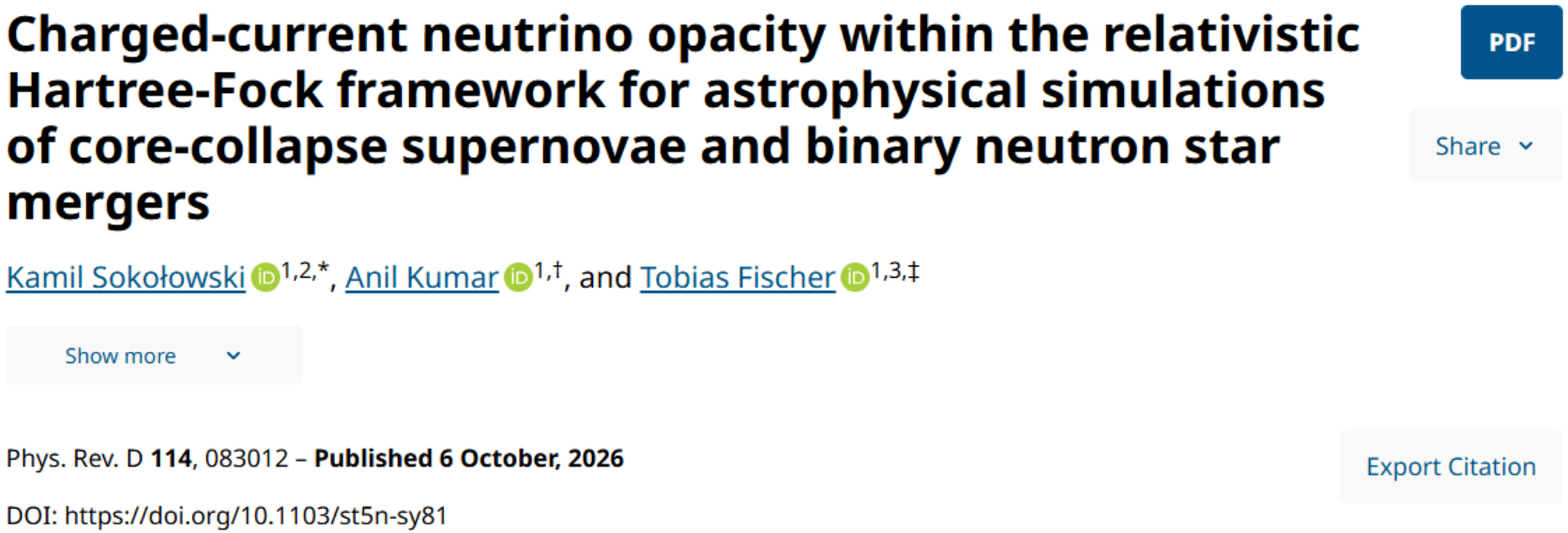
CHARGED-CURRENT NEUTRINO OPACITY WITHIN THE RELATIVISTIC HARTREE-FOCK APPROXIMATION

Kamil Sokołowski

Wroclaw University of Science and Technology



Charged-current neutrino opacity within the relativistic Hartree-Fock framework for astrophysical simulations of core-collapse supernovae and binary neutron star mergers

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CC weak rates ($\nu(p_1) + N_1(p_2) \rightarrow l(p_3) + N_2(p_4)$) within RMF approach

$$\frac{d^2\Gamma(E_1)}{dE_3\, d\cos\theta_{13}} = \frac{(1-f_3)p_3}{8\pi^2 E_1} \int \frac{d^3p_2}{(2\pi)^3} \int \frac{d^3p_4}{(2\pi)^3} \frac{|\mathcal{M}|^2}{4E_2^* E_4^*}$$
$$\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) f_2(1 - f_4)$$

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Mean-field energies:

$$\begin{cases} E_n = \sqrt{p^2 + (m_n^*)^2} + U_n \\ E_p = \sqrt{p^2 + (m_p^*)^2} + U_p \end{cases}$$

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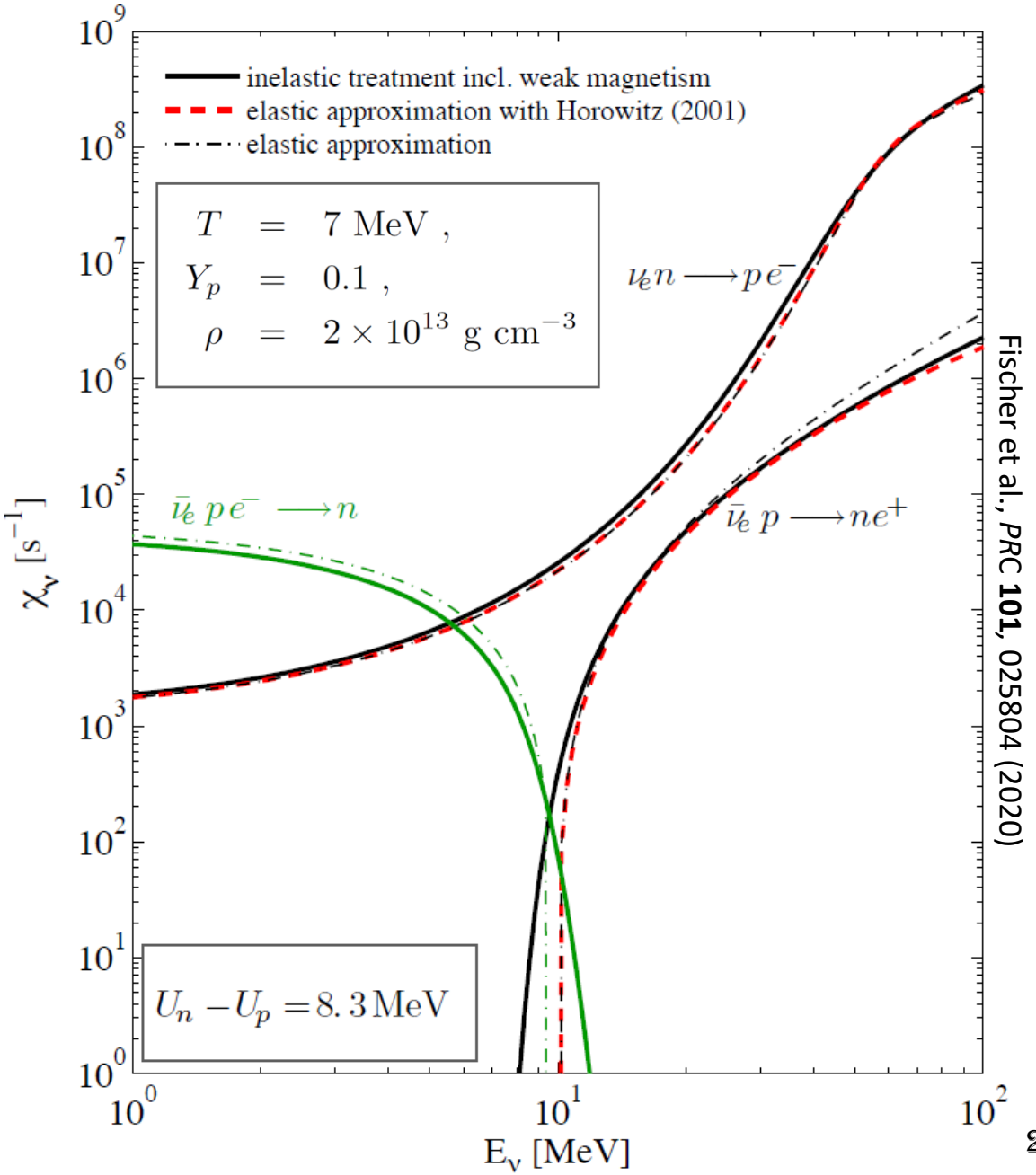
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Energy thresholds:

$$\begin{cases} E_{e^+} = E_{\bar{\nu}_e} - (E_n - E_p) \\ E_{e^-} = E_{\nu_e} + (E_n - E_p) \\ E_{e^-} = -E_{\bar{\nu}_e} + (E_n - E_p) \end{cases}$$

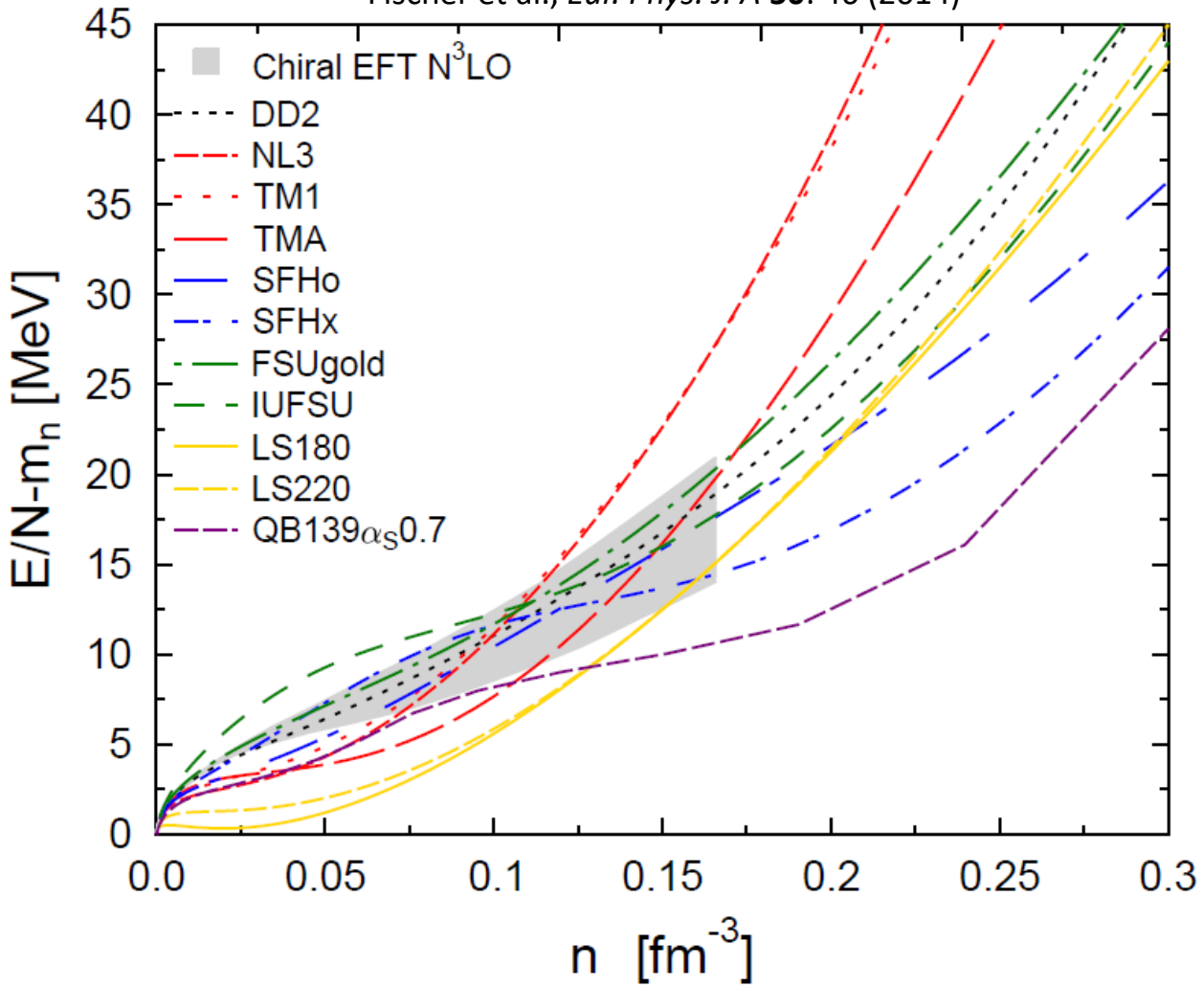


Fischer et al., *PRC* **101**, 025804 (2020)

E_{sym} vs. CC weak rates within RMF

$$\mu_n - \mu_p = 4(1 - 2Y_p)E_{sym} \propto U_n - U_p$$

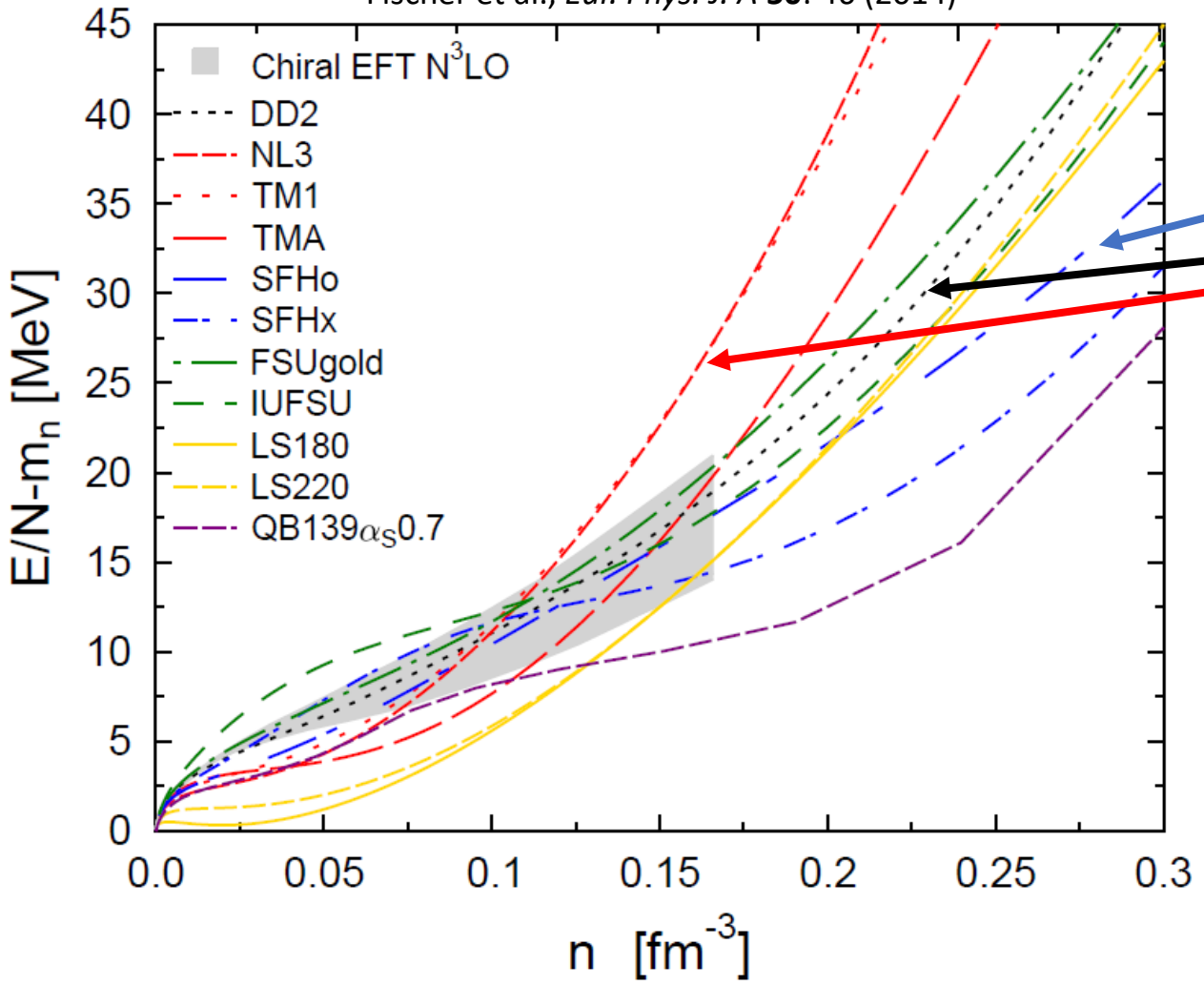
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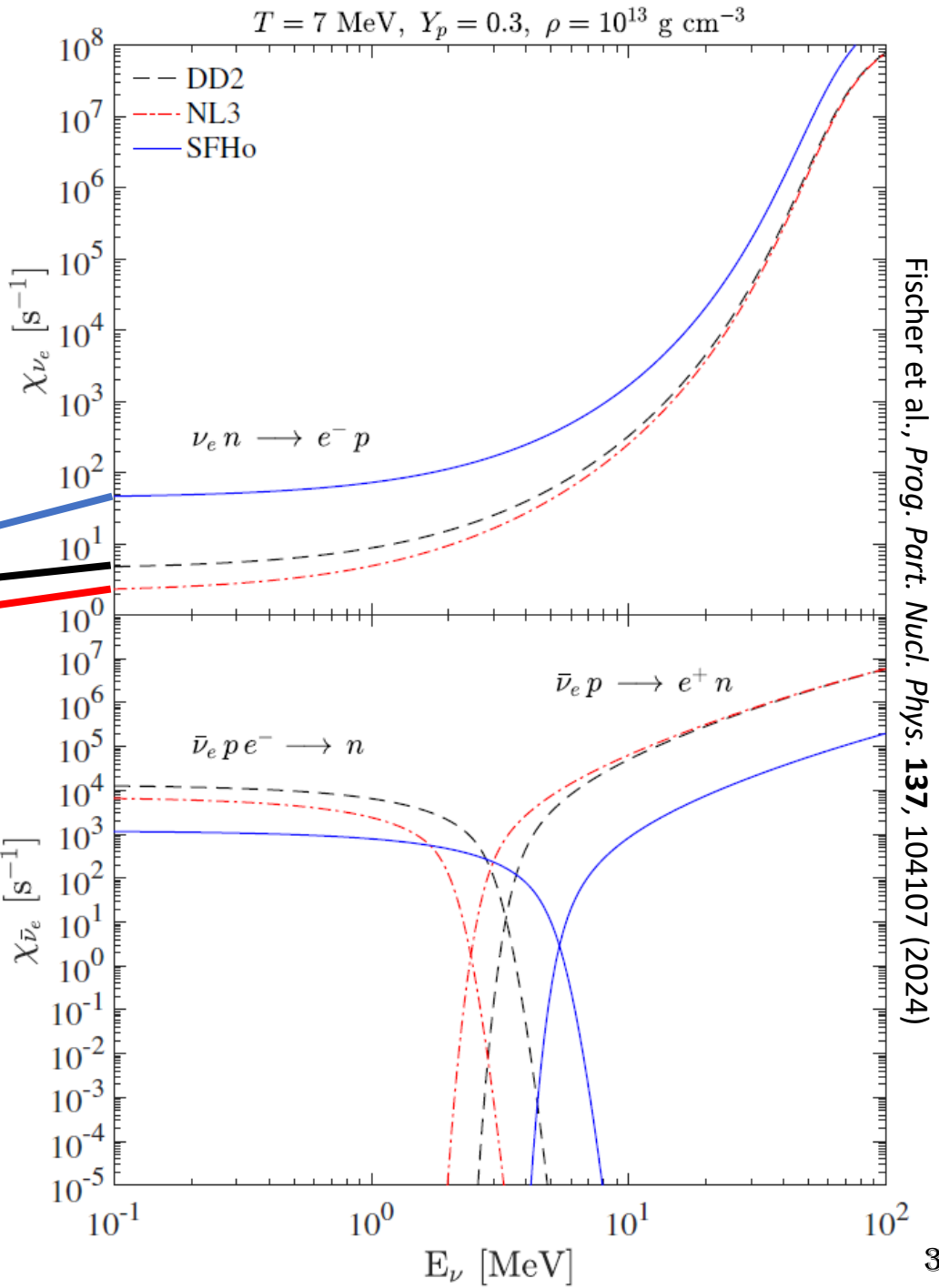


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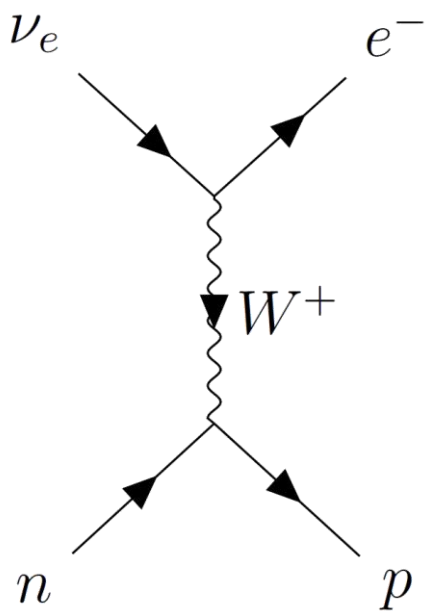
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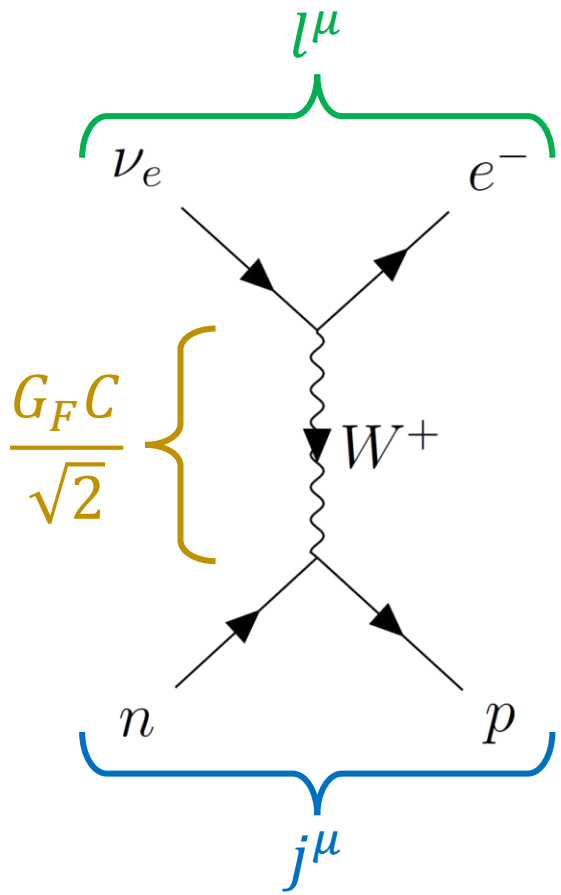
Fischer et al., *Prog. Part. Nucl. Phys.* **137**, 104107 (2024)

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Sokołowski et al., *PRD* **114**, 083012 (2026)
Roberts and Reddy, *PRC* **95**, 045807 (2017)
Le Bellac, *Thermal Field Theory* (2000)

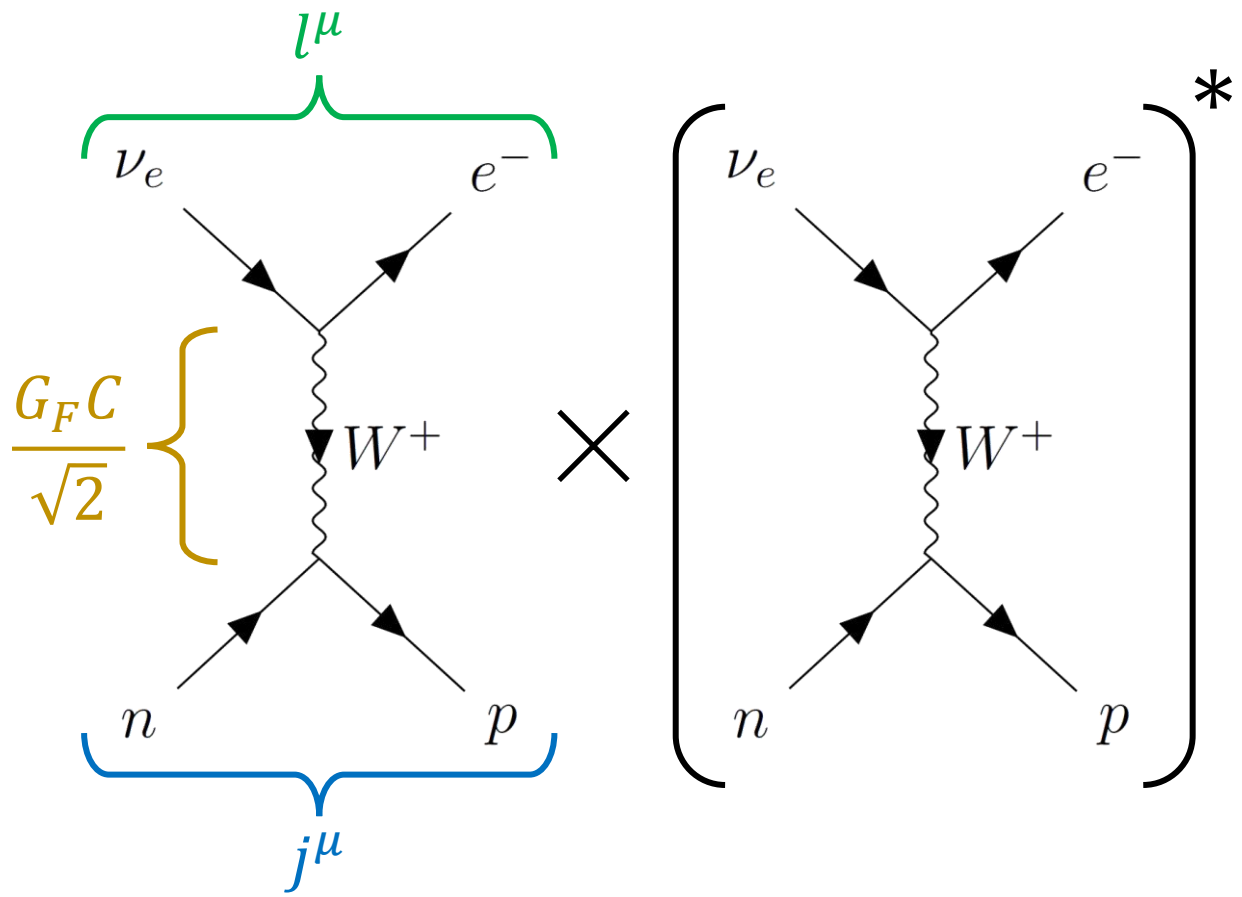
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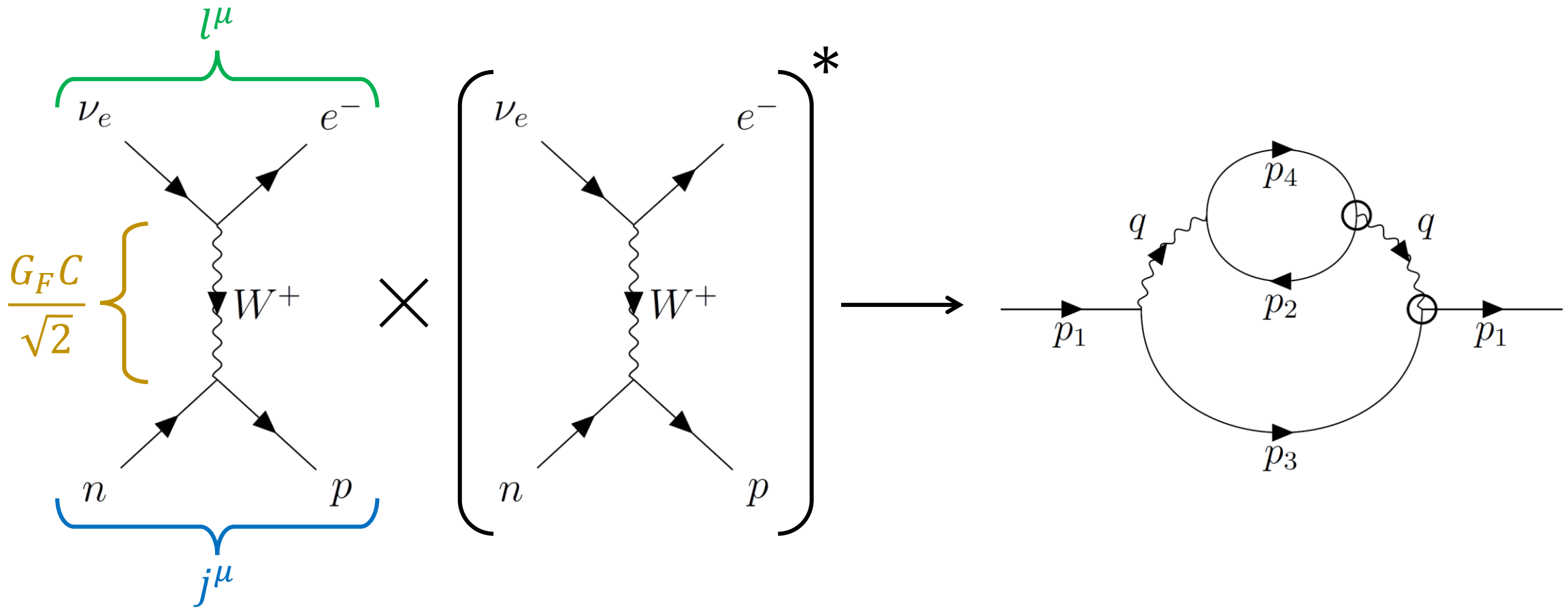
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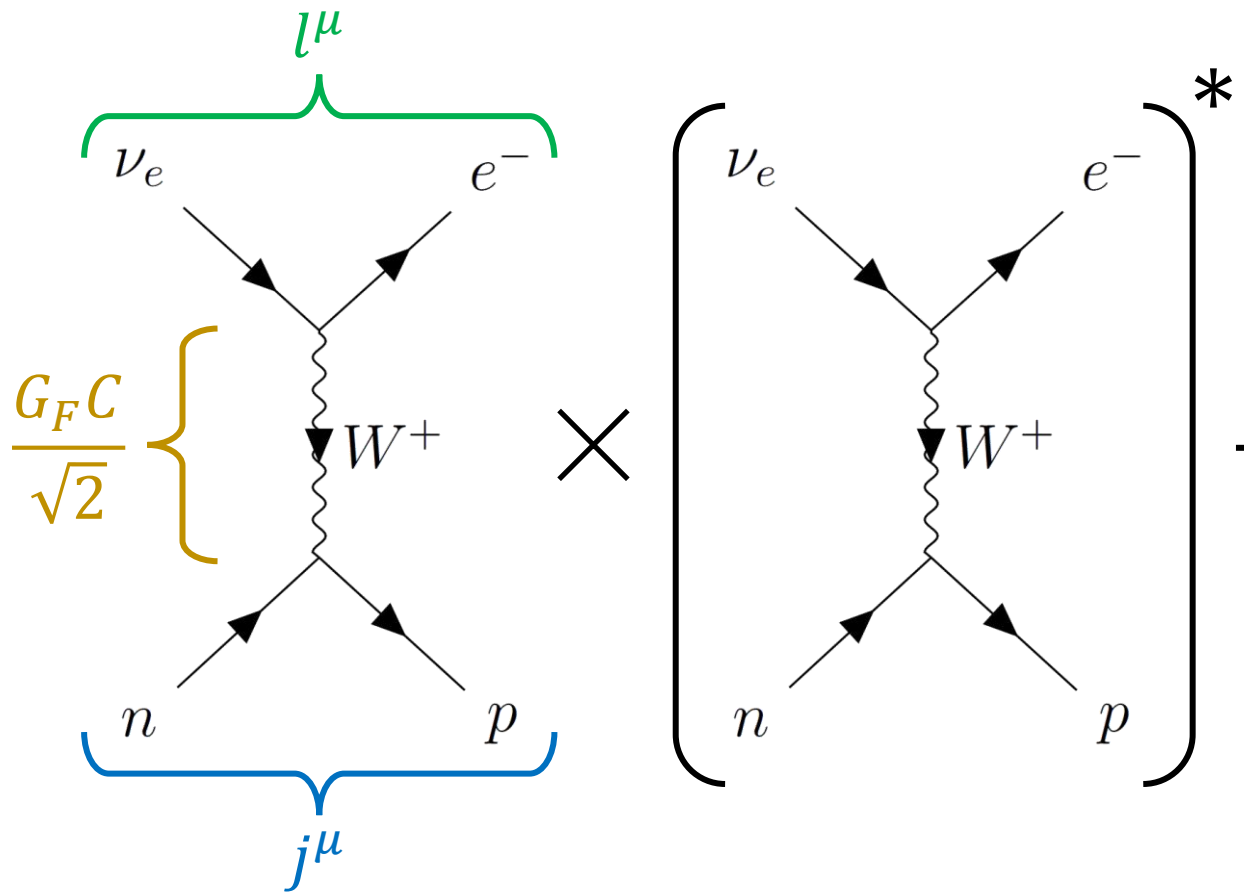
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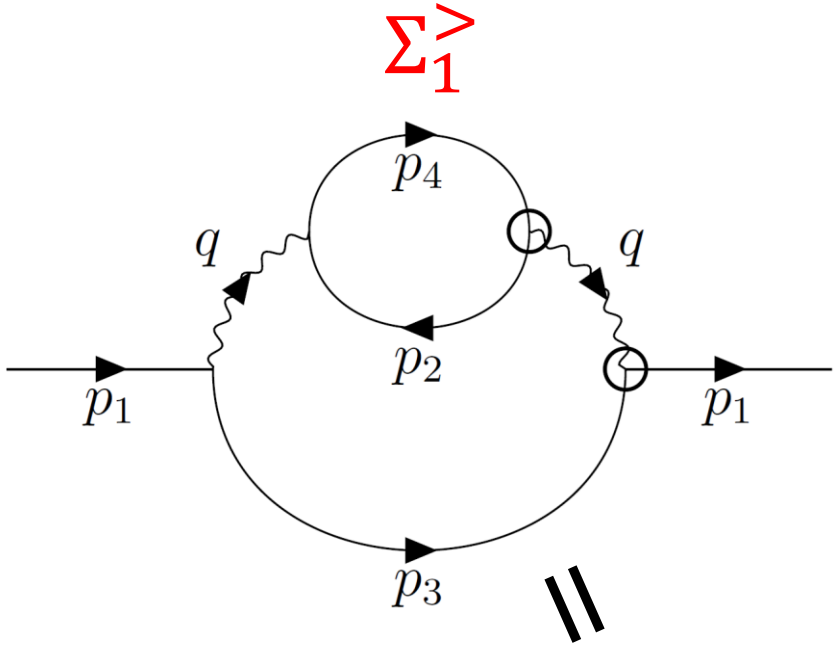


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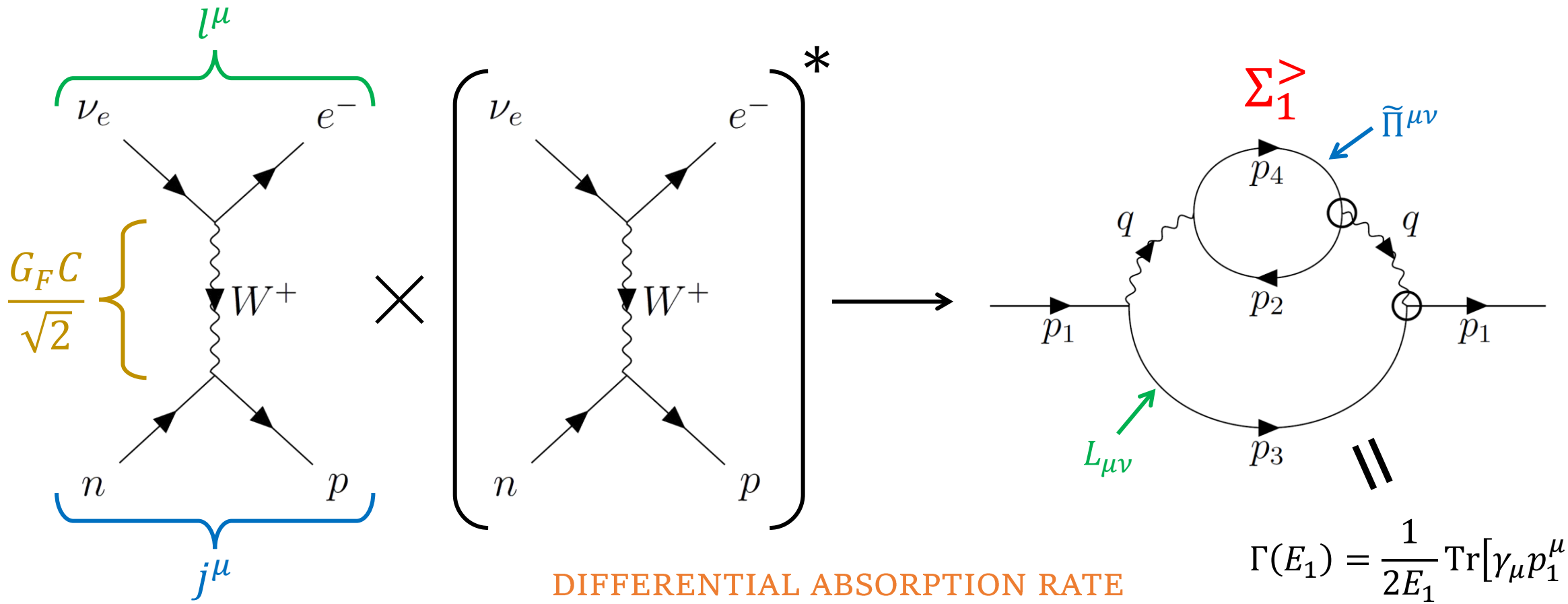


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DIFFERENTIAL ABSORPTION RATE

$$\mathcal{L} = \frac{G_F C}{\sqrt{2}} l^\mu j_\mu$$

$$\frac{d\Gamma(E_1)}{dE_3 d\Omega_{13}} = \frac{G_F^2 C^2}{32\pi^3}\frac{p_3}{E_1}\left(1 - f_3\left(E_3\right)\right)\int \frac{d^3 p_2}{(2\pi)^3} L_{\mu\nu}\,\mathrm{Im}\,\widetilde{\Pi}^{\mu\nu}$$

lepton & hadron tensor

Sokołowski et al., *PRD* **114**, 083012 (2026)
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Hadron tensor ($1 + 2 \rightarrow 3 + 4$)

$$\begin{cases} \mathrm{Im}\,\widetilde{\Pi}^{\mu\nu} = -\pi \dfrac{f_2(E_2) - f_4(E_4)}{4E_2E_4} \Lambda^{\mu\nu} \delta(q_0 + E_2 - E_4) \\ \Lambda^{\mu\nu} = \mathrm{Tr}\left[(\gamma_\mu p_2^\mu + m_2)\bar{\Gamma}^\mu(\gamma_\mu p_4^\mu + m_4)\Gamma^\nu\right] \\ \Gamma^\mu = \gamma^\mu(g_V - \gamma_5 g_A) + F_2 \dfrac{i\sigma^{\mu\alpha}q_\alpha}{2M_N} - \dfrac{G_P}{M_N}\gamma_5 q^\mu \end{cases}$$

Guo et al., *PRD* **102**, 023037 (2020)
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EFFECTIVE QUANTITIES $\quad E_i \to E_i^*, \qquad m_i \to m_i^*, \qquad p_i \to p_i^*$

Guo et al., *PRD* **102**, 023037 (2020)
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Infinite nuclear matter / effective models

$$\mathcal{L} = \mathcal{L}_{Dirac} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{\pi} + \mathcal{L}_{int}$$

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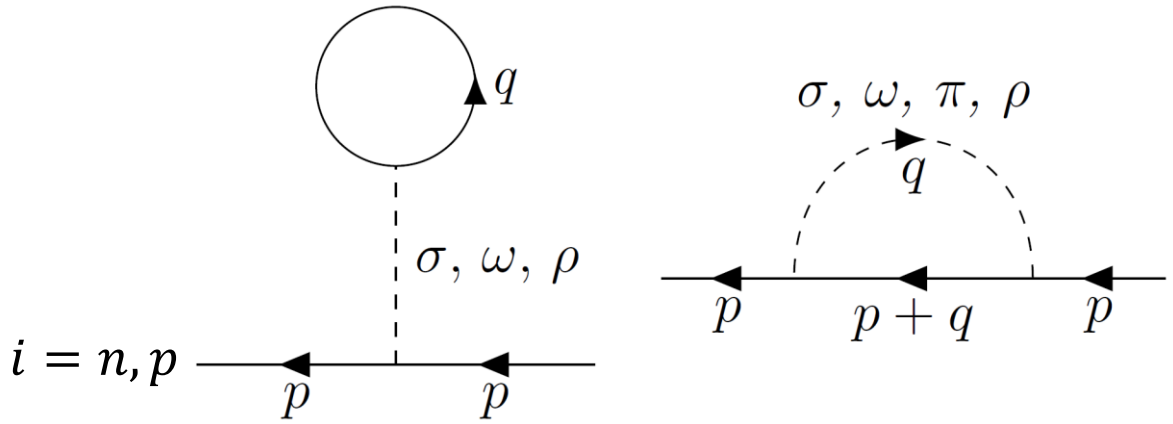
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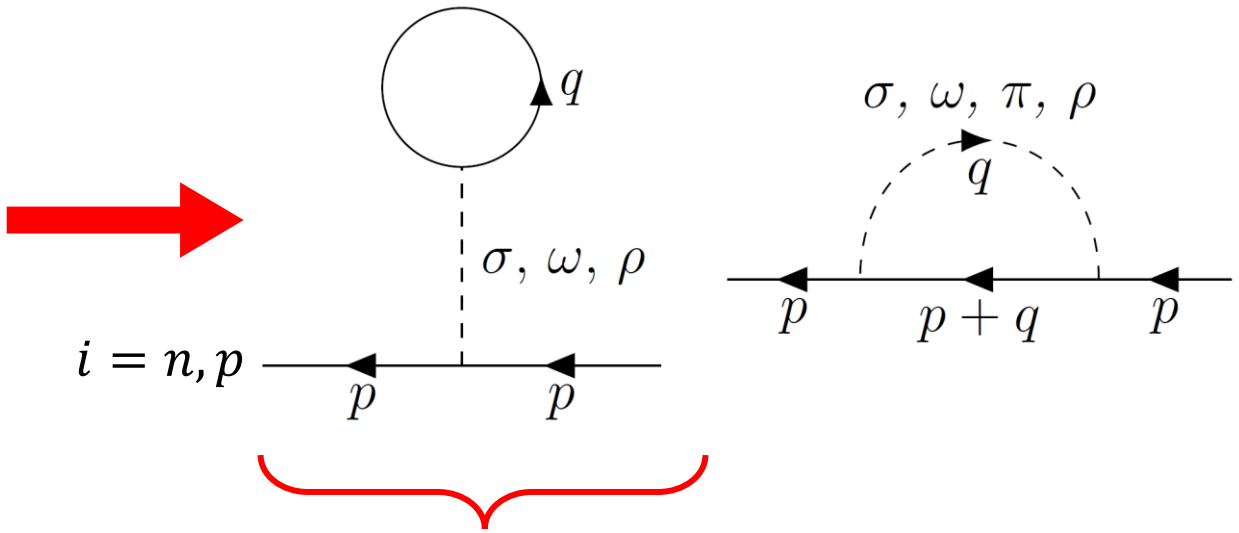
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HARTREE

e.g. DD2

Typel et al., *PRC* **81**, 015803 (2010)
Kumar et al., *Mon. Not. Roy. Astron. Soc.* **530**, 501 (2024)



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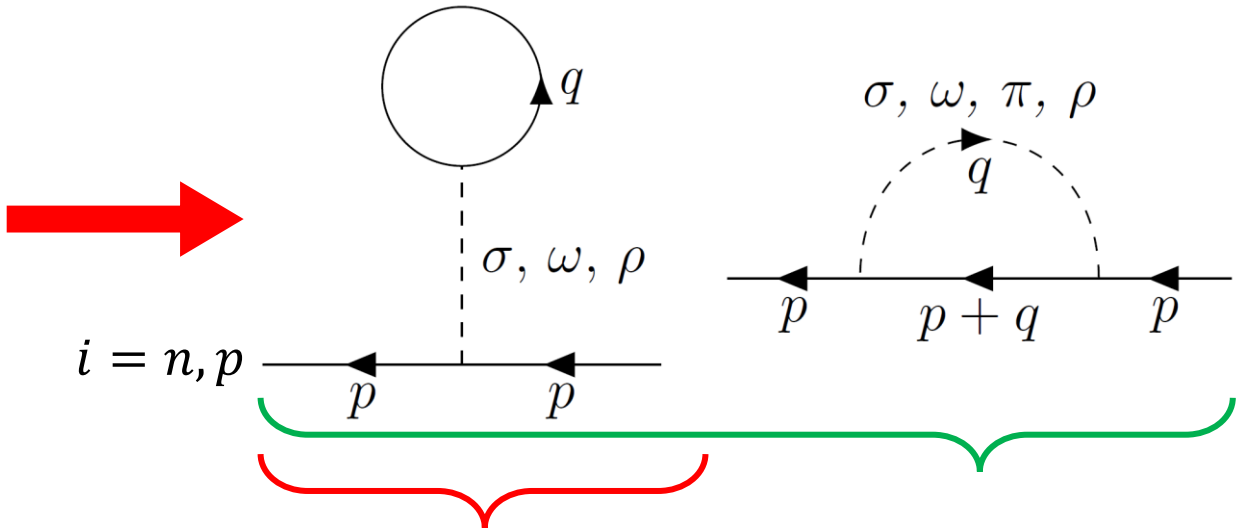
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HARTREE-FOCK

RHF

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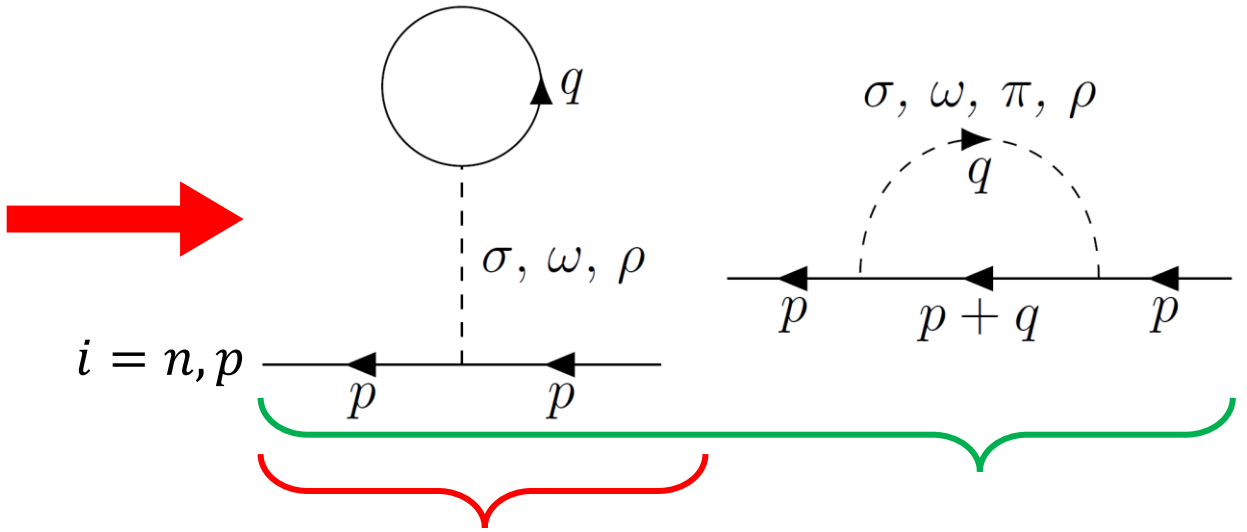
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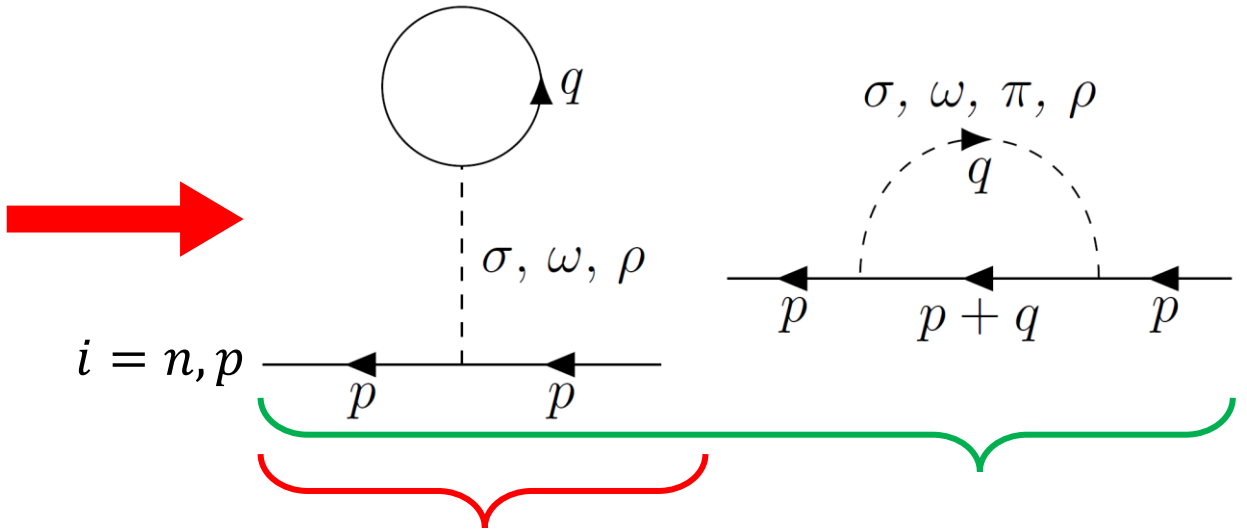
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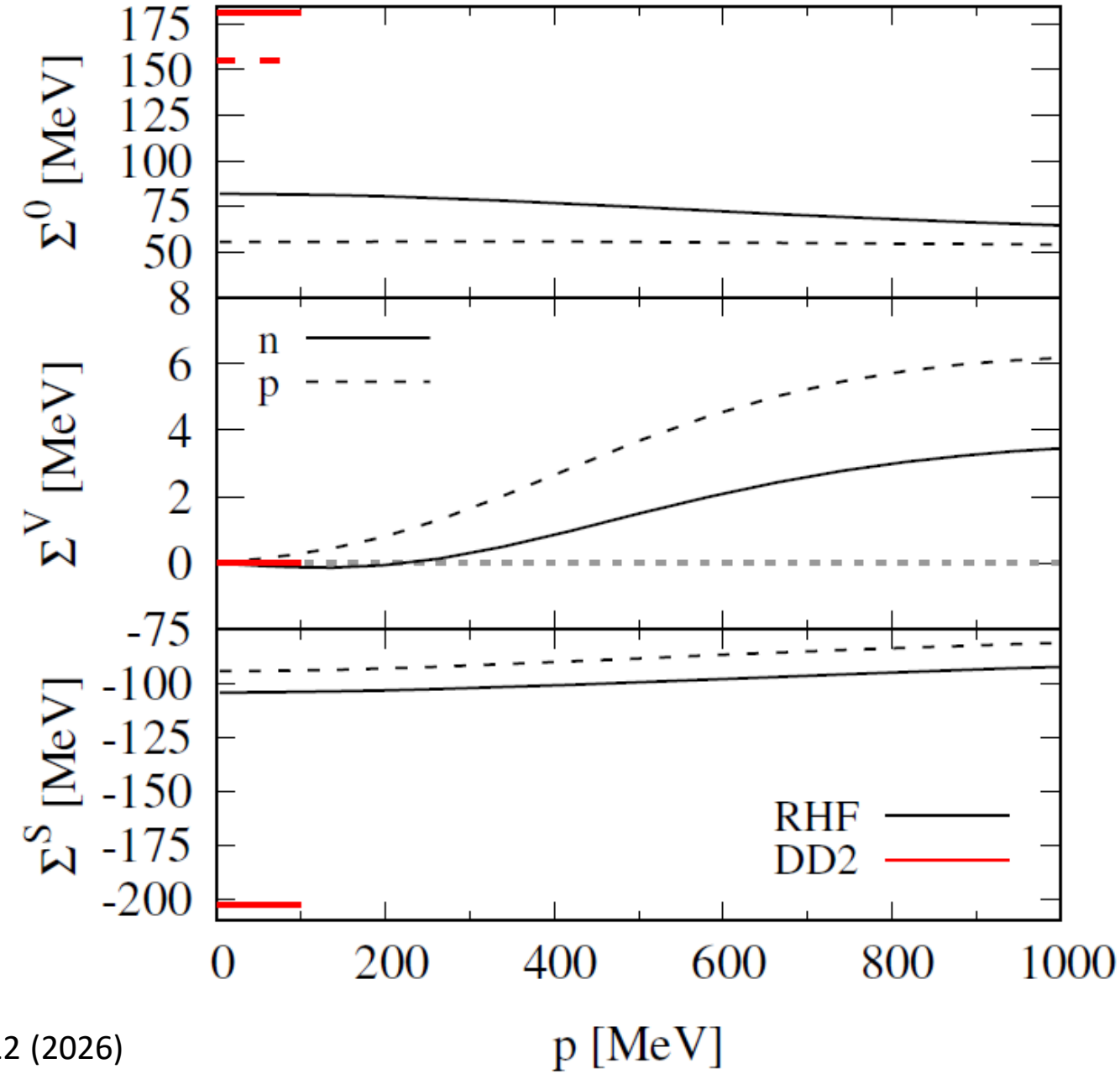
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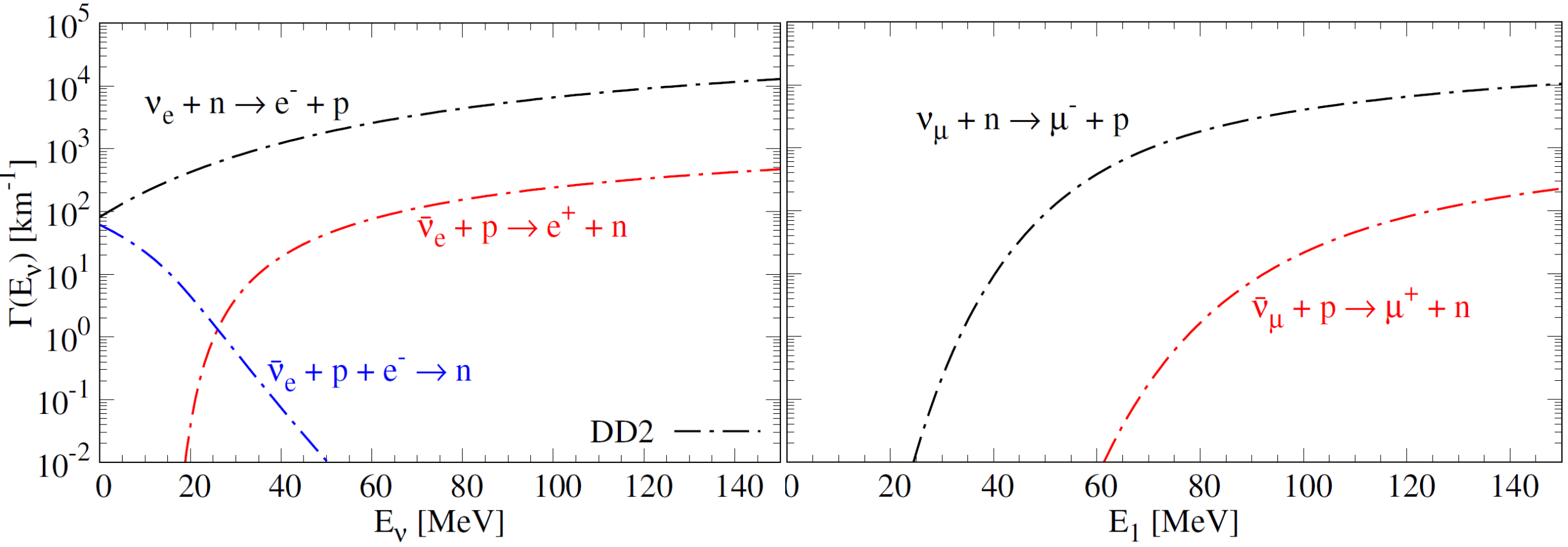
Nucleon self-energy



Sokołowski et al., *PRD* **114**, 083012 (2026)

Neutrino opacity vs E_ν

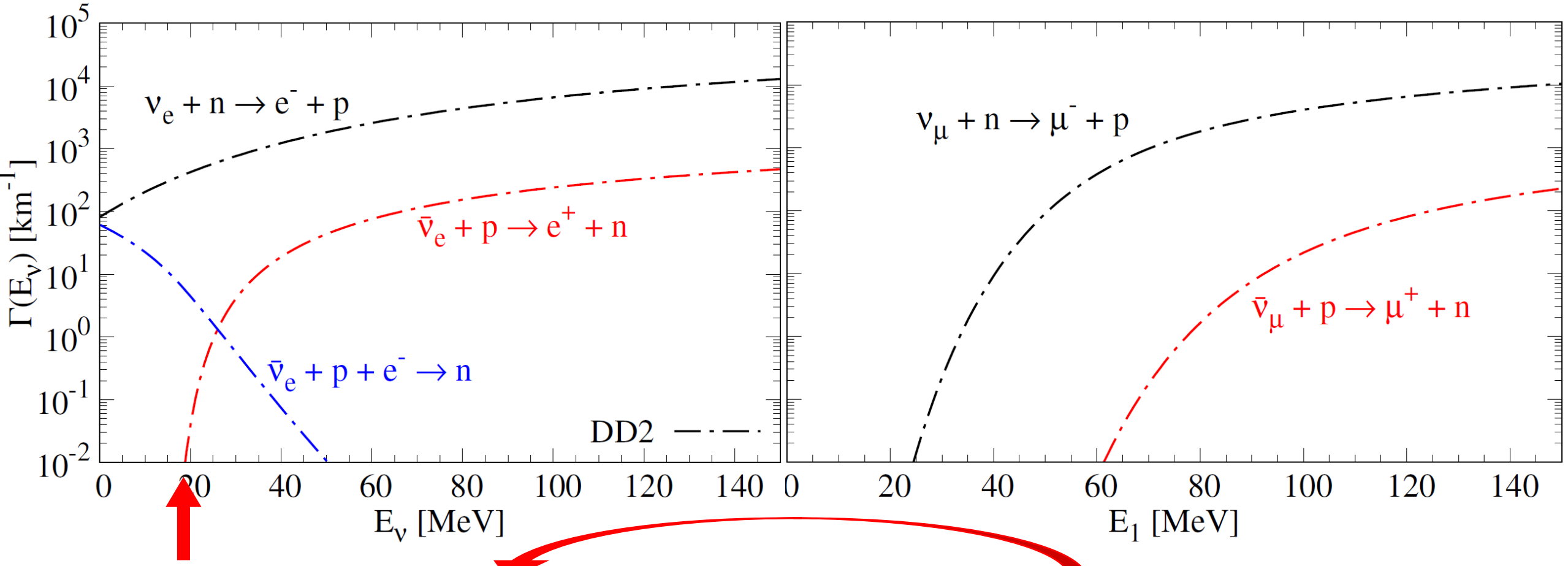
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Sokołowski et al., *PRD* **114**, 083012 (2026)

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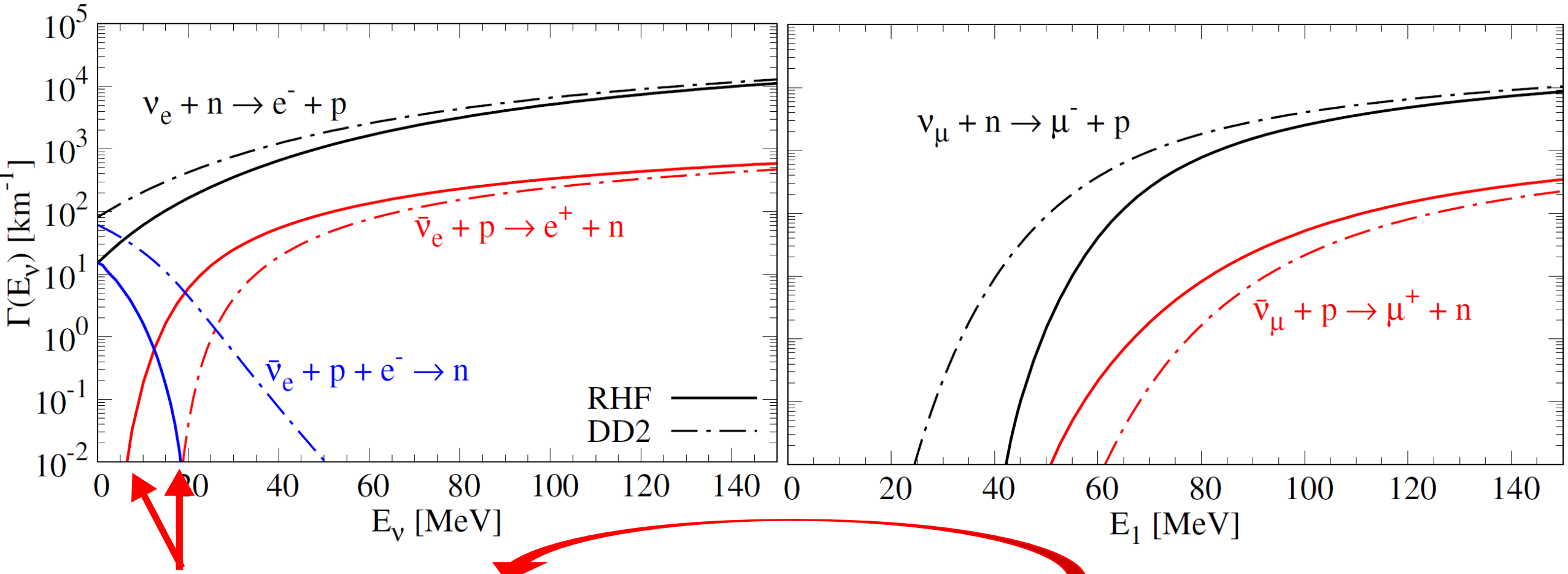
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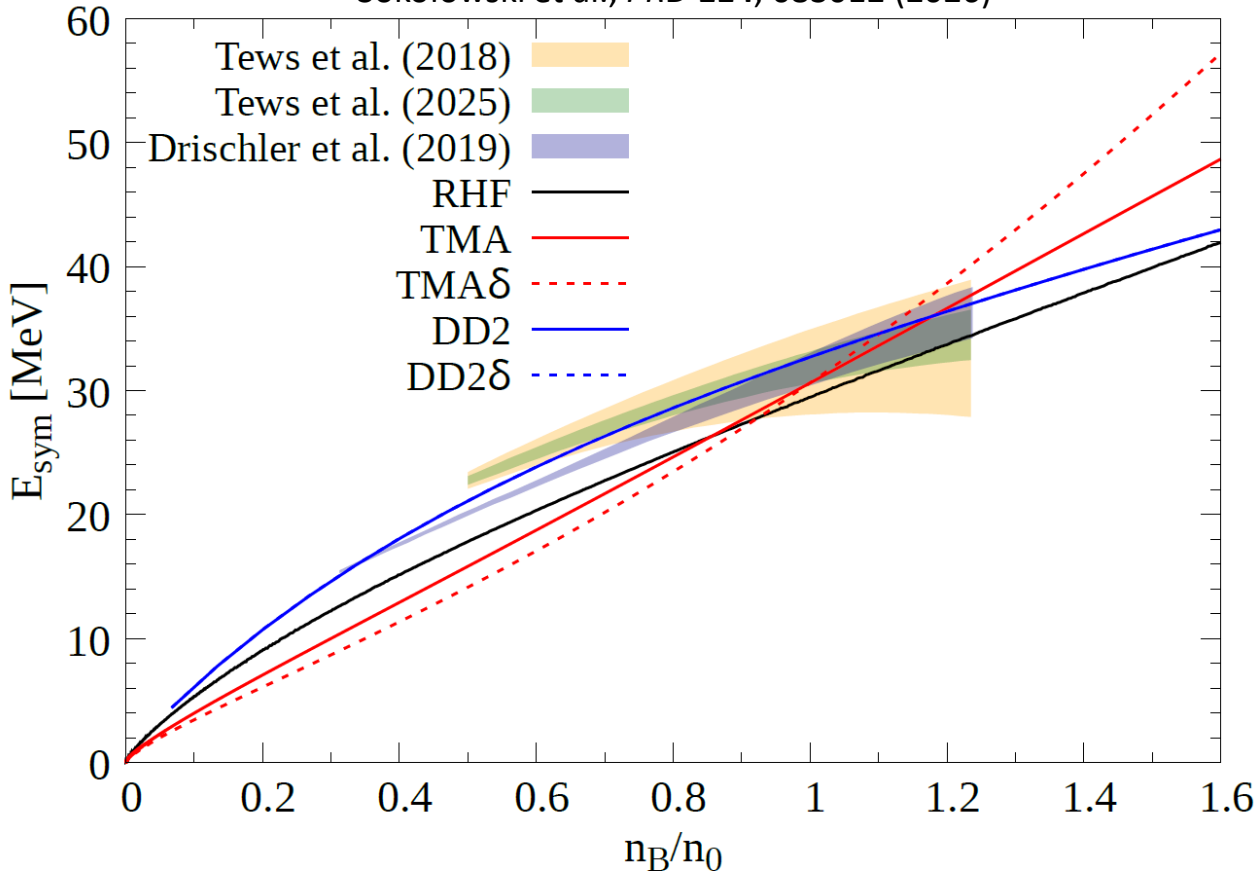
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Impact of δ mesons

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Sokołowski et al., *PRD* **114**, 083012 (2026)

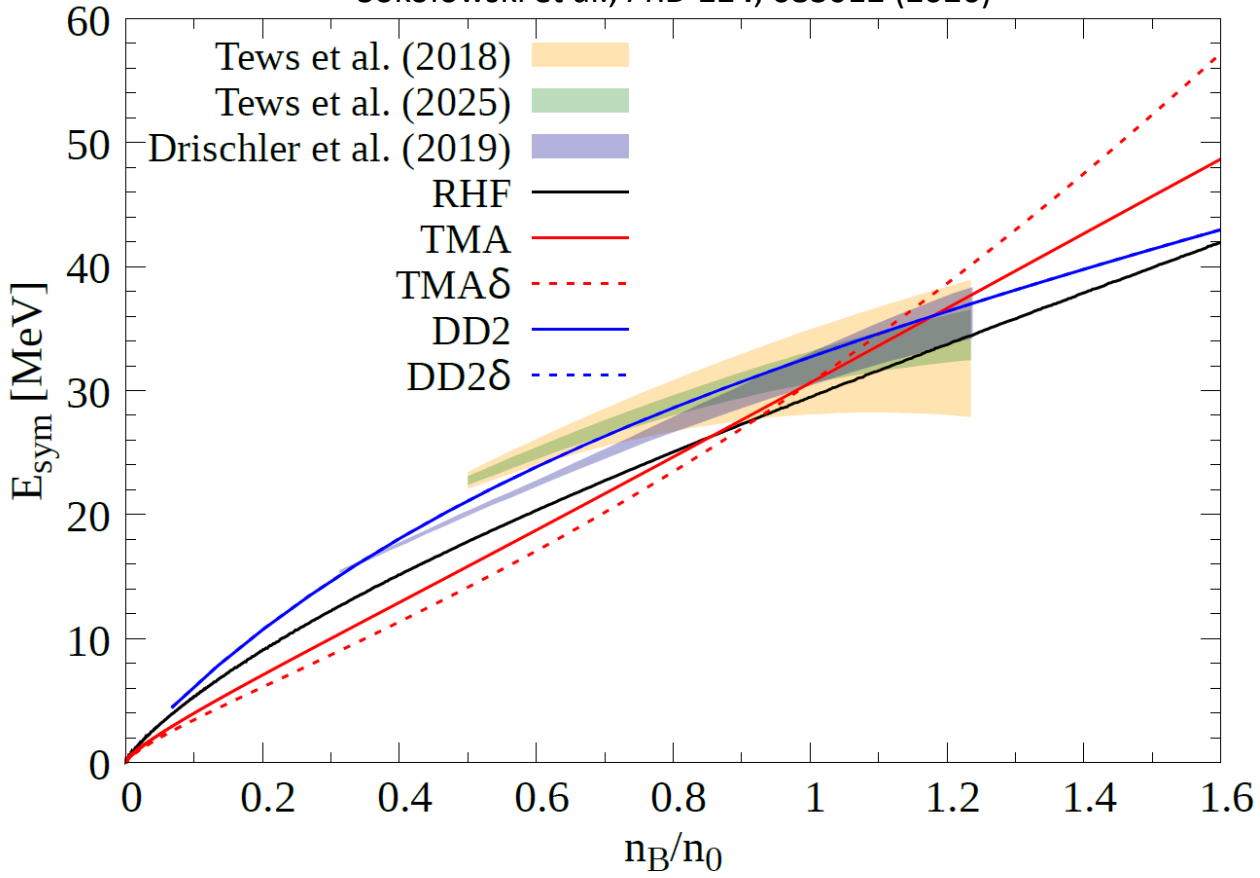


Liu et al., *PRC* **65**, 045201 (2002)

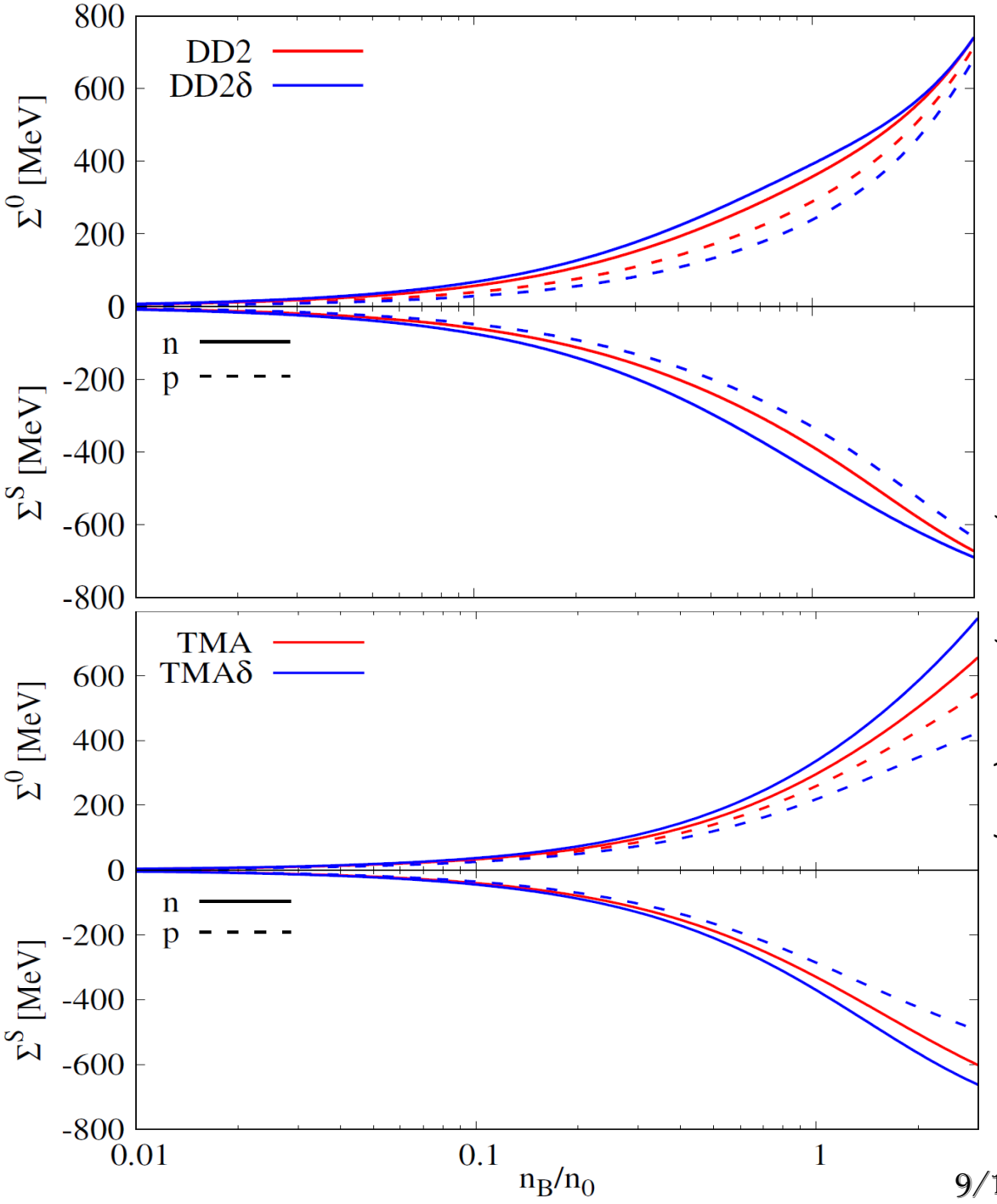
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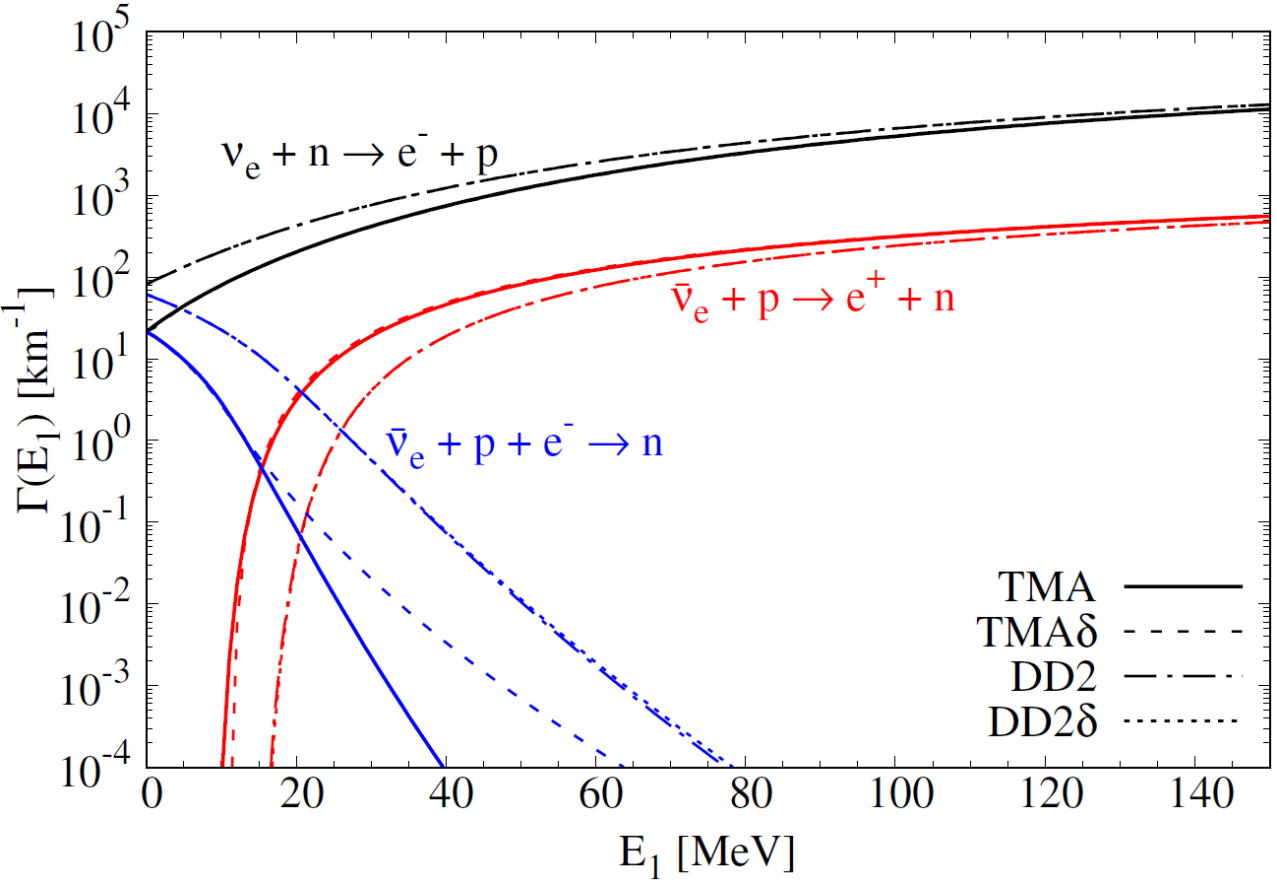


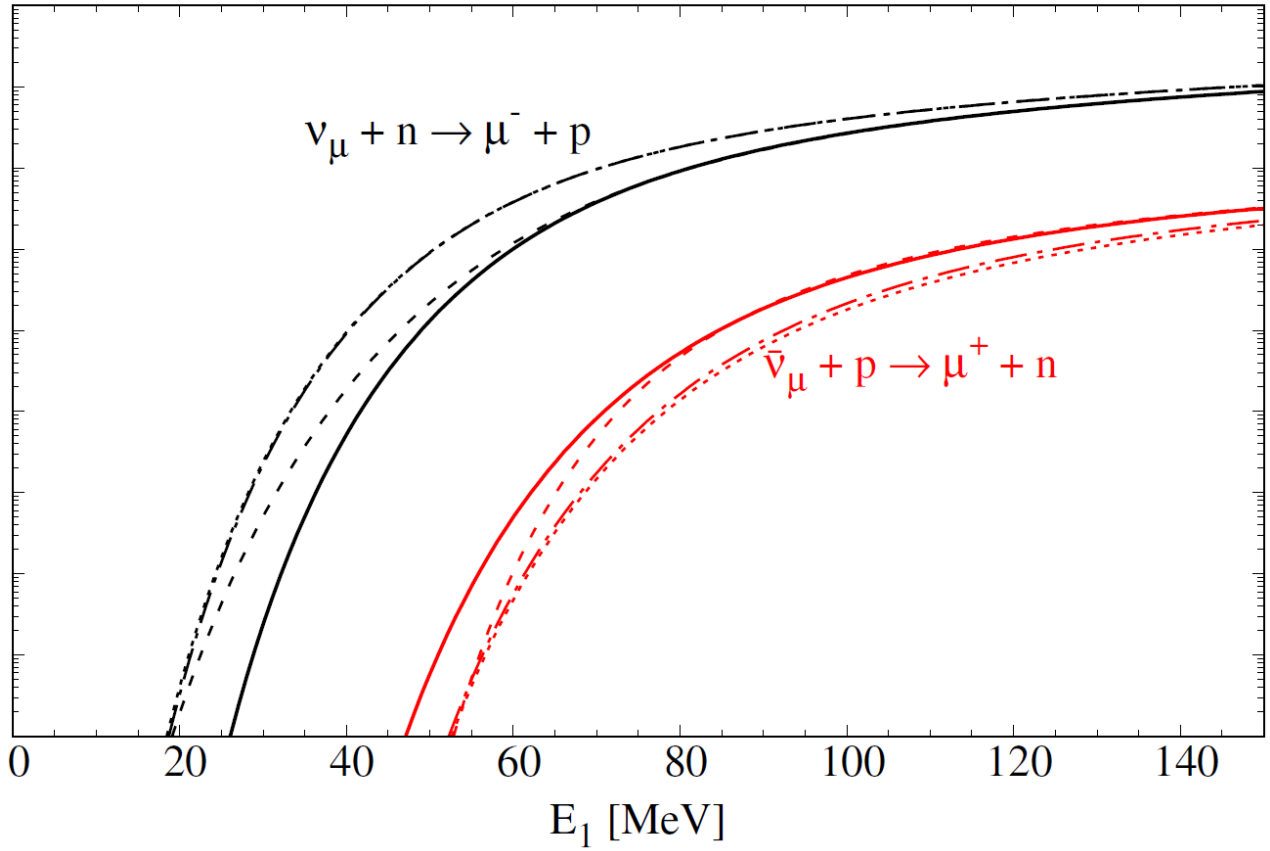
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Sokołowski et al., *PRD* **114**, 083012 (2026)

CC weak rates with δ mesons





Sokołowski et al., *PRD* **114**, 083012 (2026)

Summary and overlook

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- Needs to be explored in supernova simulations
- Impact on nucleosynthesis in neutrino-driven wind ?

$$Y_e \approx \frac{\Gamma_{\nu_e n}}{\Gamma_{\nu_e n} + \Gamma_{\bar{\nu}_e p}} \approx \left(1 + \frac{L_{\bar{\nu}_e}}{L_{\nu_e}} \frac{E_{\bar{\nu}_e} - 2\Delta + 1.2\Delta^2/E_{\bar{\nu}_e}}{E_{\nu_e} + 2\Delta + 1.2\Delta^2/E_{\nu_e}}\right)^{-1} = \begin{cases} < \mathbf{0.5} & \text{if } E_{\nu_e} - E_{\bar{\nu}_e} > 5 \text{ MeV} \\ > \mathbf{0.5} & \text{else} \end{cases}$$

$$\Delta = m_n - m_p$$

Qian et al., *ApJ* **471**, 331 (1996)

Thank you for your attention