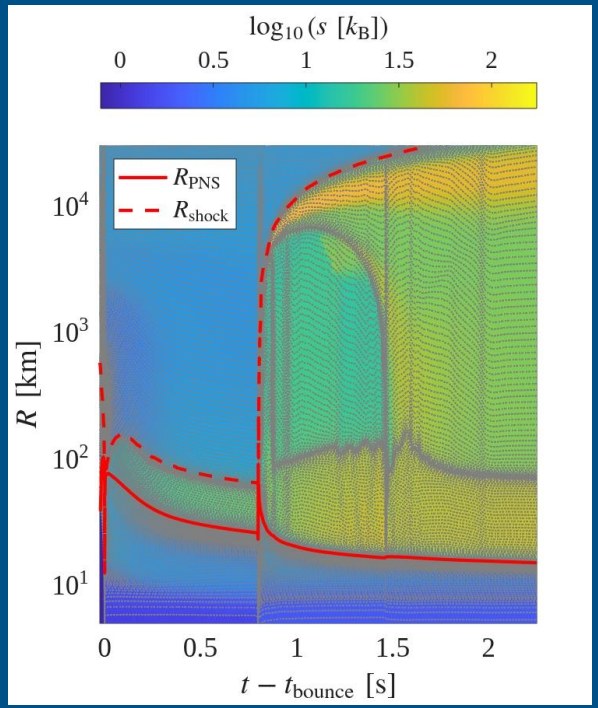
DD2 MEV (Gibbs)



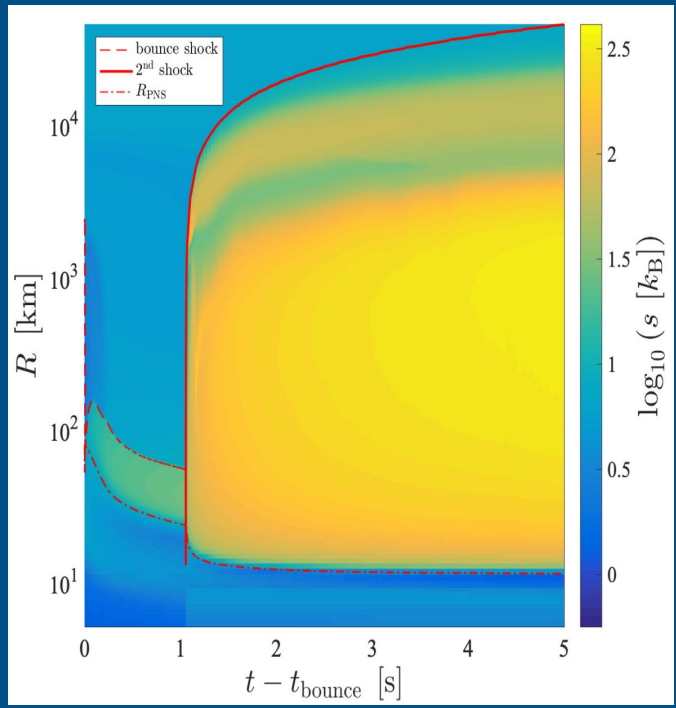
DD2 MEV



DD2 MEV
(Gibbs)

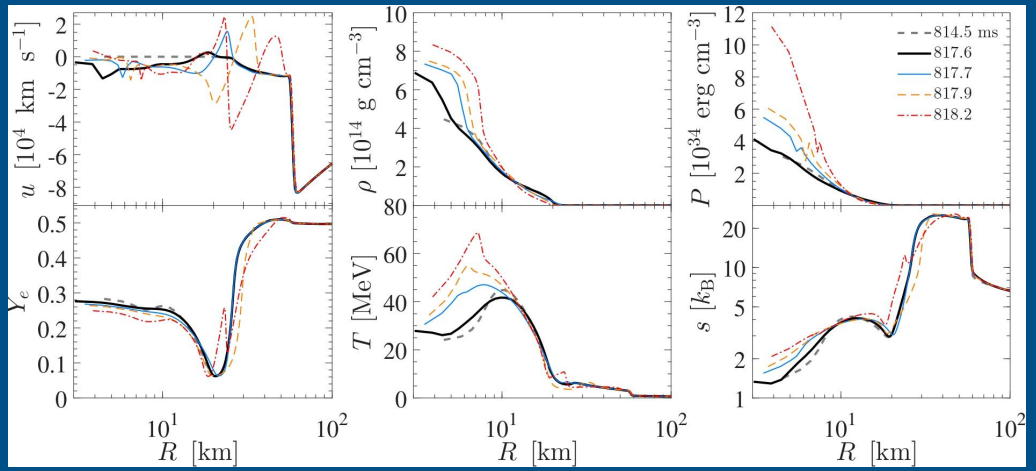


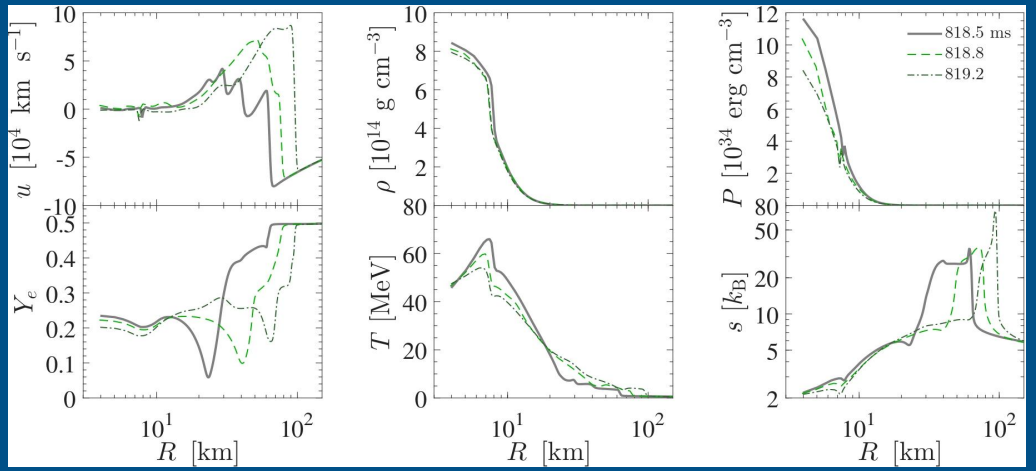
DD2 MEV



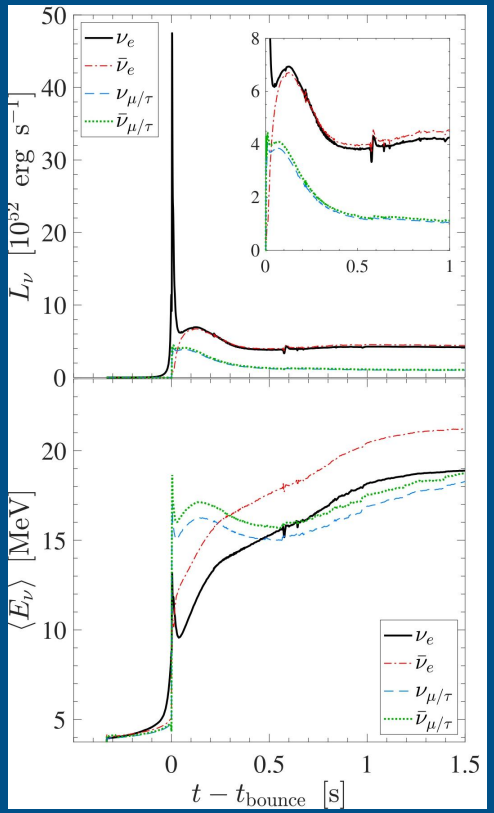
DD2F-RDF

N. K. Largani et al. APJ, 946, 143 (2024)

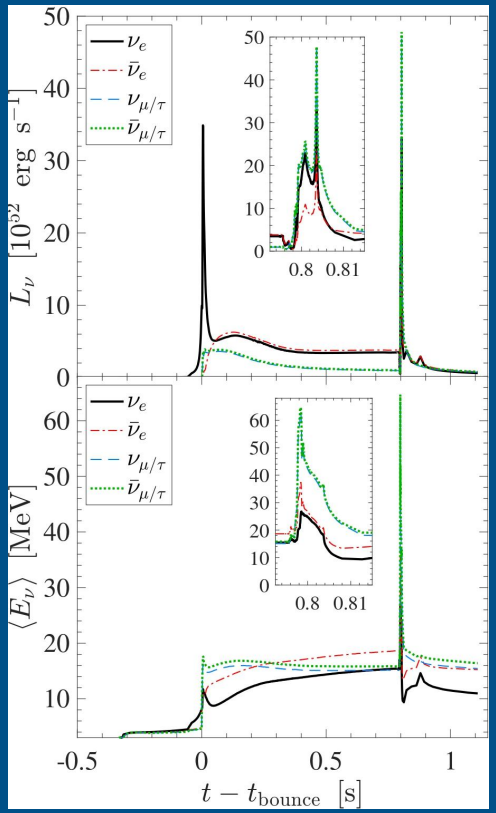

814.5 ms
817.6
817.7
817.9
818.2
u [10^4 km s^{-1}]
Y_e
ρ [10^{14} g cm^{-3}]
T [MeV]
P [10^{34} erg cm^{-3}]
s [k_B]
R [km]


818.5 ms
818.8
819.2
u [10^4 km s^{-1}]
Y_e
ρ [10^{14} g cm^{-3}]
T [MeV]
P [10^{34} erg cm^{-3}]
s [k_B]
R [km]

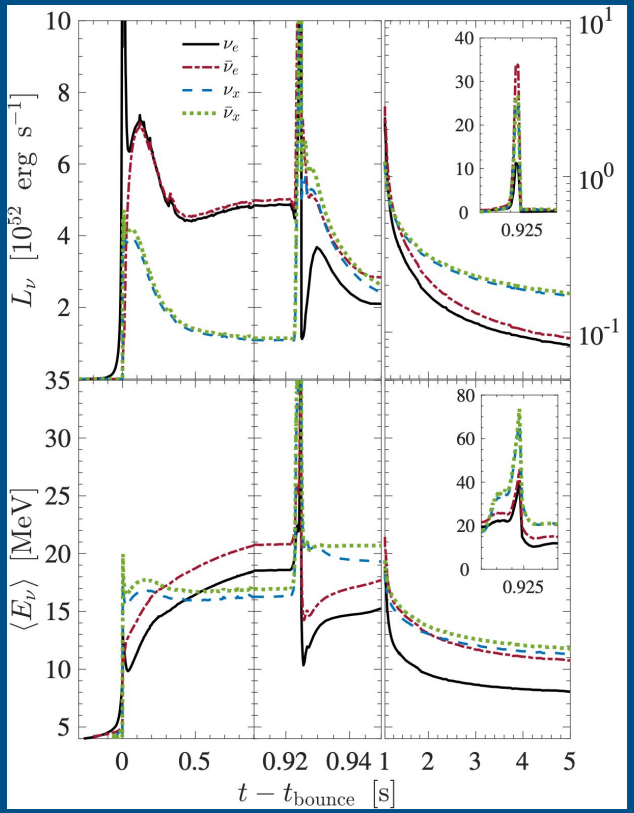
DD2 MEV



DD2 MEV (Gibbs)



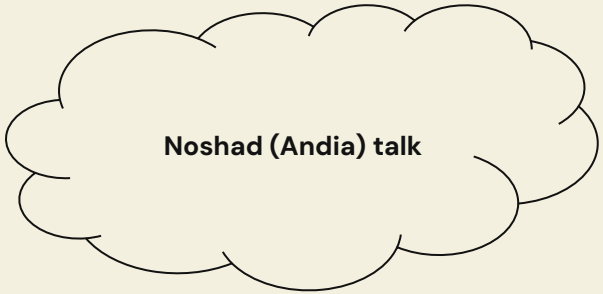
DD2 MEV



DD2F-RDF

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Asteroseismology



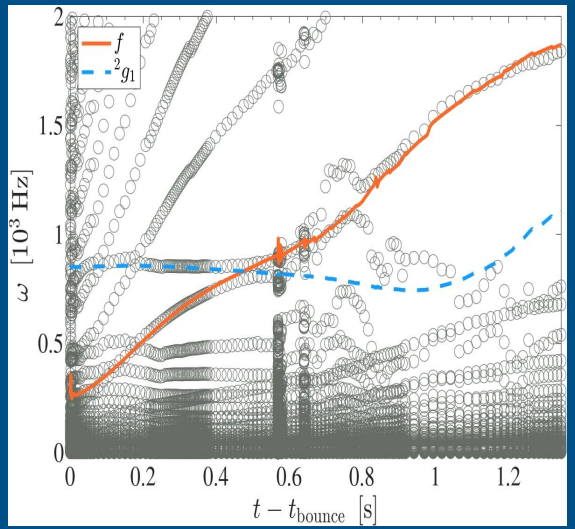
$$\omega_{\rm fit}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

f-mode

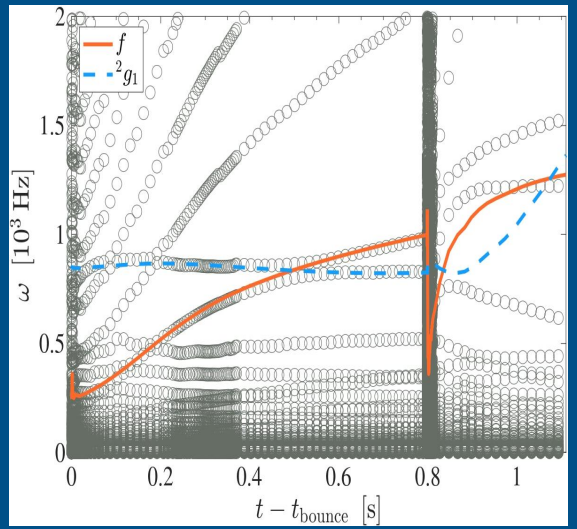
$$x = \sqrt{M_{\rm PNS}/R^3_{\rm PNS}}$$

g-mode

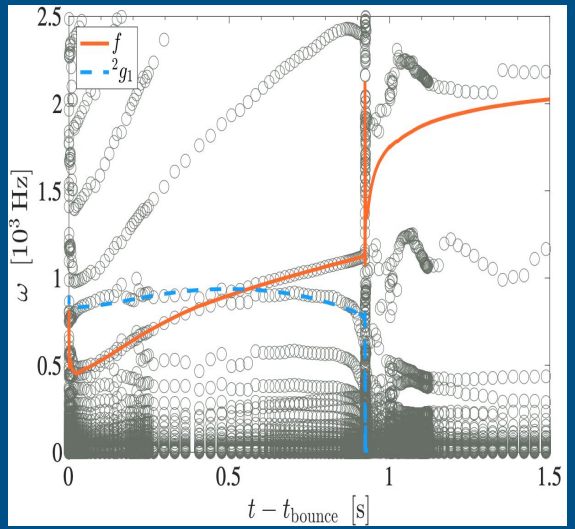
$$x = M_{\rm PNS}/R^2_{\rm PNS}$$



DD2 MEV (Gibbs)



DD2 MEV



DD2F-RDF

N. K. Largani et al. APJ, 946, 143 (2024)

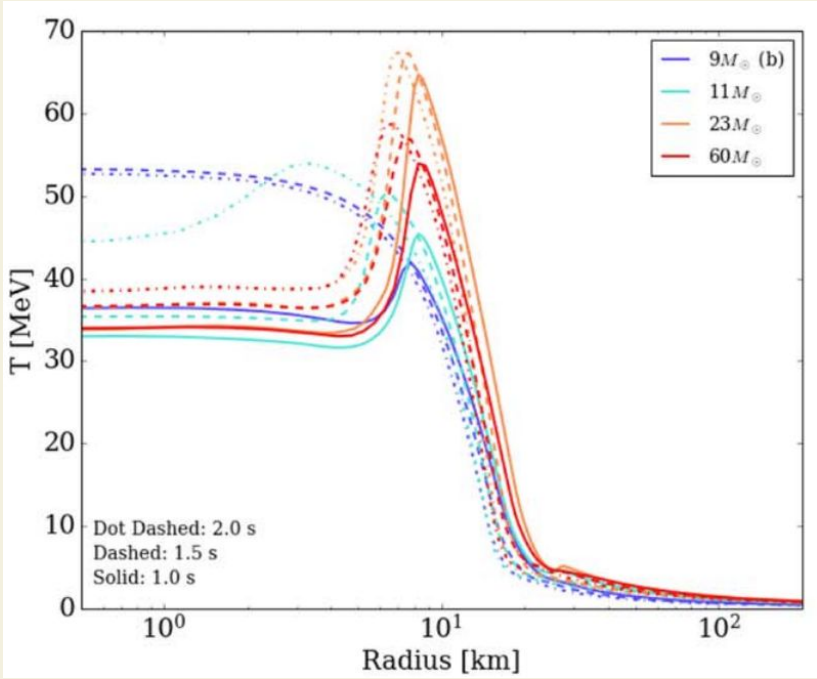
Future perspectives

$$\mu_{\pi^-} = -\mu_{\pi^+} = \mu_n - \mu_p$$

$$\mu_{\pi^0} = 0$$

$$n_p + n_{\pi^+} - n_{\pi^-} - n_e = 0$$

Tobias' talk



Souce: T. Wang and A. Burrows 2023 ApJ 954 114

interaction between pions and nucleons

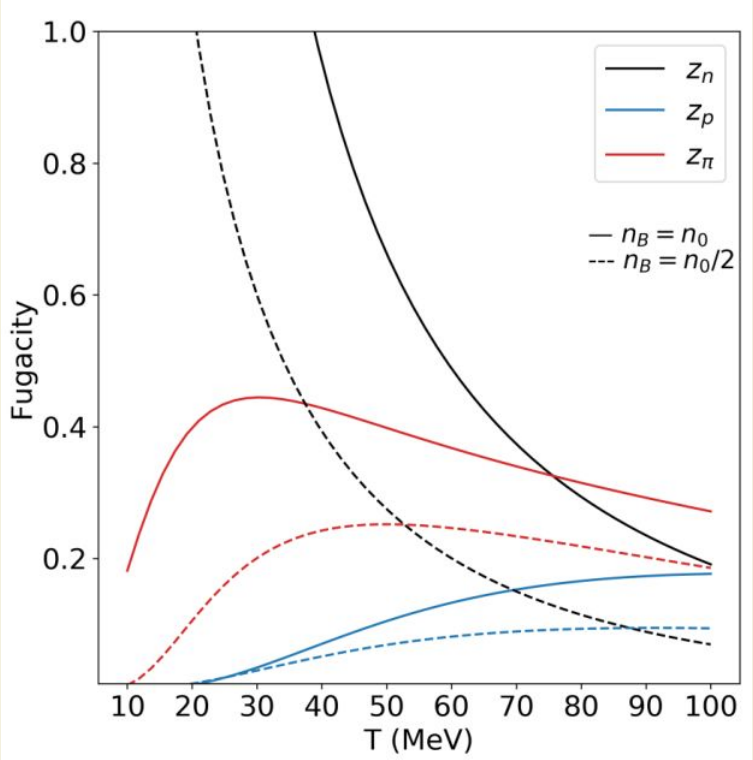
$$\frac{\Omega(T,\{\mu_i\})}{V} = -T\left(\sum_i \frac{b_1^i}{\lambda_i^3} z_i + \sum_{i,j} \frac{b_2^{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} z_i z_j + \cdots\right)$$

$$fugacity: z_i = \exp\left(\frac{\mu_i - m_i}{kT}\right)$$

$$b_2^{n\pi^-} = \frac{e^{\beta M}}{2\pi^3}\int_M^\infty dE\, E^2 K_1(\beta E)\sum_{l,\nu}(2l+1)\delta_{l,\nu}^{3/2}$$

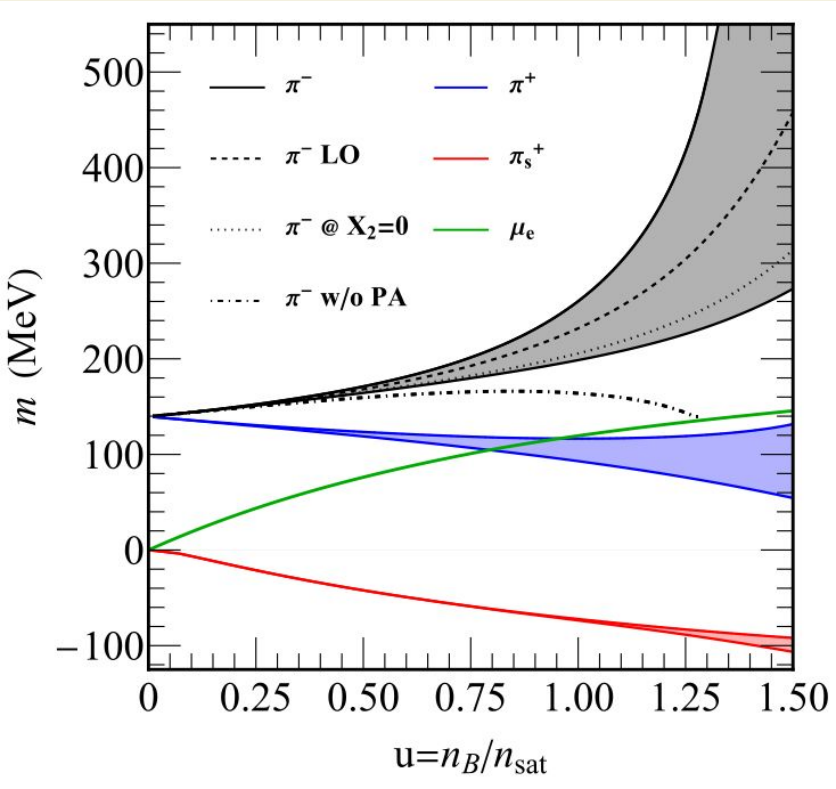
$$b_2^{p\pi^-} = \frac{e^{\beta M}}{2\pi^3}\int_M^\infty dE\, E^2 K_1(\beta E)\sum_{l,\nu}(2l+1)\left(\delta_{l,\nu}^{3/2} + \delta_{l,\nu}^{1/2}\right)$$

$$P_{\pi^-} = T z_{\pi^-}\int \frac{dk\, k^2}{2\pi^2}\exp\left[-\beta\epsilon_\pi(k)\right] + T z_{\pi^-}\sum_{N=n,p} z_N b_2^{N\pi^-}$$



B. Fore and S. Reddy PRC 101, 035809 (2020)

- χPT calculation of the pion self-energy
- Dressed nucleons account for density-dependent
- Large error bands due to uncertainty in low energy constants from experiments



B. Fore et al PRC 110, 025803 (2024)

Conclusions

- ➔ Supernova simulation outcomes are highly sensitive to EOS/phase transitions
- ➔ We need improvements in quark matter models
- ➔ Excluded volume EOS with Gibbs construction leads to black hole formation
- ➔ Explosion energy without phase construction MEV EOS is very large as a hypernova
- ➔ The neutrino spectra has higher luminosities and broader peak than RDF models
- ➔ Asteroseismology of PNS with MEV EOS is very uncertain after phase transition
- ➔ pion-nucleon interaction can be explored with some modifications in Virial EOS or RPA

Thanks



Backup-RMF

$$\mathcal{L} = \sum_N \bar{\psi}_N (i\gamma_\mu D_N^\mu - M_N^*)\psi_N + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2)$$
$$- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2 \boldsymbol{\rho}_{\boldsymbol{\mu}} \cdot \boldsymbol{\rho}^{\boldsymbol{\mu}},$$