Landau-de Gennes-Skyrme model to recreate electromagnetism for liquid crystal-like quantized topological charges ... → SM

Lattice QCD has **probabilities** in ψ – let's go **deeper**: **only ~Yang-Mills + Higgs**
quarks: excitations of **quark strings** **modeled as topological vortices** 
~Einstein's teleparallelism to unify EM + gravity by dynamics of 4D tetrad

Electron: effectively described as perfect point in **perturbative approximation**

in non-perturbative: at least $\boldsymbol{E} \propto \boldsymbol{r}^{-2}$ field of **quantized**(!) charge

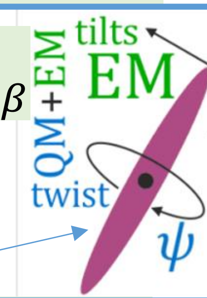
Gauss law — Gauss-Bonnet

Electric charge $= \oint_S E\cdot dA = \frac{e_0}{4\pi}\oint_{S(u,v)} du\,dv\,(\partial_u\vec{n}\times\partial_v\vec{n})\cdot\vec{n} =$ **topological**

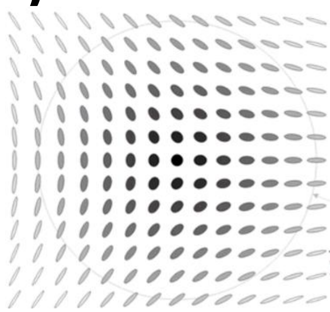
~Skyrme/EM/YM: $\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} + V_{\text{Higgs}}(M)$ for $R \sim F^{\text{dual}}_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

SO(1,3) vacuum $V_{\min}$: $M = O\,\mathrm{diag}(g,1,\delta,0)\,O^T$, weights: $g_{\text{grav.}} \gg 1_{\text{EM}} \gg \delta_{\sim\hbar} > 0$

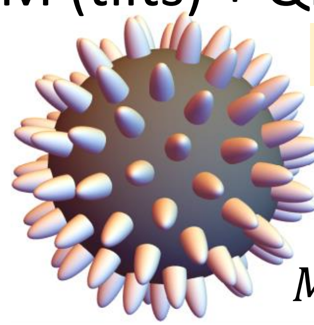
Rotations + boosts of ellipsoid field M of (Higgs) preferred ax lengths $(g,1,\delta,0)$



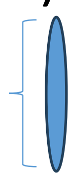
1. **1D sine-Gordon → 3D Faber's model**: **Coulomb** long-range interaction
2. **~Superfluid (~QGPlasma) liquid crystal** agreeing with **Standard Model**
3. **4D, SO(1,3) vacuum:** EM (tilts) + QM (twists) + GEM (boosts) + ⊥gluons



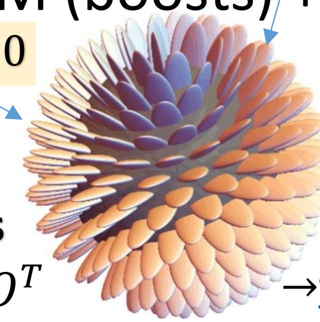
$\delta = 0$



1



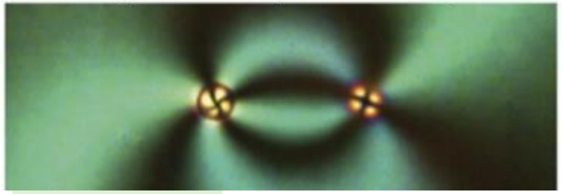
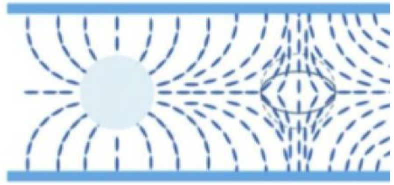
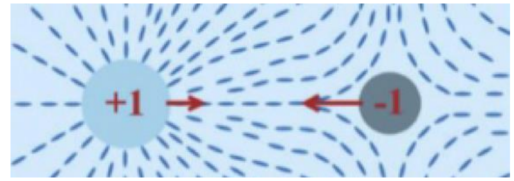
$\delta > 0$

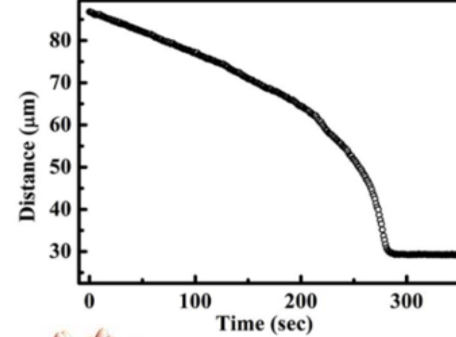


Landau-de Gennes
$M = O\mathrm{diag}(1,\delta,0)O^T$

Jarek Duda
(github, video, intro, article)

→**teleparallelism**

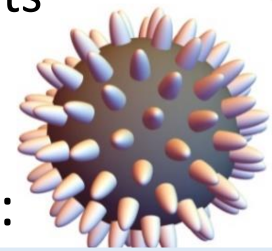






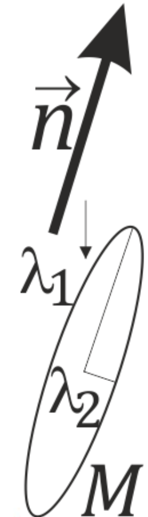
Coulomb: "**Coulomb-like** interaction in nematic emulsions (…)" PRE

"**Coulomb-like** elastic interaction (…) in nematic (…)" Scientific Reports

Standard liquid crystal models (review1, review2)



Uniaxial: $\vec{n} \in S^2$ (Higgs potential?), **Oseen-Frank** model (1958):



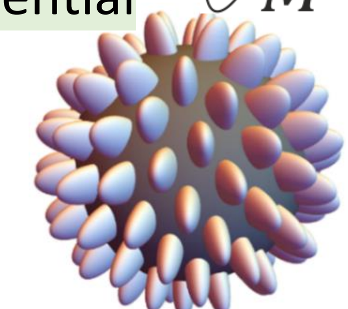
$$E[\vec{n}] = \int_\Omega K_1 |\nabla \cdot \vec{n}|^2 + K_2 |\vec{n} \cdot (\nabla \times \vec{n})|^2 + K_3 |\vec{n} \times (\nabla \times \vec{n})|^2 + (|\vec{n}|^2 - 1)^2$$

Biaxial: **Landau-de Gennes** (1974, Nobel Prize in 1991) of **preferred shape**

$(Q \equiv)$ $M = \sum_{i=1}^{3} \lambda_i \vec{n}_i \vec{n}_i^{\,T}$ **preferred** $(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}, \boldsymbol{\lambda_3})$: $V(M)$ Higgs-like potential

$$E[M] = \int_\Omega |\nabla M|^2 + \frac{A}{2}\mathrm{Tr}(M^2) - \frac{B}{3}\mathrm{Tr}(M^3) + \frac{C}{4}\left(\mathrm{Tr}(M^2)\right)^2$$

(or $V(M) = \sum_i (\lambda_i - \Lambda_i)^2$ or $\sum_i \left(\mathrm{Tr}(M^i) - c_i\right)^2$ tough choice!)



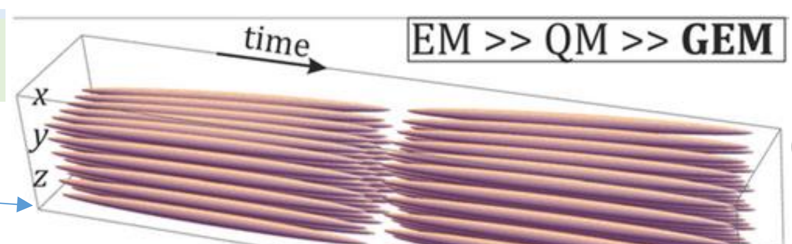
Coulomb-like – repair to EM: real Coulomb + Lorentz invariance:

$$\boldsymbol{\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V_{Higgs}(M)}$$

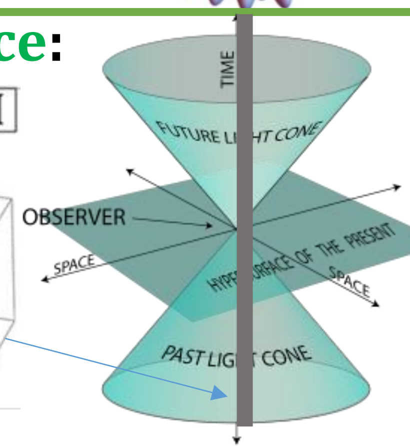
$$M_{\text{vacuum}} = O \,\mathrm{diag}(g, 1, \delta, 0)\, O^T$$

$$F_{\mu\nu\alpha\beta} = \left[\partial_\mu M, \partial_\nu M\right]_{\alpha\beta}$$

→teleparallelism, SO(1,3), Minkowski

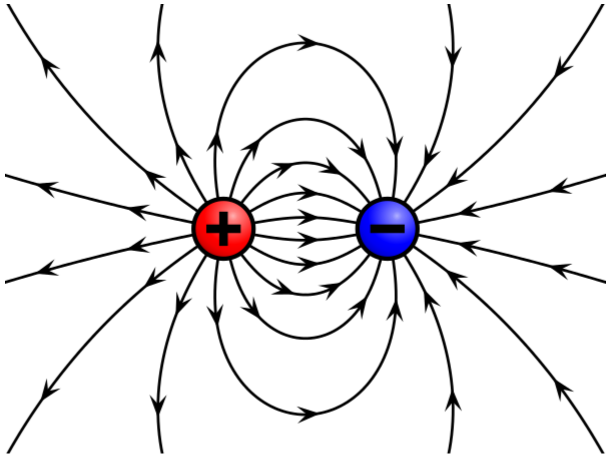


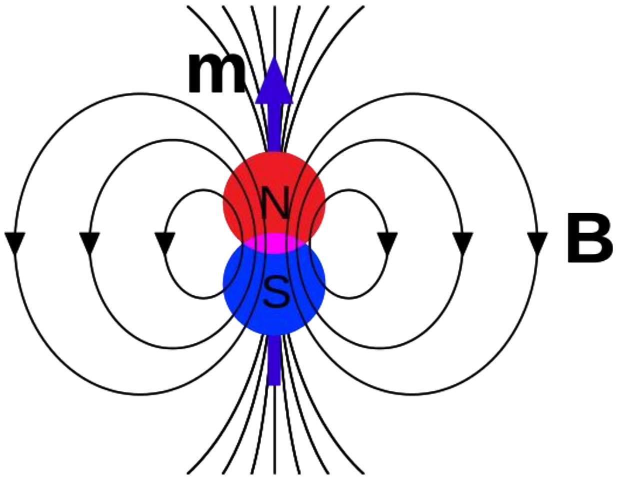
+ GEM approx. of GR
from tiny perturbations (boost) of 0th axis

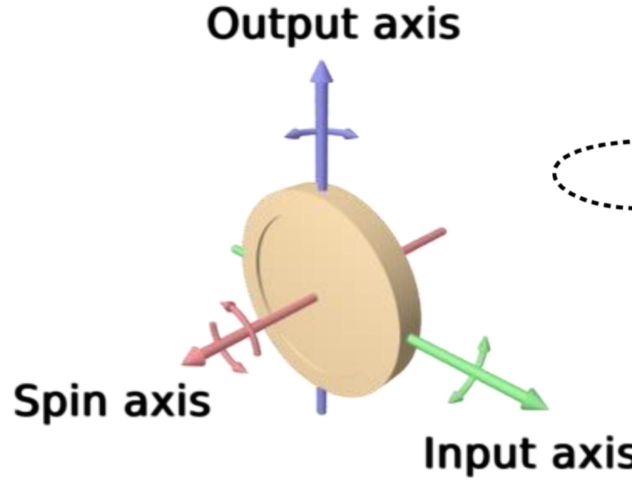


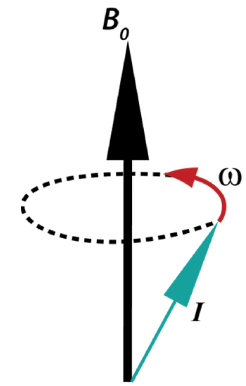
non-pertubative

Electron – (at least) a complex configuration of electromagnetic field … Larmor





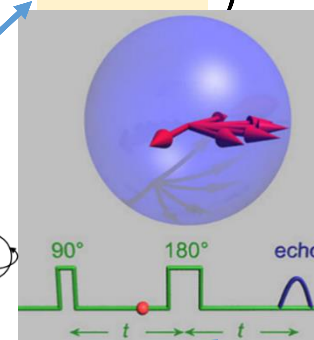




quantized!

Electric charge $\left(E \propto \frac{1}{r^2}\right)$ + magnetic dipole $\left(B \propto \frac{1}{r^3}\right.$, magnets$\left.\right)$ + "gyroscope" $\left(L = \frac{\hbar}{2}\right.$ of field? $\left.\right)$

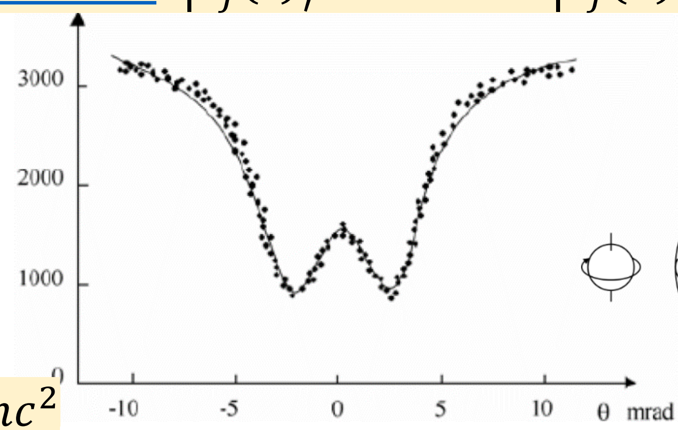
+ $\approx 10^{21} Hz$ zitterbewegung (observ.)/de Broglie's clock $(E = mc^2 = \hbar\omega)$:



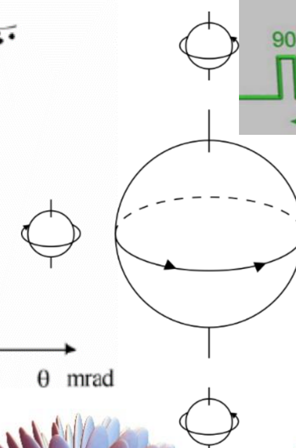
some internal periodic process ~neutrino oscillations: $|\nu_j(t)\rangle = e^{-iE_j t/\hbar}|\nu_j(0)\rangle$

Along <110> axis of silicon crystal atomic spacing correspond to 'internal clock' period for $E \approx 81MeV$ electrons – observed resonance:

P. Catillon, N. Cue, M.J. Gaillard, R. Genre, M. Gouanère, R.G. Kirsch, J.-C. Poizat, J. Remillieux, L. Roussel, M. Spighel, *A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling*, Found Phys (2008) 38: 659–664



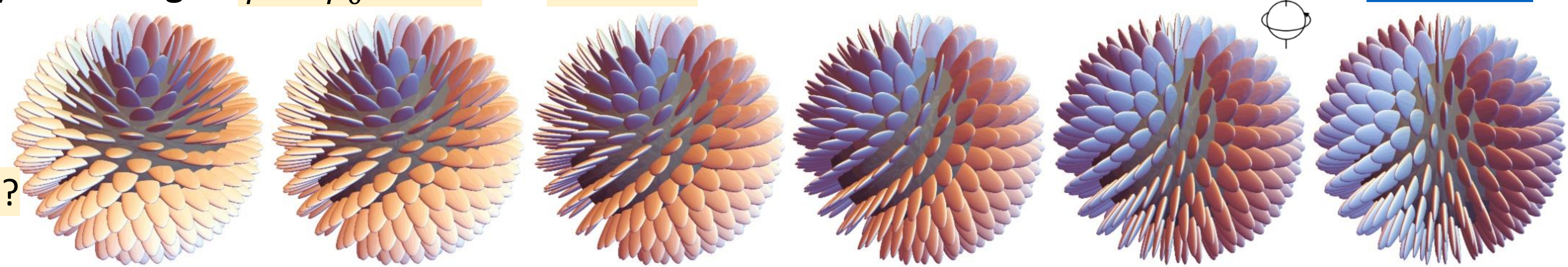
spin echo



"fluid drag"

stationary **Schrödinger**: $\psi = \psi_0 e^{-iEt/\hbar}$ for $E = mc^2$

twist as phase ψ spinning/ evolution?



What are **non-perturbative field configurations** of e.g. electron, quark, **neutrino** for its weak interactions?

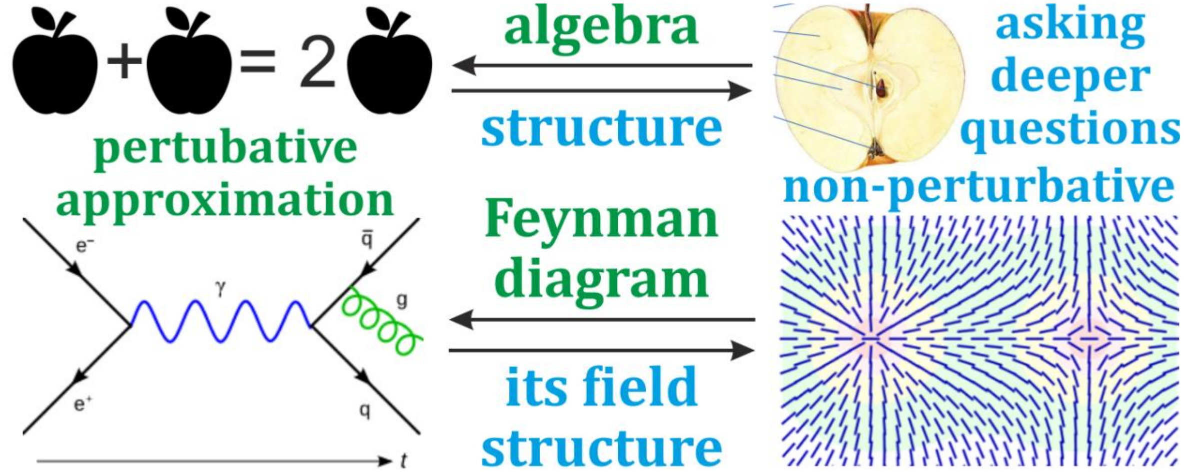
More stable than EM?

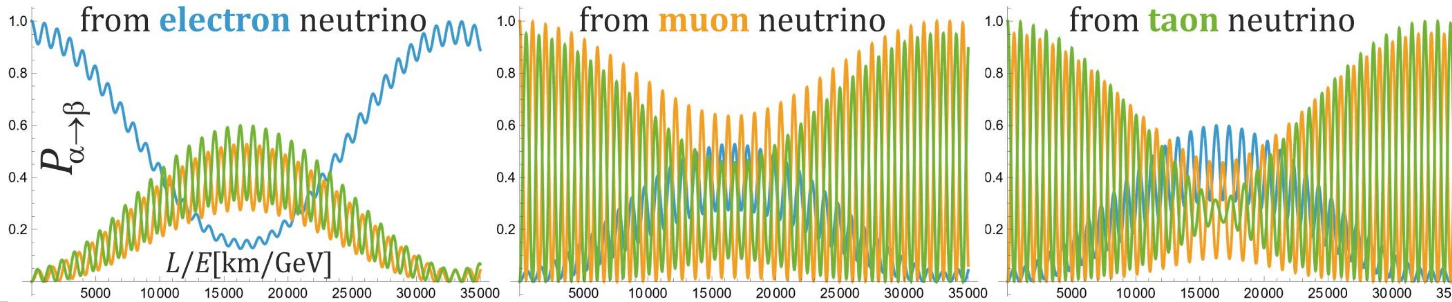
3 types? In 3D …

Resting [oscillates]

$$|\nu_j(t)\rangle = e^{-iE_j t/\hbar}|\nu_j(0)\rangle$$

("[time crystal]")?

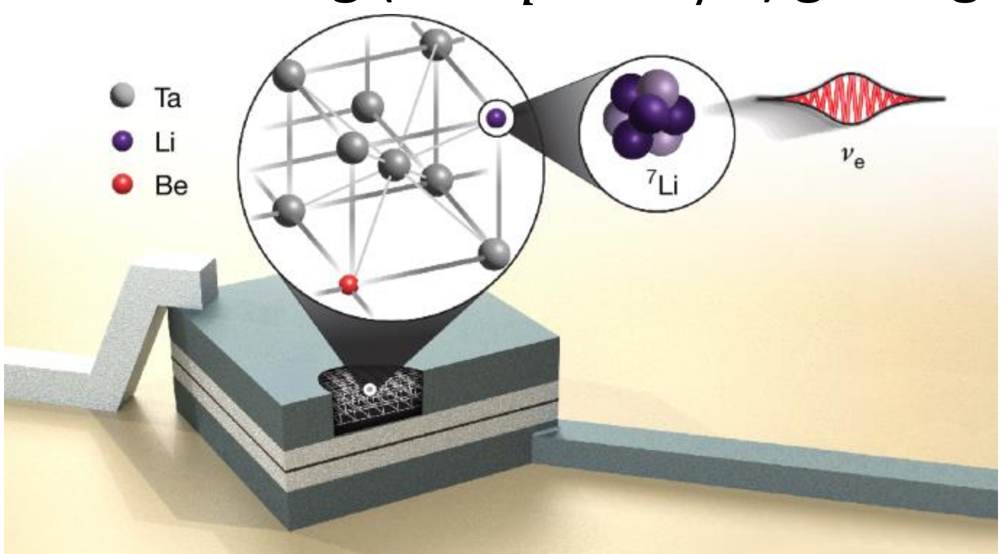


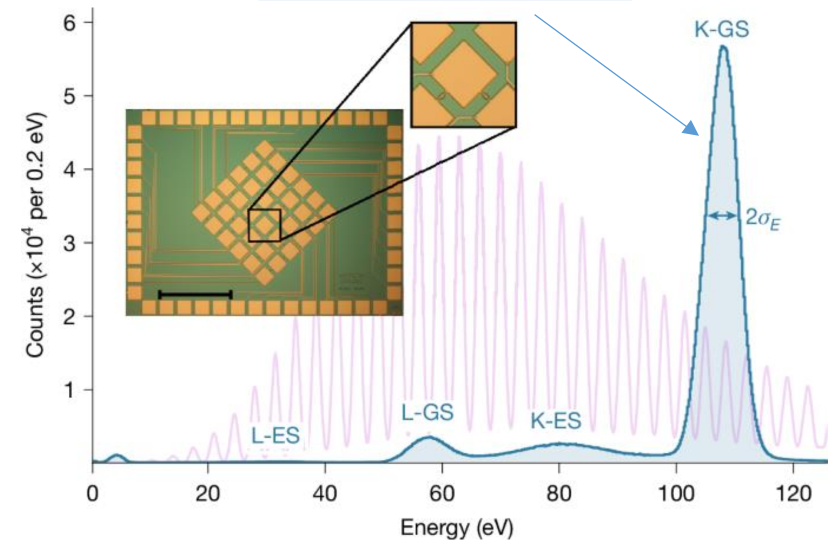


nature.com/articles/s41586-024-08479-6: **neutrino > 1000x lager than nucleus**

$^7\mathbf{Be} + \mathbf{e}^- \to {}^7\mathbf{Li} + \boldsymbol{\nu}_\mathbf{e}$ measure $^7\mathbf{Li}$ **recoil energy due to emitted neutrino**

From Heisenberg ($\Delta x\, \Delta p \geq \hbar/2$) **getting wavefunction** $\boldsymbol{\Delta x > 6.2}$ **pm**





electron "clock of field" → coupled wave → quantum phenomena

Electron clock forms coupled "pilot" waves for wave-particle duality

already classical "walking droplets" leading to QM-like phenomena

Single-Particle **Diffraction** and **Interference** at a Macroscopic Scale, **PRL** 2006

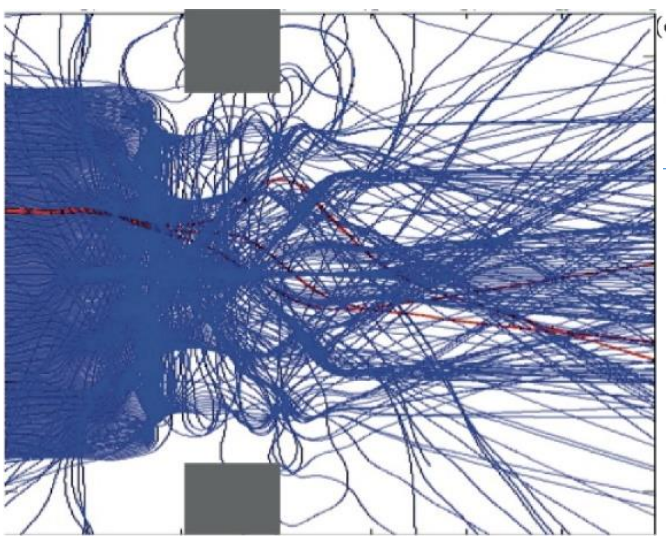
Unpredictable **Tunneling** of a Classical Wave-Particle Association, **PRL** 2009

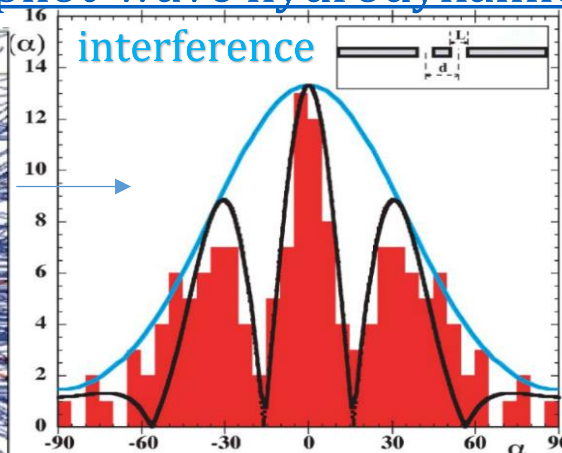
Path-memory induced **quantization of** classical **orbits**, PNAS 2010

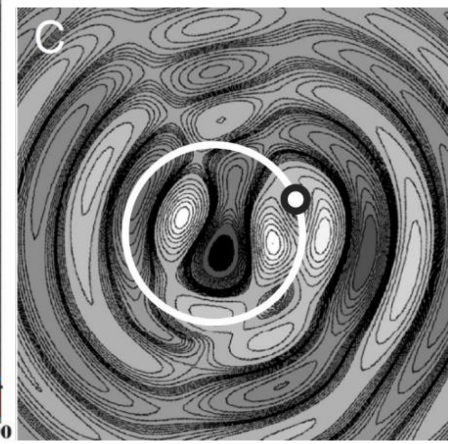
Self-organization into **quantized eigenstates** of a classical wave-driven particle, **Nature** 2014

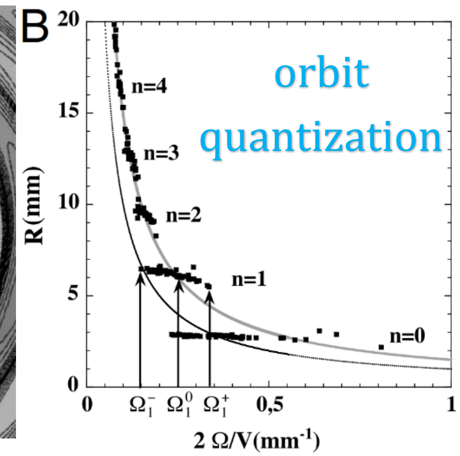
Wavelike statistics from pilot-wave dynamics in a circular corral, PRE 2013

Static **Bell test** in pilot-wave hydrodynamics, PRF 2024 - **violate CHSH**

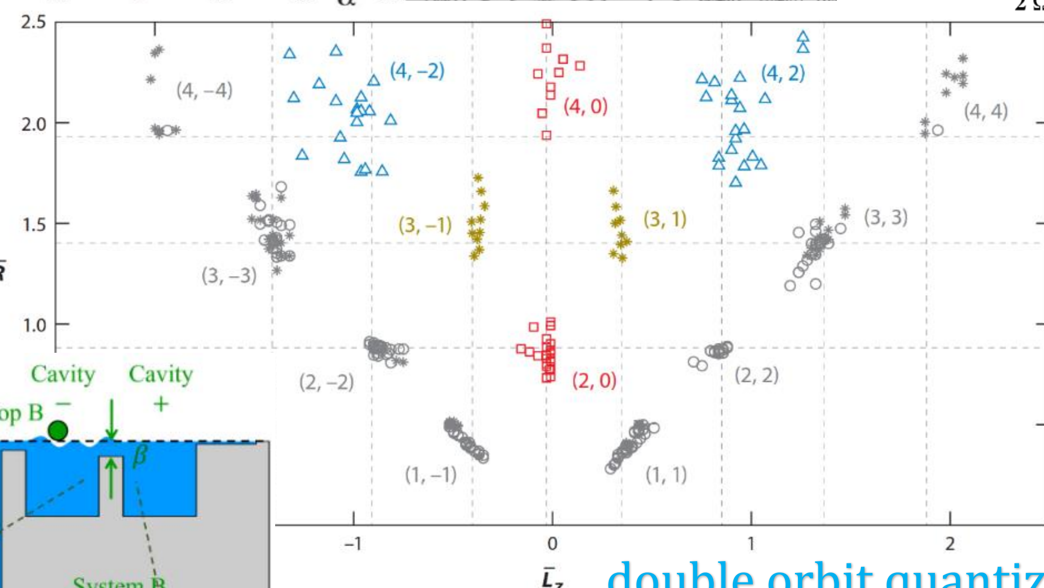


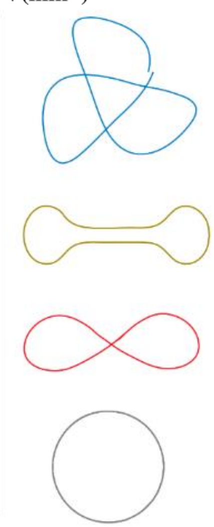


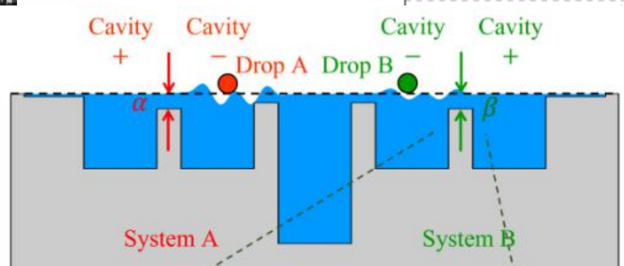




needs shaker...







double orbit quantization (n, l)

1D sine-Gordon – e.g. lattice of coupled pendula, just $\varphi_{tt} - \varphi_{xx} = -\sin\varphi$

angle wave in gravity: pendula prefer to be down, but "down" is every $2\pi k$

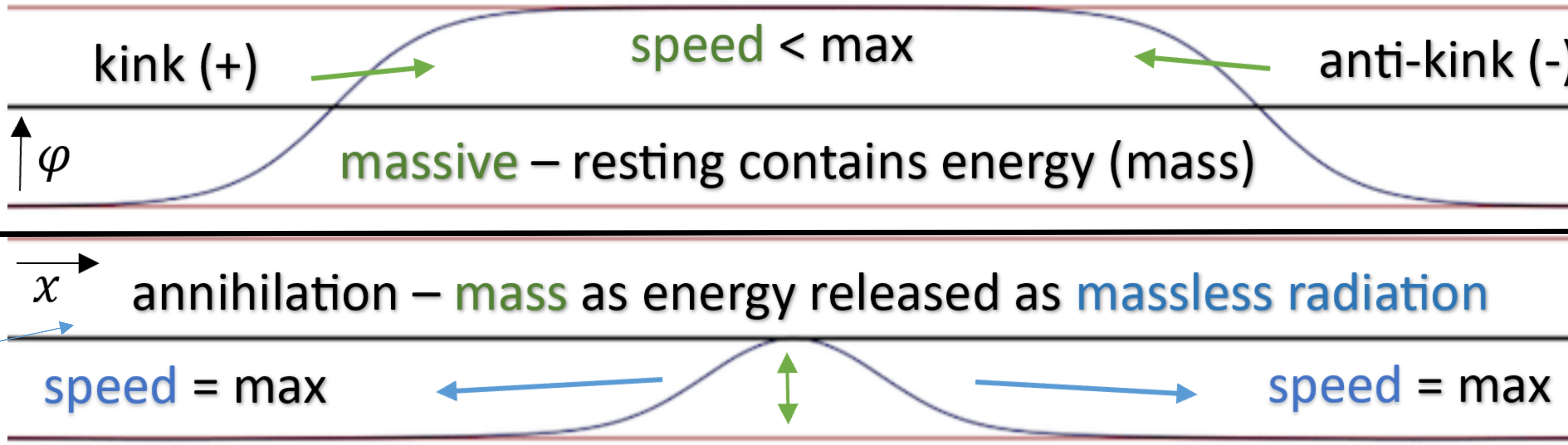
Pair creation/annihilation of **massive particles:** $\boldsymbol{m = \int_{-\infty}^{+\infty} \frac{1}{2}(\varphi_x^2 + \varphi_t^2) + V\, dx}$

special relativity: velocity $\leq$ max, Lorentz contraction, time dilation

(anti)kink: massive particle, quantized

vid., anim.

img



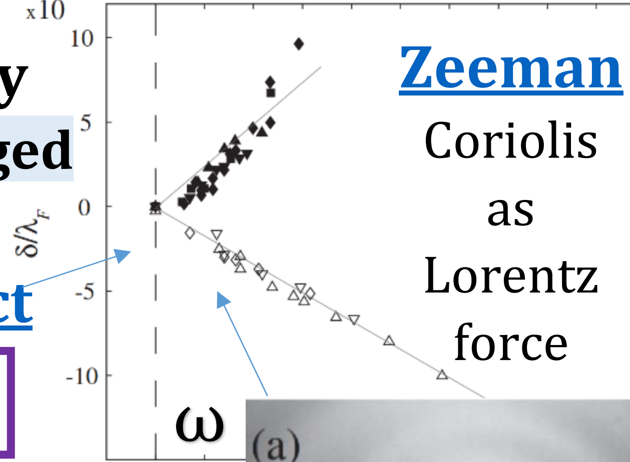
electromagnetism(c) ↔ hydrodynamics(c_s) analogy

(superconducting) … **[Barnett effect]: B from ω for uncharged**

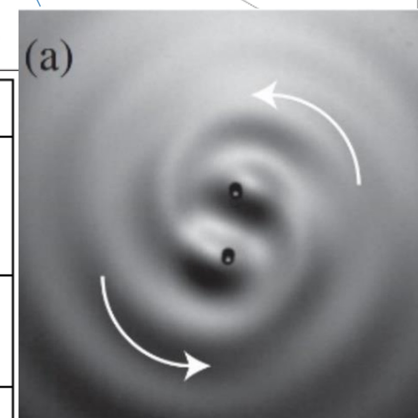
$\boxed{\boldsymbol{B} = \nabla \times \boldsymbol{A} \leftrightarrow \boldsymbol{\omega} = \nabla \times u}$ for $A \leftrightarrow u$ [velocity], [vorticity] ω

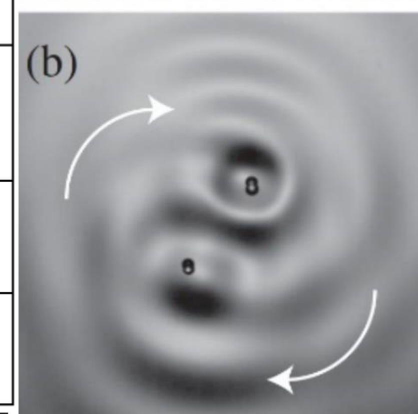
Lorentz force ↔ Coriolis force e.g. for [Zeeman effect]

[current/flow e.g. in superfluid:] $\vec{J} \sim \boxed{\vec{v}_s m = \hbar \vec{\nabla} \varphi - q\vec{A}}$



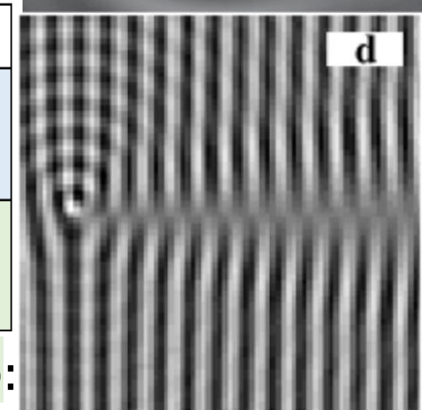
[Zeeman]
Coriolis
as
Lorentz
force





Hydrodynamics superfluid $\nu = 0$		**Electromagnetism**		analogs	
Navier-Stokes equation	$\frac{\partial u}{\partial t} = -l - \nabla\phi_p + \nu\nabla^2 u$ $\phi_p = p/\rho + u^2/2$	$\frac{\partial A}{\partial t} = -E - \nabla\phi$ E - electric field	vector and scalar potential	u l	A E
vorticity Lamb vector $l \equiv \omega \times u$	$\omega = \nabla \times u$ $\quad \nabla \cdot \omega = 0$	$B = \nabla \times A$ $\quad \nabla \cdot B = 0$ magnetic induction	Coulomb Thomson	ω	B
vorticity tendency	$\frac{\partial \omega}{\partial t} = -\nabla \times l + \nu\nabla^2\omega$	$\frac{\partial B}{\partial t} = -\nabla \times E$	Faraday's law		
Lamb vector tendency and turbulent current (j)	$\frac{\partial l}{\partial t} = \nabla \times \eta - j$	$\frac{\partial(\epsilon_0 E)}{\partial t} = c^2 \nabla \times H - \mu_0 J$ (zero polarization)	Amper's law	η	H
vorticity field strength (η) and magnetization	$\eta = \lvert u^2 \rvert \omega - M$ $M = u(u \cdot \omega) + \nu\nabla^2 u$	$H = \mu_0^{-1} B - M$	magnetic field strength and magnetization		
turbulent charge density	$\nabla \cdot l = u \cdot \nabla \times \omega - \lvert\omega^2\rvert = \rho_n$	$\nabla \cdot (\epsilon_0 E) = \rho_e$ [link]	electric charge density	ρ_n	ρ_e

Theory	Gauge fields	Circulation	Gauge condition	Matter field
Electro-dynamics	φ, $\vec{A}$ four-potential	$\vec{B} = \vec{\nabla} \times \vec{A}$ magnetic f.	$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0$	$\vec{E}_e = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi$
Hydro-dynamics	$\chi = v^2/2, \vec{v}$ flow velocity	$\vec{\omega} = \vec{\nabla} \times \vec{v}$ vorticity	$\vec{\nabla} \cdot \vec{v} + \frac{1}{c_s^2}\frac{\partial \chi}{\partial t} = 0$	$\vec{E}_h = -\frac{\partial \vec{v}}{\partial t} - \vec{\nabla}\chi$



[link] [Aharonov-Bohm] (Berry) electron~hydro:

Liquid crystal long-range interactions due to **nontrivial vacuum** like for **Higgs** $V(\vec{n}) = (|\vec{n}|^2 - 1)^2$

$F \sim 1/D$: "**Annihilation** dynamics of topological defects induced by microparticles in nematic liquid crystals" Soft Matter

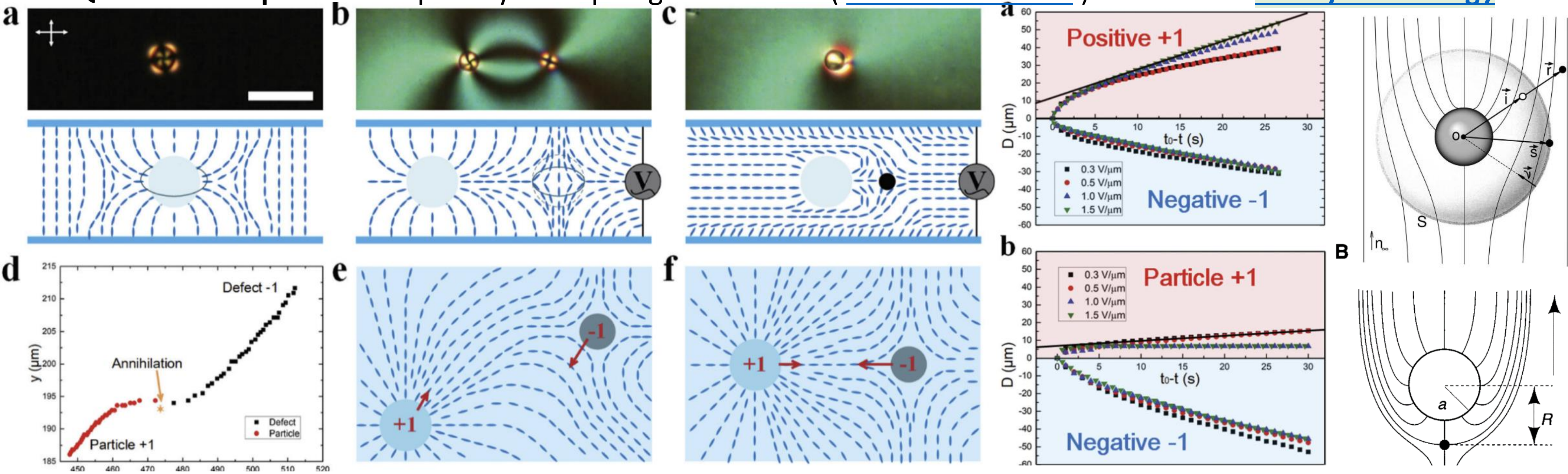
Coulomb: "**Coulomb**-like interaction in nematic emulsions induced by external torques exerted on the colloids" PRE

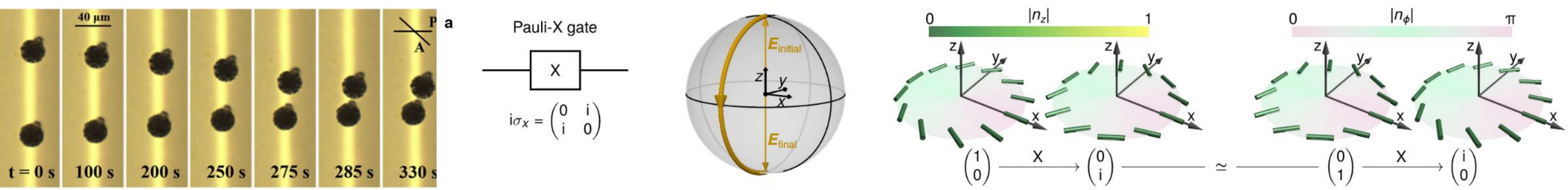
"**Coulomb**-like elastic interaction induced by symmetry breaking in nematic liquid crystal colloids" Scientific Reports

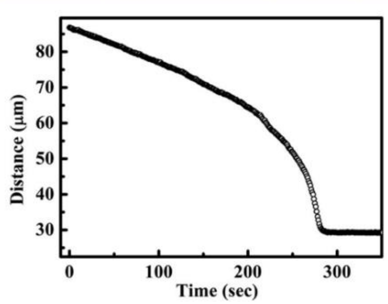
dipole-dipole: "Novel Colloidal Interactions in Anisotropic Fluids" Science **repulsion** Nature

quadrupole-quadrupole: "**Long-range forces** and aggregation of colloid particles in a nematic liquid crystal" PRE

Quantum computers on liquid crystal topological defects? (Science Advances) **EM-hydro analogy**







Theory	Gauge fields	Circulation	Gauge condition	Matter field
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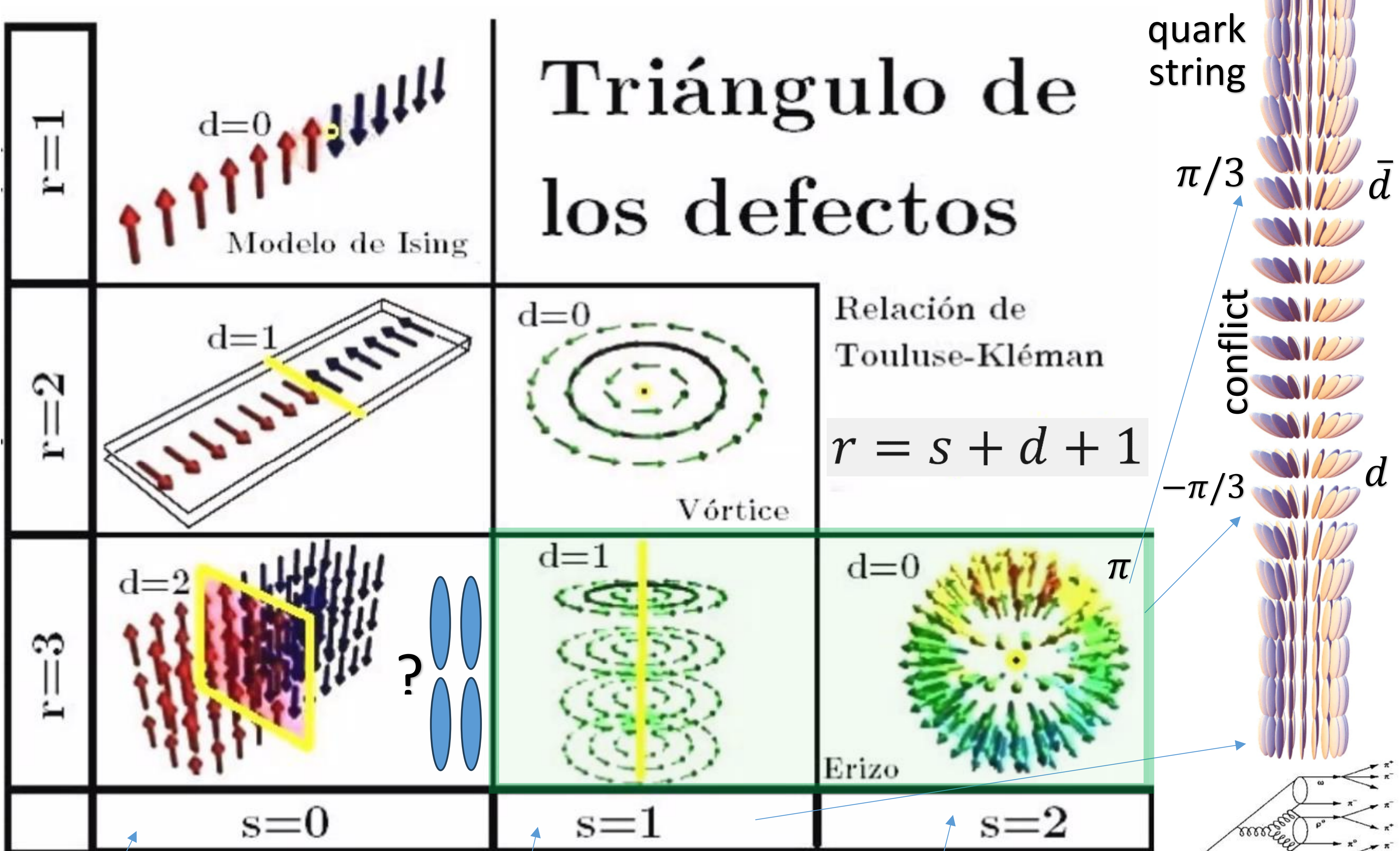
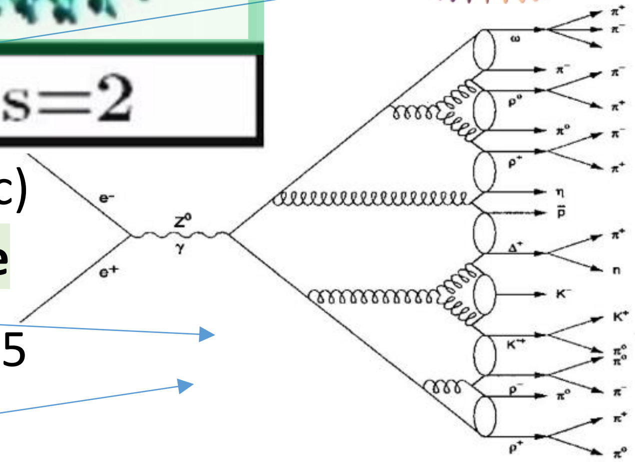
Hydro-dynamics	$\chi = v^2/2, \vec{v}$ flow velocity	$\vec{\omega} = \vec{\nabla} \times \vec{v}$ vorticity	$\vec{\nabla} \cdot \vec{v} + \frac{1}{c_s^2}\frac{\partial \chi}{\partial t} = 0$	$\vec{E}_h = -\frac{\partial \vec{v}}{\partial t} - \vec{\nabla}\chi$





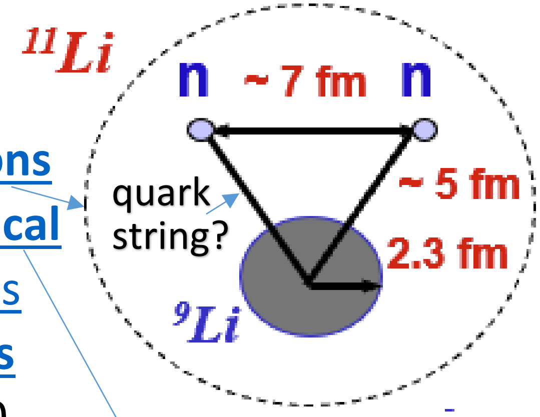
Particle physics in 3D: dual $B \leftrightarrow E$ (not magnetic)

domains?, **quark string/gluon flux tube**, **quantized electric charge**

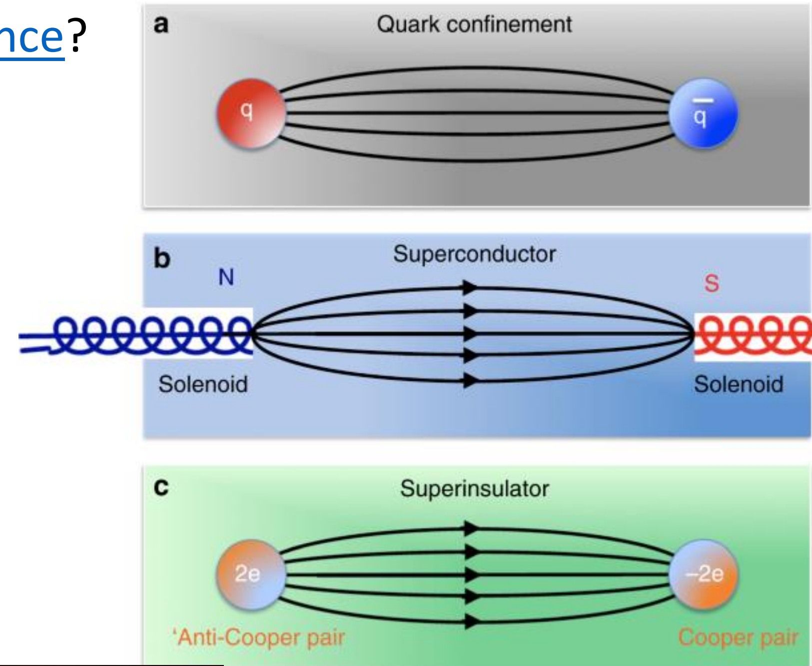
Gareth P. Alexander et al., **Liquid crystal** reviews, 10, 1, 91-97, 2005

String hadronization – its decay into particles in LHC collisions

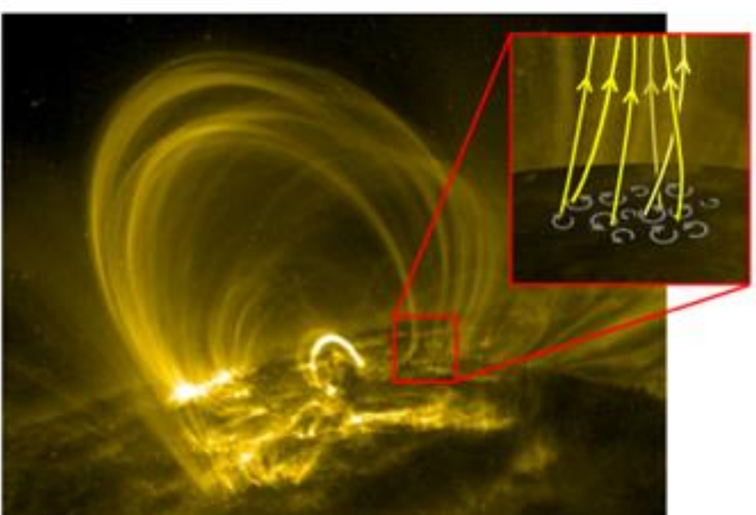
Required: **vacuum quark strings/flux tubes** – with energy density:

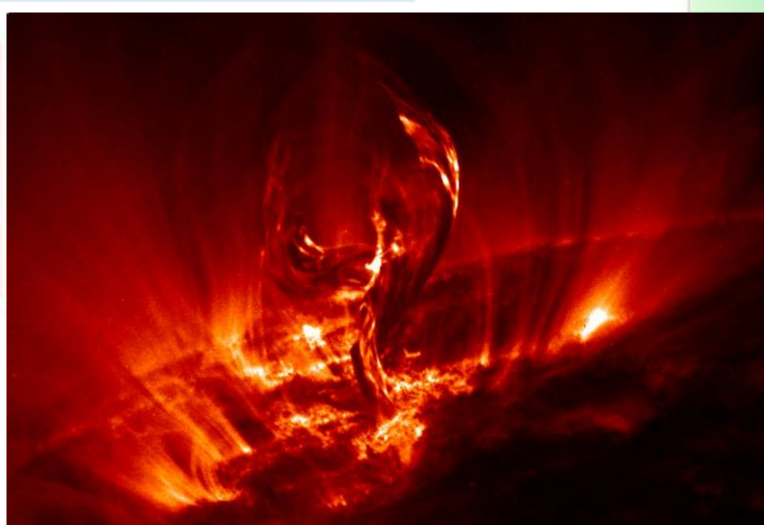


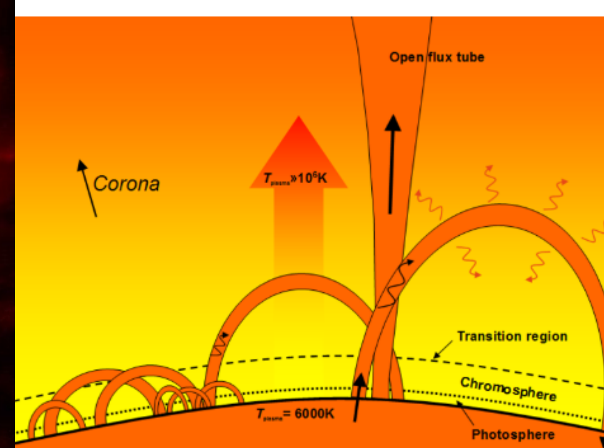
- **Holding nucleus together against Coulomb repulsion (?), halo neutrons**
- **popular quark string model, Nature article suggesting being topological**
- **coronal** heating problem (surface 6000K, corona 10^7K), reconnections
- **electrons in anti-parallel alignment – in 100-1000nm for Cooper pairs**
- Brawley et. al., "Electron-like scattering of positronium", Science 2010
- **Cosmic strings** – 1D topological defects of $\arg(\psi)$, evidence?

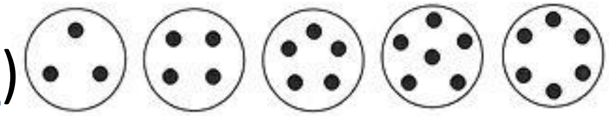


"Physics of **Magnetic Flux Tubes**" book by Ryutova:
"Vortices in superfluid Helium and superconductors, magnetic flux tubes in solar atmosphere and space, filamentation process in biology and chemistry have probably a common ground, which is to be yet established. One conclusion can be made for sure: **formation of filamentary structures in nature is energetically favorable and fundamental process**."



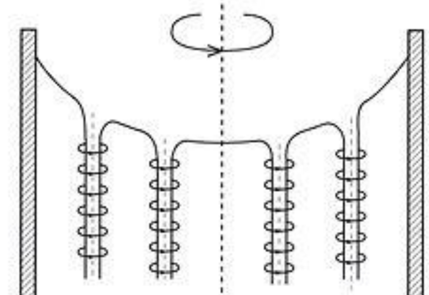






wiki/**Macroscopic_quantum_phenomena** (v1, v2, photo)

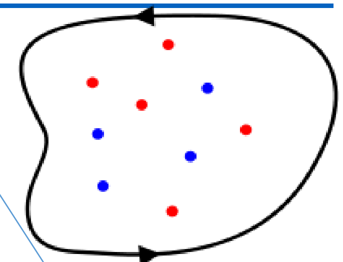
Superfluid: quantized angular momentum (of field alone)



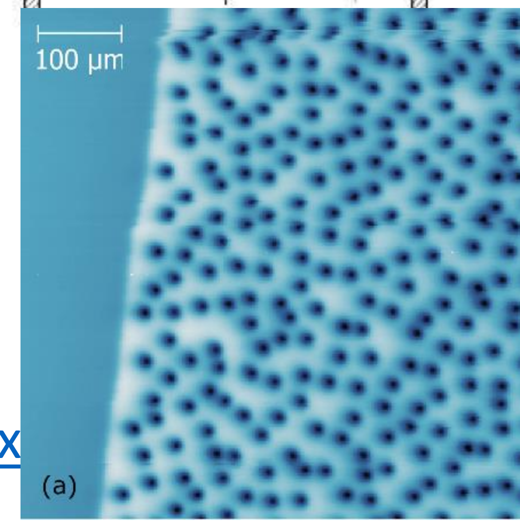
"'Perfect liquid' **quark-gluon plasma** is the most vortical fluid"

Superconductor: quantized magnetic field

$$\boldsymbol{2\pi n = \Delta\varphi} = \frac{q}{\hbar}\oint_{\partial S} A\cdot dl = \frac{q}{\hbar}\int_S B\cdot dS$$



~fluxon

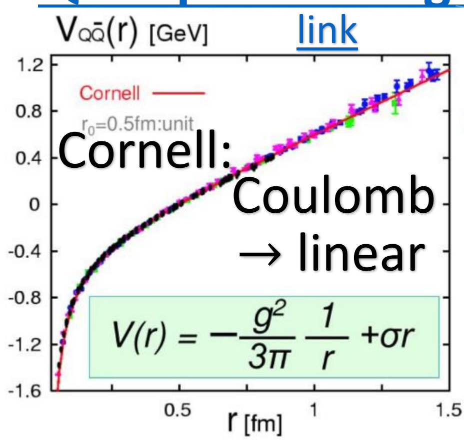


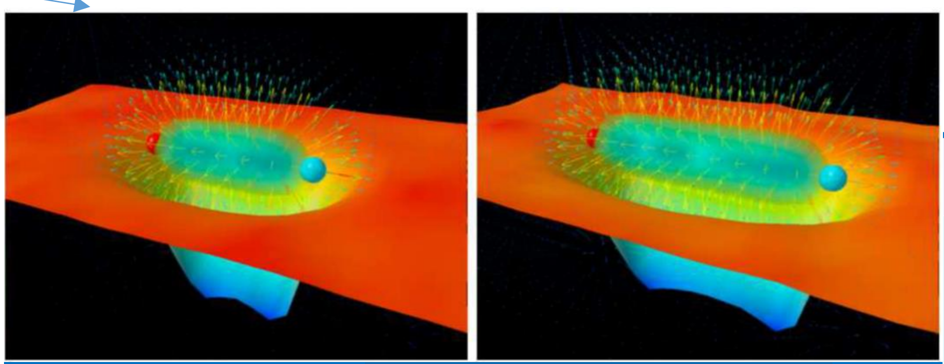
current/velocity: $\vec{J} \sim \vec{v}_s = \left(\hbar\vec{\nabla}\varphi - q\vec{A}\right)/m$

2D: argument principle in complex analysis:

$$\oint_C \frac{f'(z)}{f(z)}dz = \oint_C (\ln f)'\,dz = 2\pi i(\#\mathrm{zeros} - \#\mathrm{poles})$$

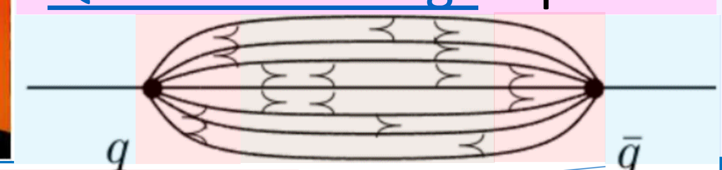
QCD quark strings/flux tubes often modeled as Abrikosov vortex

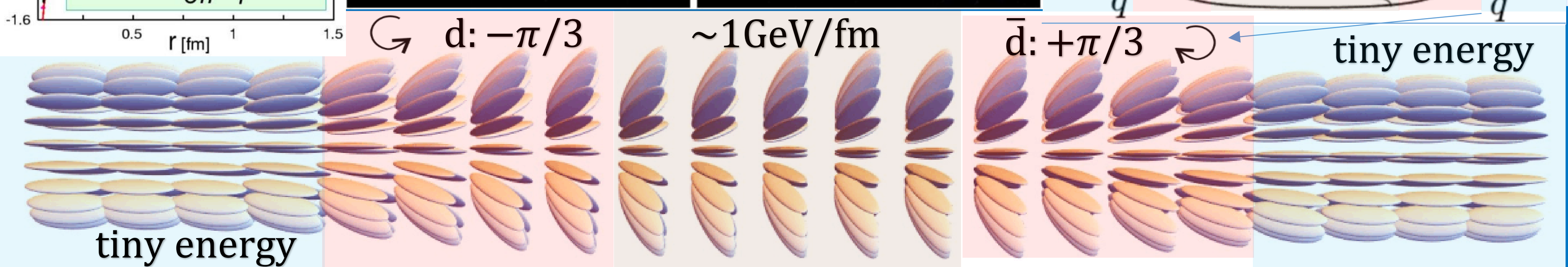




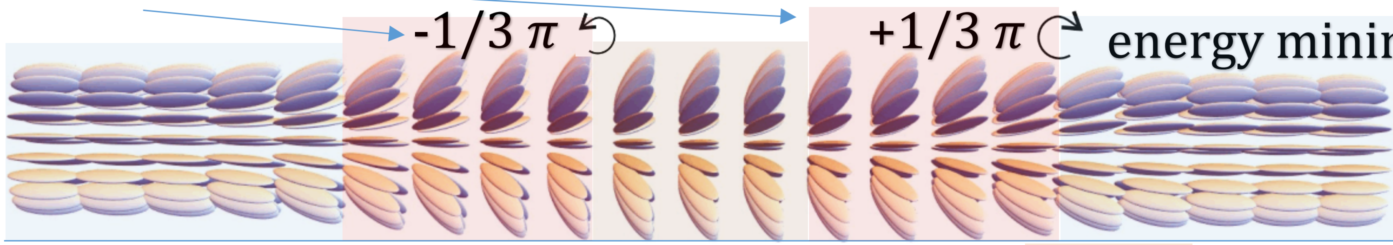
String hadronization?

~QCD color strings...quarks?

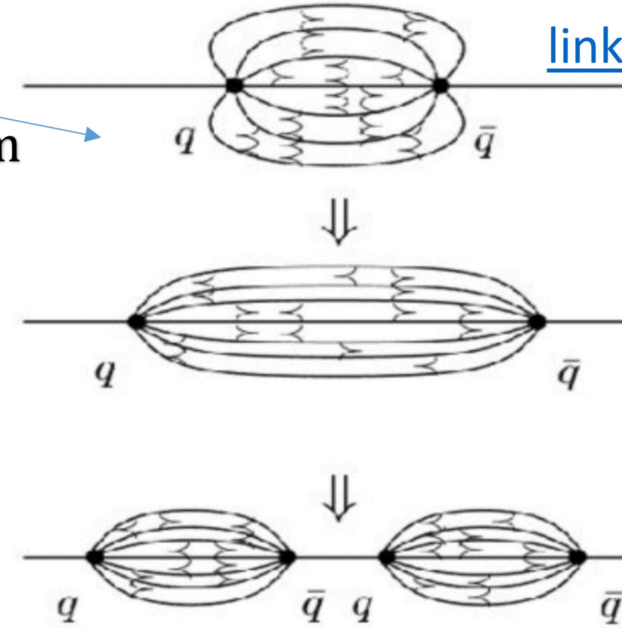




Quark string/gluon flux tube – shouldn't leave baryons?



link



Binding of halo nuclei like Boron-8 with **proton** for 770ms

Ne-17: 2 halo **protons** 109ms, 1p P-26, 2p S-27 44,16ms

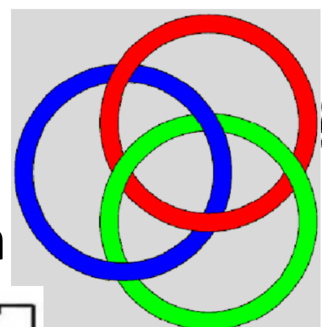
Li-11**:** 8.6 ms lifetime in ~5 fm distance

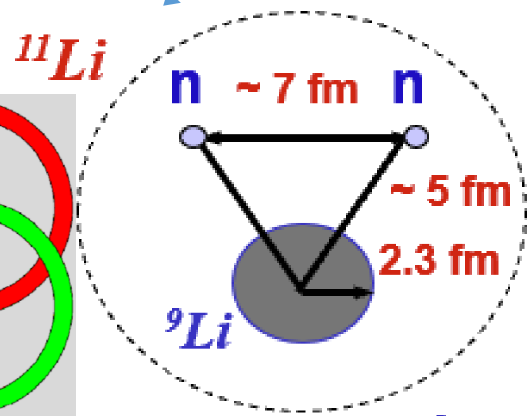
Efimov state **field conf.?**

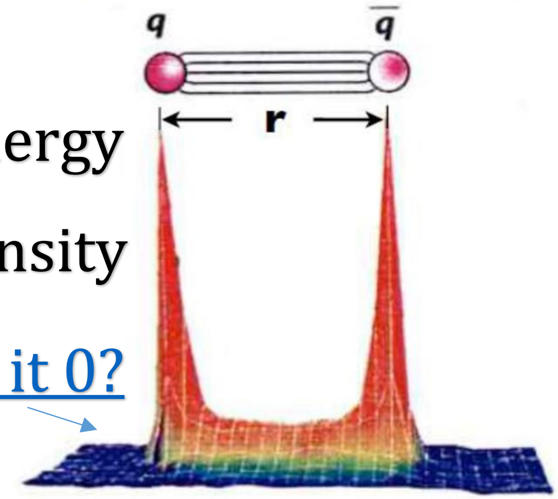
Borromean halo nuclei

requiring 3-body forces

diagrams - He6: He4+nn

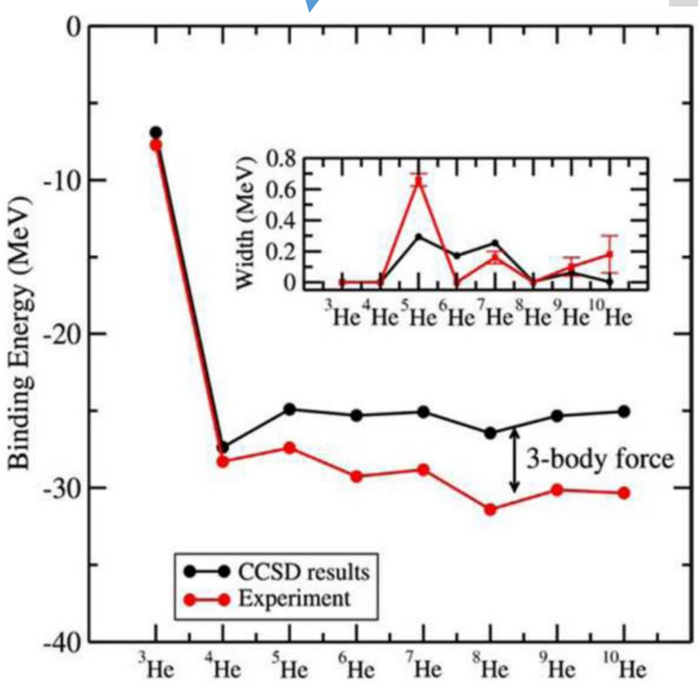


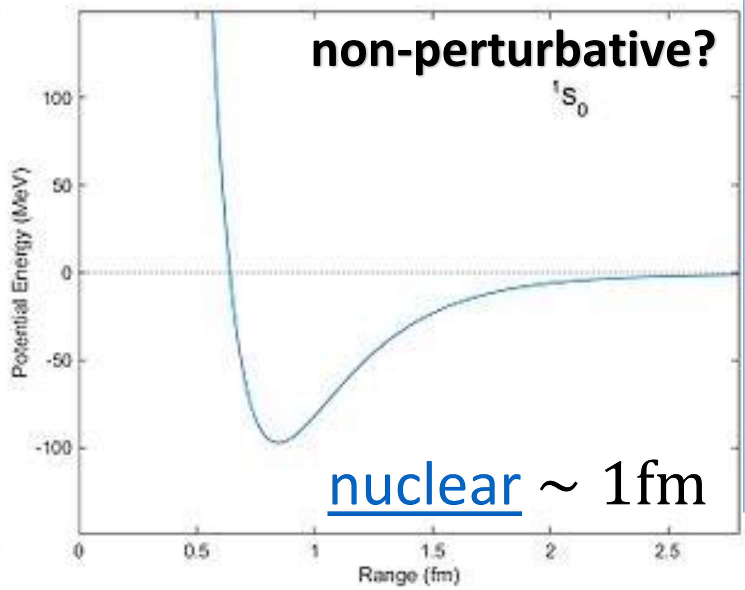




Energy density

is it 0?

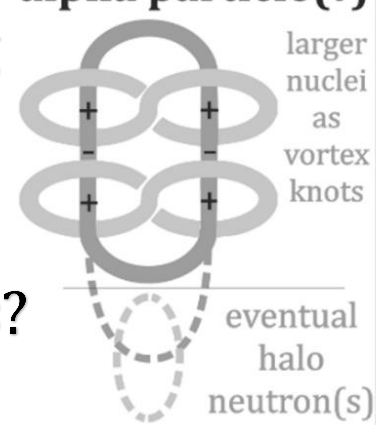




nuclear ~ 1fm

alpha particle(?)

binding **larger nuclei** as **knots**?



2D topological charge

fluxon – quant of magnetic field

resembles **spin** ($\to \mu$) also **½**

$$2\pi k = \Delta\varphi = \frac{q}{\hbar}\oint_{\partial S} A\cdot dl = \frac{q}{\hbar}\oint_{S} B\cdot dS$$

quantum rotation operator

spin s particle by θ angle rotates

quantum phase: $\psi \to \psi\, e^{-is\theta}$

spin, charge: structural, **dynamic**:

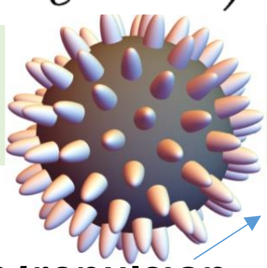
$\mu_z \sim \ell_z$ twisting **lepton $g \approx 2$**

g-factor: $\mu_z = g\, ½ \left(\frac{\hbar e}{2m}\right) \approx (\mu_B)$

(½ spin) bispinor rotation by ϕ:

$$S[\Lambda_{\text{rot}}] = \begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2} \end{pmatrix}$$

3D topological charge: electric



$V(r)$ distance dep.

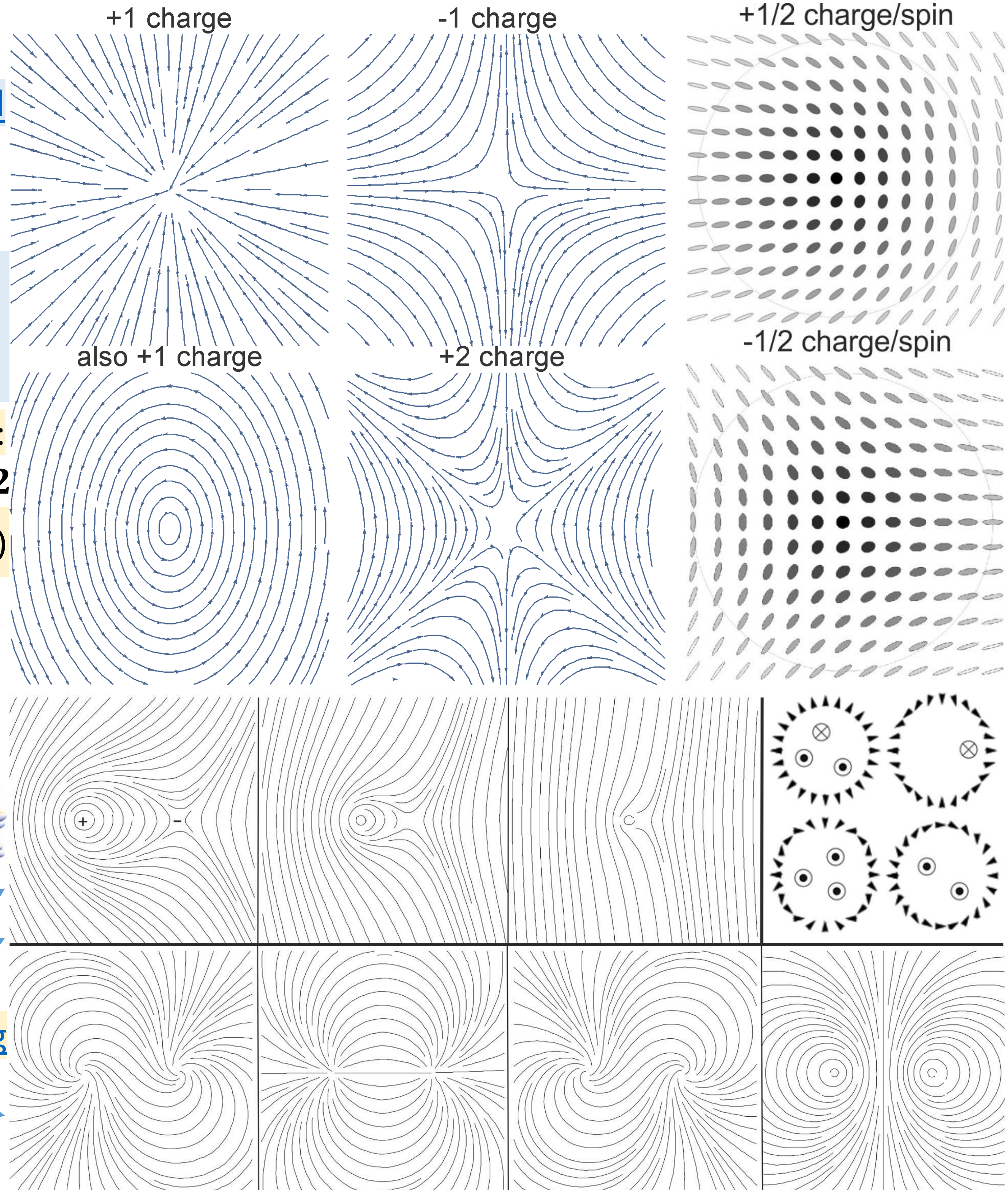
long-range attraction/repulsion:

virtual particle: perturbation toward

de Broglie clock/zitterbewegung

twisting of **quantum phase**

around - forming pilot wave

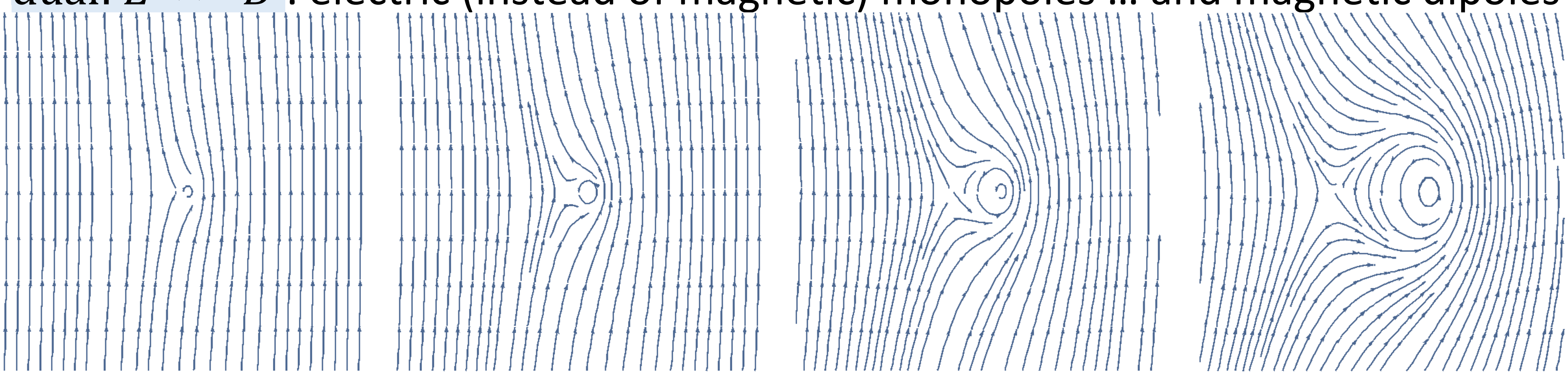


electromagnetism	**topological charge/index/degree**
charge conservation	the number of **covering** of $\boldsymbol{f}: \boldsymbol{S^{D-1}} \to \boldsymbol{S^{D-1}}$ **Hopf thm**: f, g are homotopic $\Leftrightarrow deg\, f = deg\, g$ **Poincare-Hopf thm.**: $\sum_i deg_{x_i} = \chi(S)$ **Riemann-Hurwitz thm.**: $\chi(n \text{ covering of } S) = n\, \chi(S)$
Gauss law (in **3D**): **charge** inside $S = \oint_S E \cdot dS$ charge is a real number	**argument principle in 2D:** $\oint_C (\ln f)' dz = 2\pi i (N-P)$ general - quantized **Gauss-Bonnet theorem**: $\oint_S R \cdot dS = 2\pi n\, \chi(S)$ (R: curvature/Jacobian)
Coulomb: same charges repel, opposite attract $F \propto 1/r^2$	to **reduce stress of the field** entire field causes attraction/repulsion

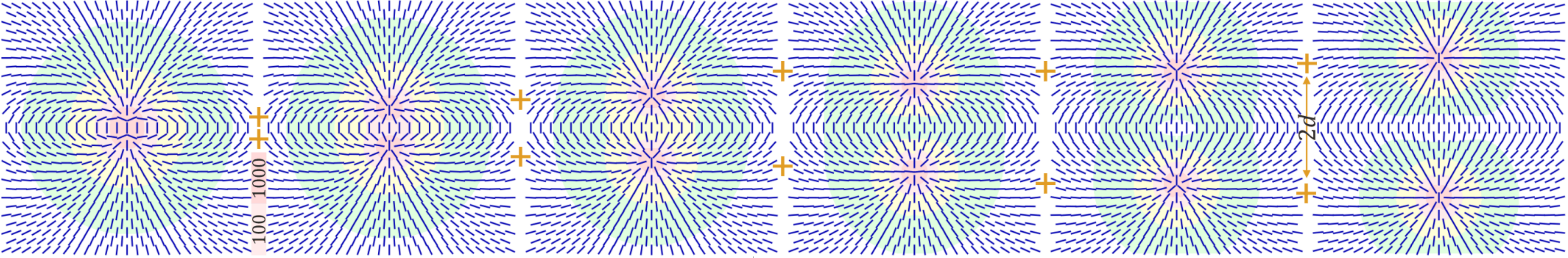
$\vec{\Gamma}_i = (\partial_i \vec{n}) \times \vec{n}$ $\quad \vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ $\quad \mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu}$ Faber's quantized EM

$|\vec{n}| = 1$, **connection** $\Gamma \sim A$, **EM Lagrangian** with **curvature** $F^*_{\mu\nu} \sim R_{\mu\nu}$ (+Higgs)

dual: $E \leftrightarrow^* B$: electric (instead of magnetic) monopoles … and magnetic dipoles



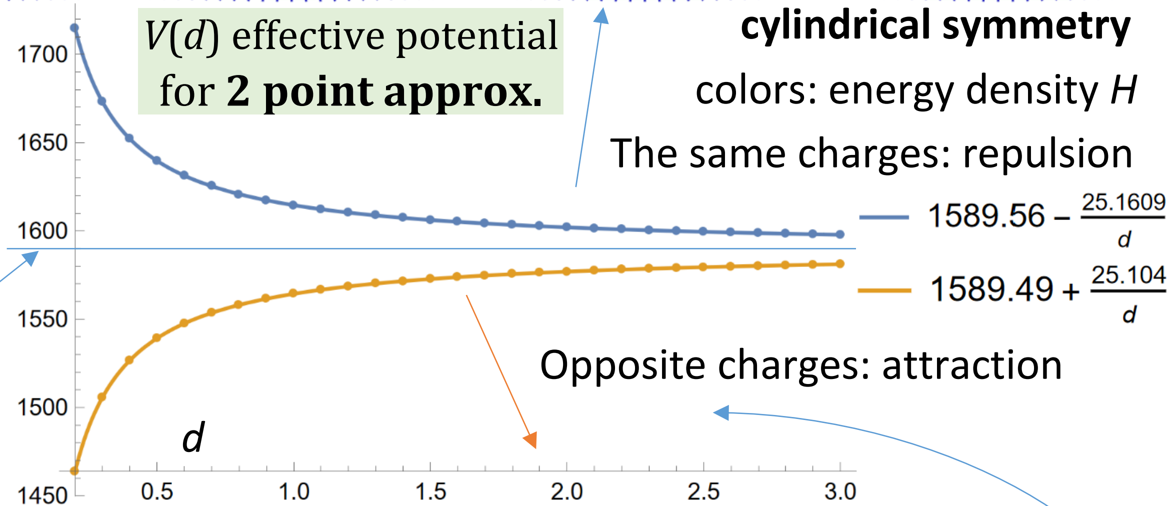
Virtual pair creation: start < 2x511 keV and return, ~perturbation toward pair creation



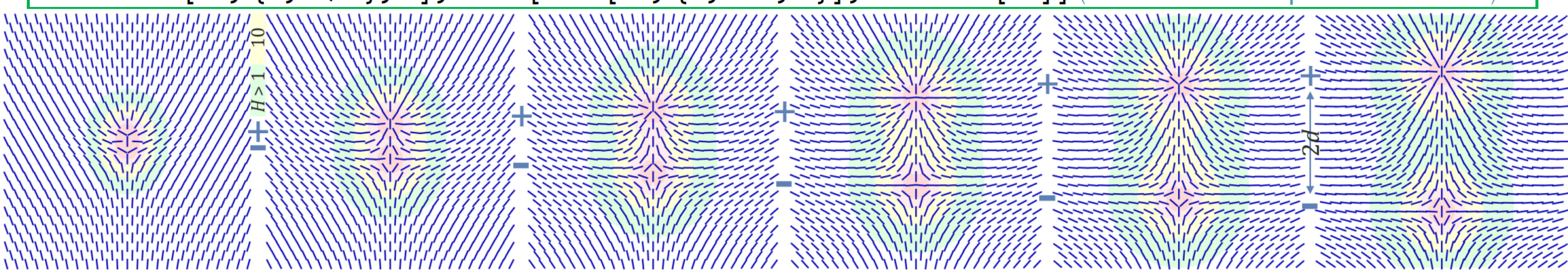
Field energy: ~Coulomb potent.

$$E \approx \frac{m_0}{\sqrt{1-v_1^2}} + \frac{m_0}{\sqrt{1-v_2^2}} + V(d)$$

cutoff ϵ around singularities:
to be **regularized** to rest masses
Lorentz inv.: SRT scaling, magnetism



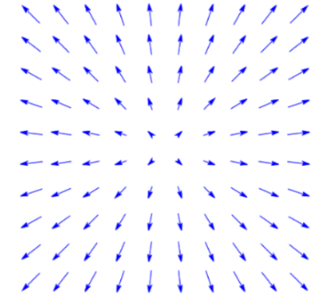

```
cos = 1 + (z - d) / Sqrt[(z - d)^2 + r^2] - (z + d) / Sqrt[(z + d)^2 + r^2];  (*Manfried Faber dipole ansatz*)
n = {Sqrt[1 - cos^2] x / r, Sqrt[1 - cos^2] y / r, cos} /. r → Sqrt[x^2 + y^2];  (* cylindrical symmetry *)
M = KroneckerProduct[n, n]; dM = {D[M, x], D[M, y], D[M, z]};  (*vector n → matrix M field*)
H = Simplify[Sum[Total[(dM[[i]].dM[[j]] - dM[[j]].dM[[i]])^2, 2], {i, 2}, {j, i + 1, 3}]]; (*Hamilitonian*)
Es = Table[{d, NIntegrate[4 Pi * x (H /. {y → 0}) * Boole[x^2 + (z - d)^2 > 0.001],  (* 0.001  cutoff *)
      {x, 0, Infinity}, {z, 0, Infinity}]}, {d, 0.1, 3, 0.1}]; (*integrate H: total field energy*)
ft = Fit[Es, {1, 1 / d}, d]; Show[Plot[ft, {d, 0.2, 3}], ListPlot[Es]] (*fit Coulomb potentiential*)
```

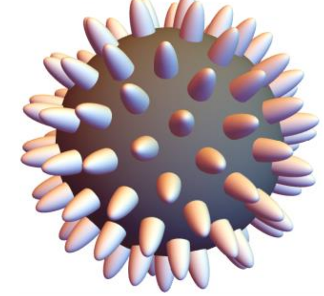


Regularization to finite energy e.g. for "hedgehog":

Asymptotically (vacuum) $||\vec{n}|| \approx 1$, but $\vec{n}(0) = 0$

thanks to **Higgs**-like potential, e.g. $V(\vec{n}) = (\|\vec{n}\|^2 - 1)^2$





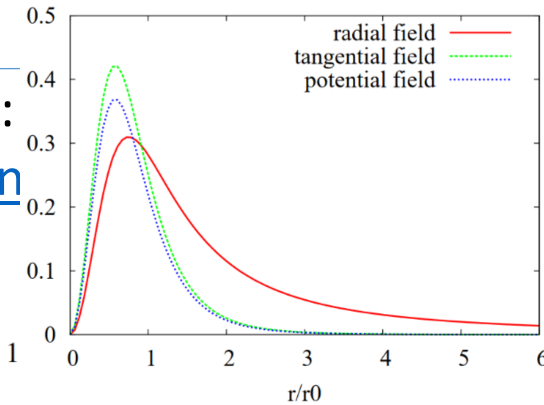
: $\vec{\Gamma}_i = (\partial_i \vec{n}) \times \vec{n} \qquad = \frac{1}{r} \quad ||\vec{n}||^2 \to \frac{1}{r}$ $\qquad \Gamma \propto r^{-1} \qquad R \propto r^{-2}$

$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu \quad \to \frac{1}{r^2}$ electric field in 3D, dual: ${}^*\vec{F}_{\mu\nu} = \frac{-e_0}{4\pi\epsilon_0 c}\vec{R}_{\mu\nu} \sim \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$

$\mathcal{L}_{EM} = -\frac{\alpha\hbar c}{16\pi}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}$ $\to$ electromagnetism in **vacuum**

Gauss law counting topological charge: $$Q_{el}(\mathcal{S}) = \frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} du\, dv\ (\partial_u \vec{n} \times \partial_v \vec{n}) \cdot \vec{n}$$

Jacobian of closed surface $\to S^2$: $\det[\vec{n}, \vec{n}_\mu, \vec{n}_\nu] = \vec{n} \cdot (\vec{n}_\mu \times \vec{n}_\nu)$



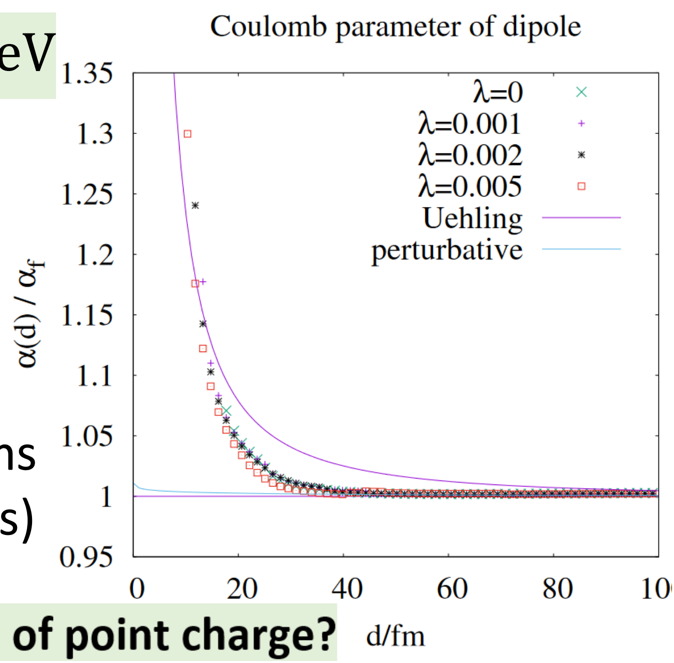
No parton structure (!) for **electron**: point-like, Faber:

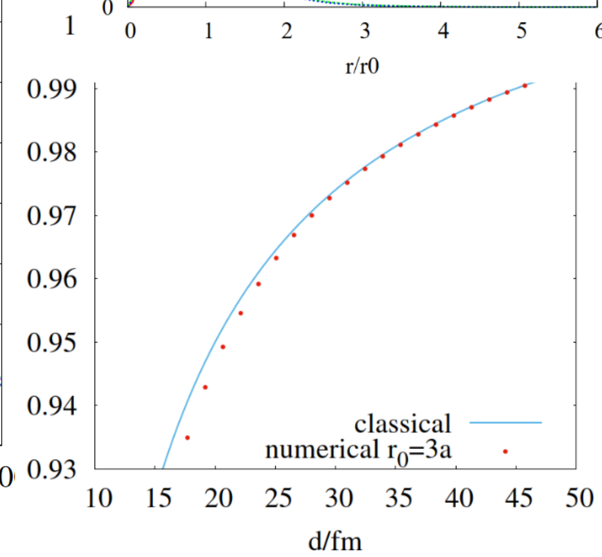
only field deformation not to exceed 511keVs, ~vacuum polarization

$$\int_{\sim 1.4\text{fm}}^{\infty} \frac{1}{2}|E|^2 4\pi r^2 dr = 511\text{keV}$$

Experimental effects of finite size?

Coulomb deformation, **Faber**:





Running coupling:

$\alpha \approx \frac{1}{137} \to \approx \frac{1}{127}$ in 90 GeVs

To hide finite size in Feynman diagrams

(+ renormalization to remove infinities)

Point-like, but with deformed electric field of point charge?

Experimental boundaries for size of electron?

Point-like, but with deformed electric field of point charge?

Dehmelt 1988, Penning trap (Nobel in 1989): $R < 10^{-22}m$

extrapolating from g-factor: "(…) electron as **three smaller much heavier new fermions** [Brodsky, Drell, 1980] (…)"

Neutron (udd): $g \approx -3.8 \qquad < r_{\mathrm{n}}^2 > \approx -0.1\ \mathrm{fm}^2$

Classically: $g = \frac{2m}{q}\frac{\mu}{L} = \frac{2m}{q}\frac{\int A dI}{\omega I} = \frac{m}{q}\frac{\int \rho_q(r)\, r^2 dr}{\int \rho_m(r)\, r^2\, dr}$

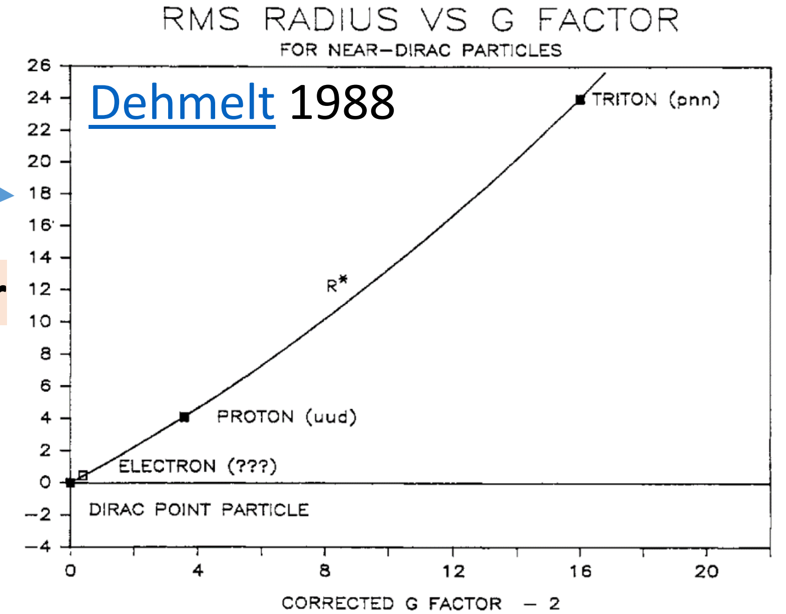


Fig. 8. Normalized RMS radius $R^* = R/\lambda\!\!\!^{-}_C$ vs. corrected g-factor minus 2 for near-Dirac particles [3]. A parabola has been fitted to the data points. Recent theories conjecture that the electron, similar to proton and triton, is composed of three smaller fermions. The data point at the origin represents a Dirac point particle of finite arbitrary charge and mass.

Electron-positron cross-section:

Which energy should we use??? (Lorentz contraction!),

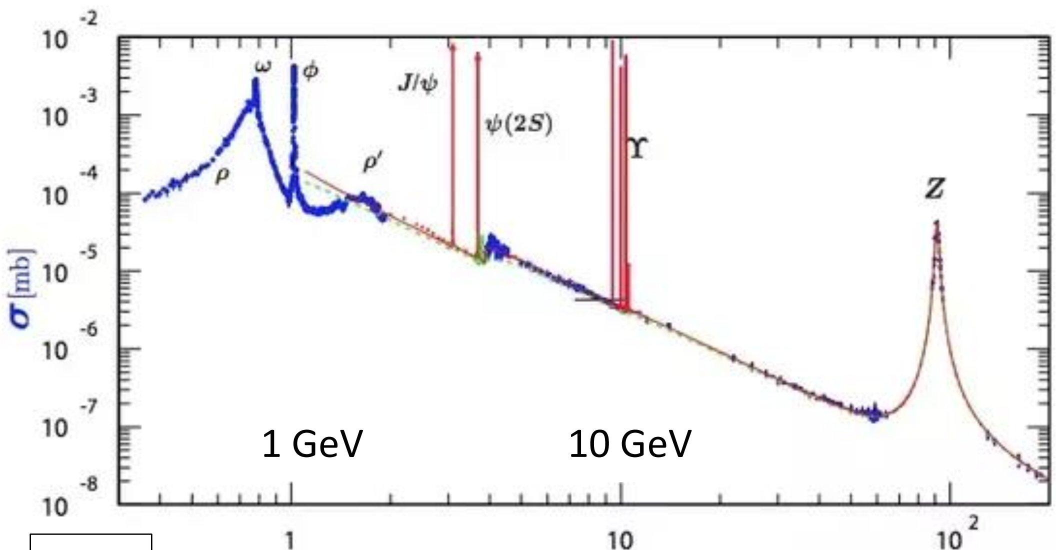
~**line** in log-log, ≈ 100nb for 1GeV $\qquad \gamma \approx 1000$

Extrapolating to resting: $\gamma = 1 \qquad \sigma \propto \gamma^{-2}$

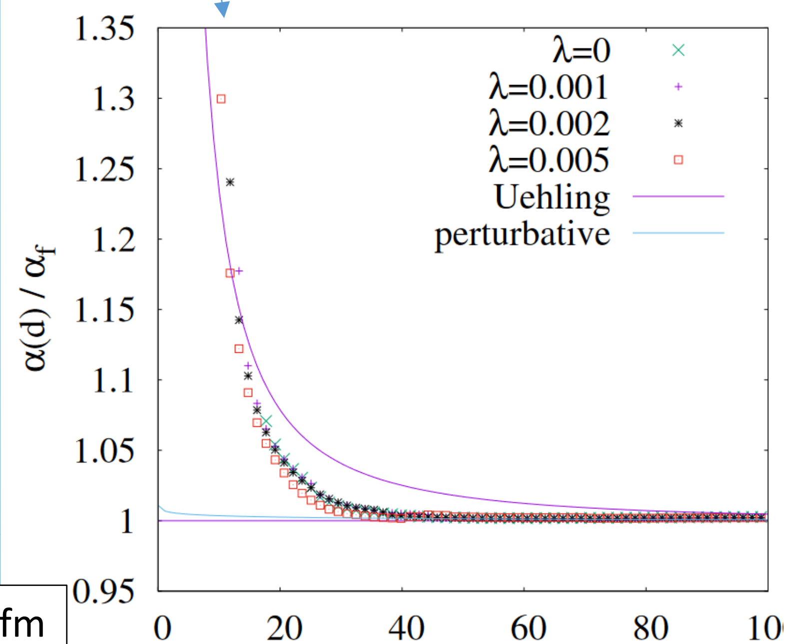
we get ≈ 100mb: $\boldsymbol{r \approx 2}$ **fm**? (energy < 511keV!)

Running coupling – deformation of α (Coulomb) **proper in Faber's model**

Coulomb parameter of dipole

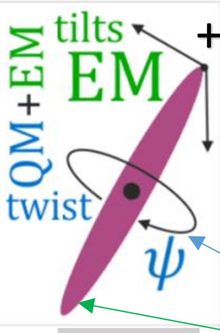


GeV



fm

$$\int_{\sim 1.4\mathrm{fm}}^{\infty} \frac{1}{2} |E|^2 4\pi r^2 dr = 511\mathrm{keV}$$



+ twists e.g. for angular momentum conservation

Affine connection $SO(1,3) \ni O \to O(I + \epsilon\Gamma_\mu)$:

local rotations, $\widetilde{\text{boosts}}$: $\boldsymbol{\Gamma_\mu = O^T \partial_\mu O} = \begin{pmatrix} 0 & \tilde\Gamma^1 \text{ boost} & \tilde\Gamma^2 & \tilde\Gamma^3 \\ \tilde\Gamma^1 & 0 & -\Gamma^3 & \Gamma^2 \\ \tilde\Gamma^2 & \Gamma^3 \text{ tilt} & 0 & -\Gamma^1 \\ \tilde\Gamma^3 & -\Gamma^2 \text{ tilt} & \Gamma^1 \text{ twist} & 0 \end{pmatrix}$

of $M = ODO^T \qquad D \approx \mathrm{diag}(g, 1, \delta, 0)$

$g_{\text{grav}} \gg 1 \gg \delta_{\sim\hbar} > 0$ shape $\qquad \boxed{L_{QED} = -\hbar c\, \bar\psi\gamma^\mu D_\mu \psi - mc^2 \bar\psi\psi - F_{\mu\nu}F^{\mu\nu}/4}$

QM: $\delta\Gamma^1\Gamma^2$, $\delta\Gamma^1\Gamma^3$, electromagnetism: $\Gamma^2\Gamma^3$, GEM: $g\tilde\Gamma^1\tilde\Gamma^2$, $g\tilde\Gamma^2\tilde\Gamma^3$, $g\tilde\Gamma^3\tilde\Gamma^1$

3D curvature EM+QM

$$\vec R_{\mu\nu} \equiv \vec R^{ee}_{\mu\nu} = \vec\Gamma_\mu \times \vec\Gamma_\nu$$

0[th] axis boost GEM

$$\vec R^{gg}_{\mu\nu} = \vec\Gamma^g_\mu \times \vec\Gamma^g_\nu$$

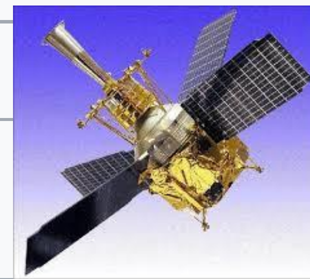
4D EMQM-GEM interact.:

$$\Gamma^g \equiv \tilde\Gamma \qquad \vec R^{eg}_{\mu\nu} = \vec\Gamma_\mu \times \vec\Gamma^g_\nu$$

light bending, time dilat.

GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

Gravity Probe B



+ **short-range** (to particles) **axes deformations $\boldsymbol{\partial_\mu D}$** $\qquad O^T M_\mu O = \bar M_\mu + \partial_\mu D$

$$O^T \bar F_{\mu\nu} O = O^T [M_\mu, M_\nu] O = [\bar M_\mu, \bar M_\nu] + [\partial_\mu D, \bar M_\nu] + [\bar M_\mu, \partial_\nu D]$$

EM + QM + GEM + Yang-Mills gluons ($L_{YM} = -G^a_{\mu\nu} G^{\mu\nu}_a / 4$) + Higgs particle

weak: $SU(2)_{LR} \sim SO(3)$ neutrino **3D rotations**, **strong**: $SU(3)$ baryon with twist

teleparallelism – tetrad field Einstein's attempt to unify EM + gravity arXiv:2501.04036 toy model of electron **clock**/neutrino oscillation mechanism

Relativistic QM: **mass of particle propels its clock** – what is its mechanism?

$\psi = \psi_0 e^{-iEt/\hbar}$ for $E = mc^2$ – electron clock, neutrino oscillations (3 masses)

Also "time crystal" – with periodic process already in the lowest energy state

Lorentz invariant R^2

$-(\phi_0\psi_1 - \phi_1\psi_0)^2$

$\phi_1 = \partial_1 \phi$

shape of particle

energetically favors

$\psi_0 = \partial_0 \psi$

phase evolution

codes to calculate

Clock of kink from mass by curvature coupling

Kink minimizing energy E for Hamiltonian: curvature R

$$\mathcal{H} = \phi_0^2 + \phi_1^2 + (1-\phi^2)^2 - (\phi_0\psi_1 - \phi_1\psi_0)^2 + (\phi_0\psi_1 - \phi_1\psi_0)^4$$

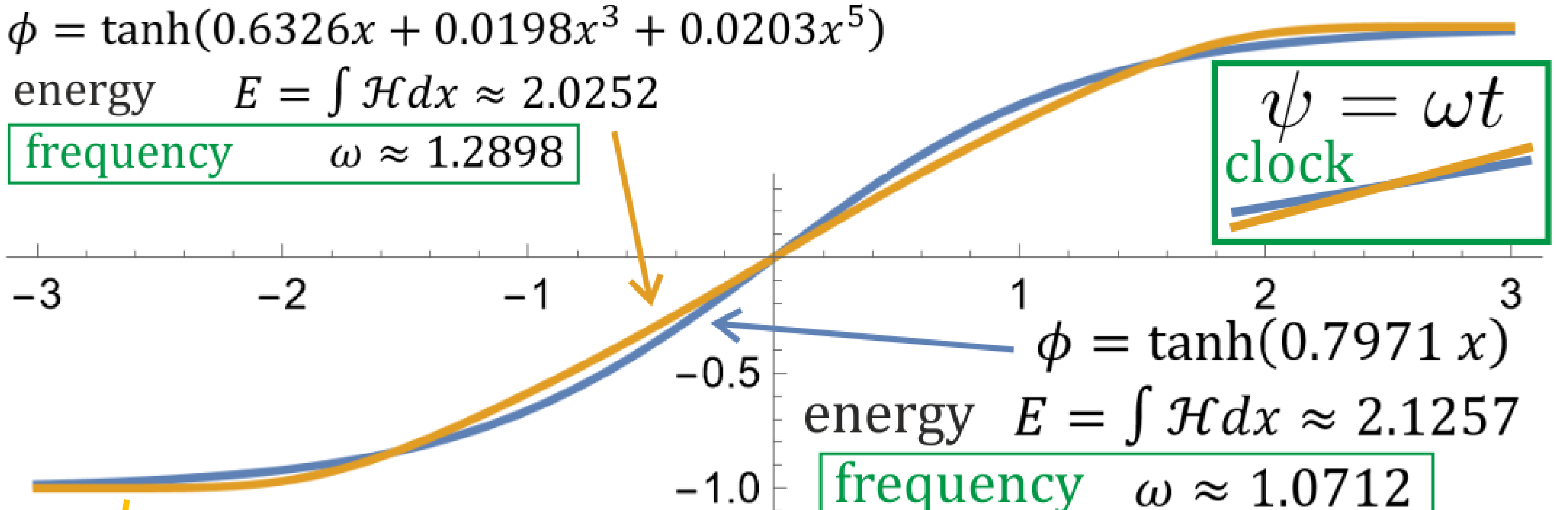
$\phi = \tanh(0.6326x + 0.0198x^3 + 0.0203x^5)$

energy $E = \int \mathcal{H}dx \approx 2.0252$

frequency $\omega \approx 1.2898$

$\psi = \omega t$

clock

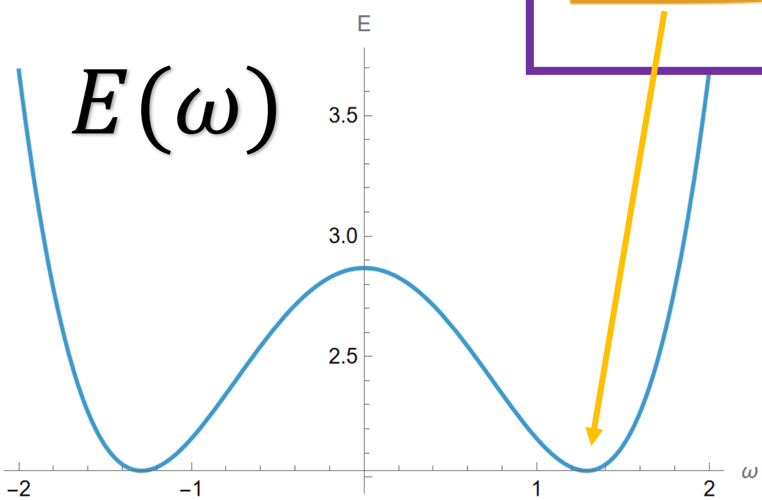


$\phi = \tanh(0.7971\,x)$

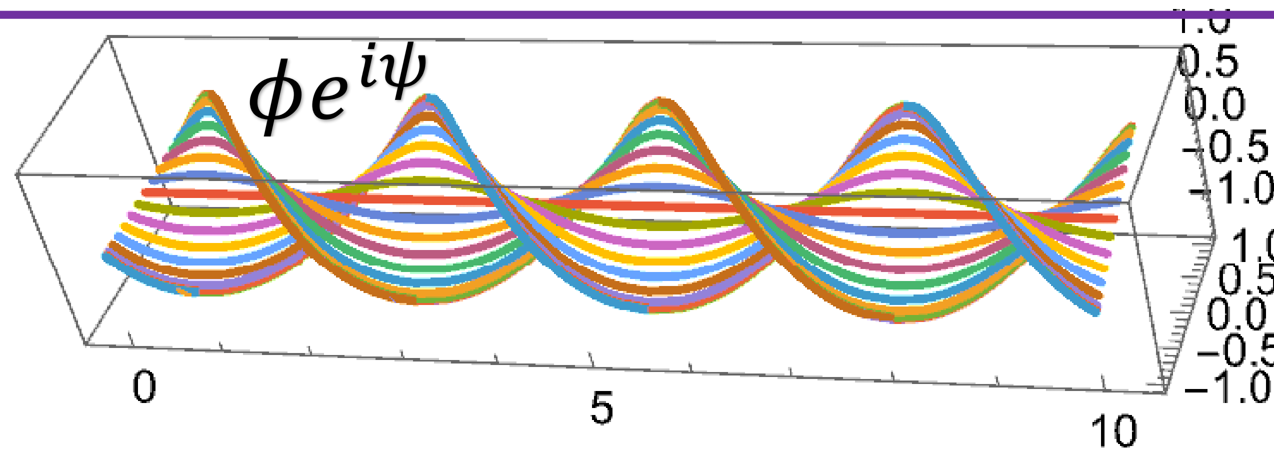
energy $E = \int \mathcal{H}dx \approx 2.1257$

frequency $\omega \approx 1.0712$

$E(\omega)$



$\phi e^{i\psi}$



Electron's clock, neutrino oscillation propulsion? $\psi = \psi_0 e^{iEt/\hbar}$ for $E = mc^2$

EM **Lagrangian** $B^2 - E^2 = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \rightarrow$

Hamiltonian $\frac{1}{2}\sum_{\mu\nu}(F_{\mu\nu})^2 = B^2 + E^2$

Full spacetime curvature/Skyrme:

4 indexes $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta} \rightarrow F_{\mu\nu\alpha\beta} F^{\alpha\beta}_{\mu\nu}$

negative curvature^2 terms in Hamiltonian

$$\mathcal{L} = -\underbrace{\sum_{\alpha\beta\mu\nu} F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}}_{\text{EM-like}} - \underbrace{V(M)}_{\text{Higgs-like}}$$

$G, \hbar$ weighted curvature

for $F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$

vacuum: $V(M) = 0$

$$\mathcal{H} = \sum_{0 \le \mu < \nu \le 3} \underbrace{F_{\mu\nu\alpha\beta} F^{\alpha\beta}_{\mu\nu} + V(M)}_{\text{energy contributions}} =$$

$$= 2 \sum_{0 \le \mu < \nu \le 3} \left(\boxed{+\sum_{1 \le \alpha < \beta \le 3} (F_{\mu\nu\alpha\beta})^2} \boxed{-\sum_{\alpha=1}^{3} (F_{\mu\nu\alpha 0})^2} \right) + V(M)$$

$$M_\mu = \partial_\mu (ODO^T) = O\overline{M}_\mu O^T \quad \text{for} \quad \overline{M}_\mu = [\Gamma_\mu, D]$$

$$A_\mu = [M, M_\mu] = O\overline{A}_\mu O^T \quad \text{for} \quad \overline{A}_\mu = [D, \overline{M}_\mu]$$

$$F_{\mu\nu} = [M_\mu, M_\nu] = O\overline{F}_{\mu\nu} O^T \quad \text{for} \quad \overline{F}_{\mu\nu} = [\overline{M}_\mu, \overline{M}_\nu]$$

$-\left(\Gamma_0^1 \tilde{\Gamma}_\mu^i - \Gamma_\mu^1 \tilde{\Gamma}_0^i\right)^2$ in Hamiltonian ($i = 2,3$) – prefer twist in time Γ_0^1: **clock**, together with $\tilde{\Gamma}_\mu^{2,3}$ local boosts – **gravitational mass** of the particle

```
(Fμν = FullSimplify[coms[M̄μ, M̄μ /. μ → ν], sub]) // MatrixForm
```

clock propulsion?

EM/QM - gravity interactions

$$\begin{pmatrix}
0 & g\left(\Gamma_\nu^3 \tilde{\Gamma}_\mu^2 - \Gamma_\nu^2 \tilde{\Gamma}_\mu^3 - \Gamma_\mu^3 \tilde{\Gamma}_\nu^2 + \Gamma_\mu^2 \tilde{\Gamma}_\nu^3\right) & g\left(\Gamma_\nu^3 \tilde{\Gamma}_\mu^1 \boxed{+ \delta \Gamma_\nu^1 \tilde{\Gamma}_\mu^3} - \Gamma_\mu^3 \tilde{\Gamma}_\nu^1 \boxed{- \delta \Gamma_\mu^1 \tilde{\Gamma}_\nu^3}\right) & g\left(-\Gamma_\nu^2 \tilde{\Gamma}_\mu^1 \boxed{+ \delta \Gamma_\nu^1 \tilde{\Gamma}_\mu^2} + \Gamma_\mu^2 \tilde{\Gamma}_\nu^1 \boxed{- \delta \Gamma_\mu^1 \tilde{\Gamma}_\nu^2}\right) \\
g\left(\Gamma_\nu^3 \tilde{\Gamma}_\mu^2 - \Gamma_\nu^2 \tilde{\Gamma}_\mu^3 - \Gamma_\mu^3 \tilde{\Gamma}_\nu^2 + \Gamma_\mu^2 \tilde{\Gamma}_\nu^3\right) & 0 & \delta R^3_{\{\mu,\nu\}} + g^2 \tilde{R}^3_{\{\mu,\nu\}} & \delta R^2_{\{\mu,\nu\}} - g^2 \tilde{R}^2_{\{\mu,\nu\}} \\
g\left(\Gamma_\nu^3 \tilde{\Gamma}_\mu^1 + \delta \Gamma_\nu^1 \tilde{\Gamma}_\mu^3 - \Gamma_\mu^3 \tilde{\Gamma}_\nu^1 - \delta \Gamma_\mu^1 \tilde{\Gamma}_\nu^3\right) & -\delta R^3_{\{\mu,\nu\}} - g^2 \tilde{R}^3_{\{\mu,\nu\}} & 0 & R^1_{\{\mu,\nu\}} + g^2 \tilde{R}^1_{\{\mu,\nu\}} \\
g\left(-\Gamma_\nu^2 \tilde{\Gamma}_\mu^1 + \delta \Gamma_\nu^1 \tilde{\Gamma}_\mu^2 + \Gamma_\mu^2 \tilde{\Gamma}_\nu^1 - \delta \Gamma_\mu^1 \tilde{\Gamma}_\nu^2\right) & -\delta R^2_{\{\mu,\nu\}} + g^2 \tilde{R}^2_{\{\mu,\nu\}} & -R^1_{\{\mu,\nu\}} - g^2 \tilde{R}^1_{\{\mu,\nu\}} & 0
\end{pmatrix}$$

squared +/- energy contributions

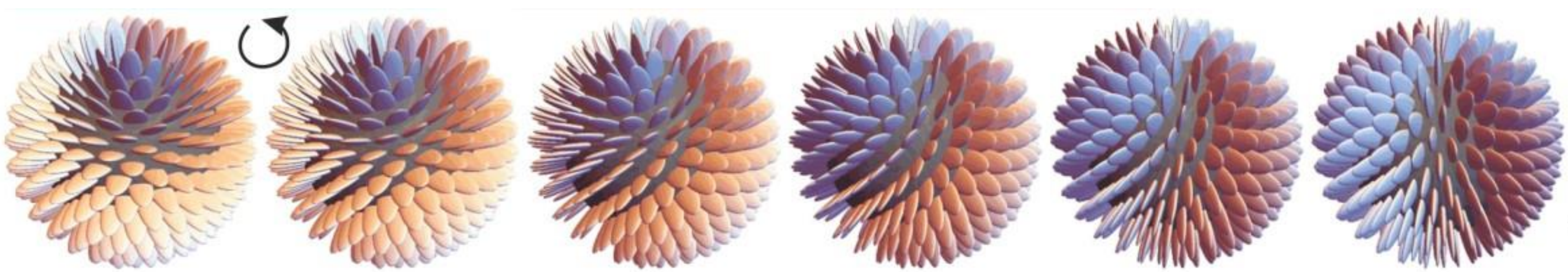
$F_{\mu\nu}$ curvatures

QM: tilt-twist

EM: tilt-tilt

gravity: boosts

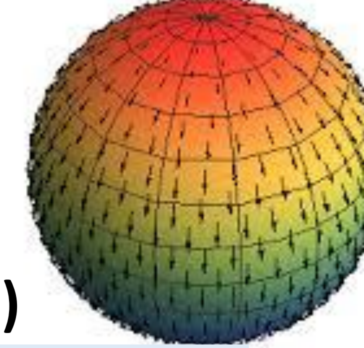
EM-QM-GEM unifi.



electron clock(?): phase ψ time evolution

Why no naked charges in physics – leptons need magnetic dipole?

Maybe **hairy ball theorem**? - no continuous vector field on sphere



Uniaxial → **biaxial** nematic: $\boldsymbol{S^2} \rightarrow \boldsymbol{SO}(\mathbf{3})$ **vacuum: +** $\boldsymbol{U}(\mathbf{1})$ **phase**

- **Required additional linear topological defect (magnetic?)**
- **Hedgehog of 1 of axes: same charge, different mass (3 leptons?)**

3x3 tensor field: $M = O \cdot \mathrm{diag}(\lambda_i) \cdot O^T$, Higgs-like: $V(M) = \sum_i \left(\lambda_i - \lambda_i^0\right)^2$

hedgehog:

+e charge

$\boldsymbol{\pi}$ **field rot.**

can be

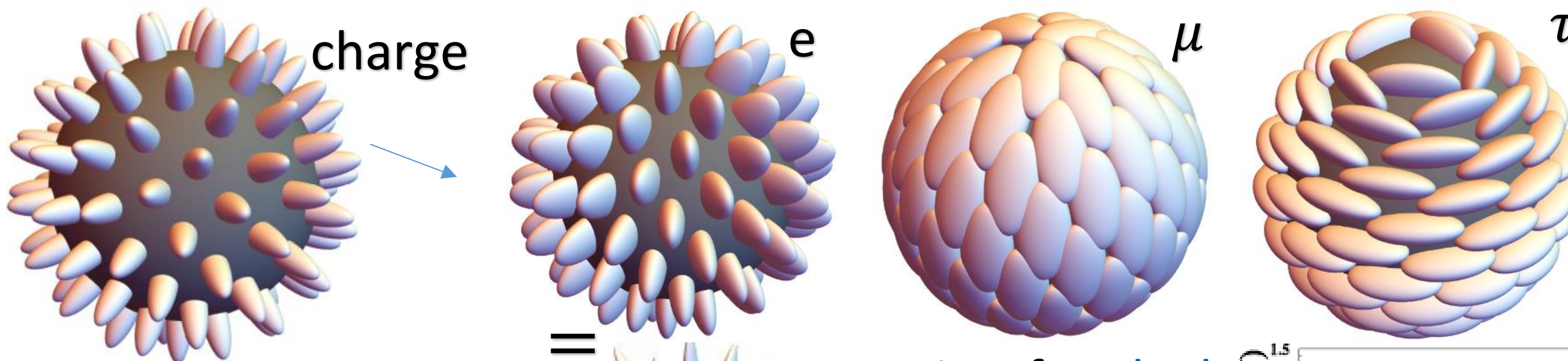
fractional

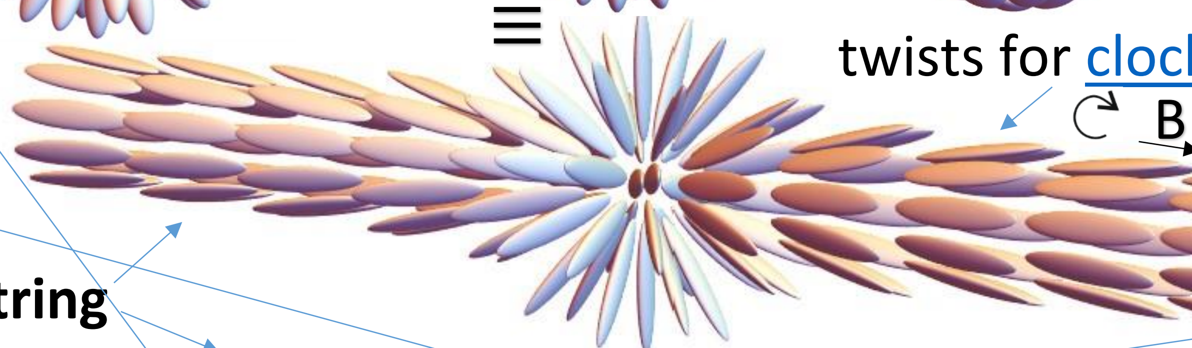
for d **quark** $-\pi/3$

antiquark $+\pi/3$

flux tube/quark string

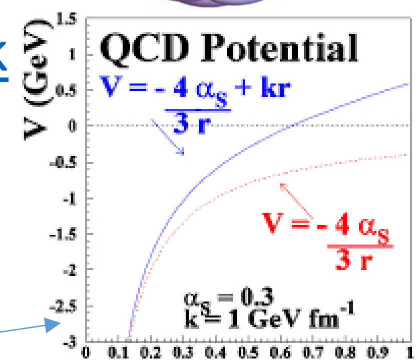
lowest energy: parallel





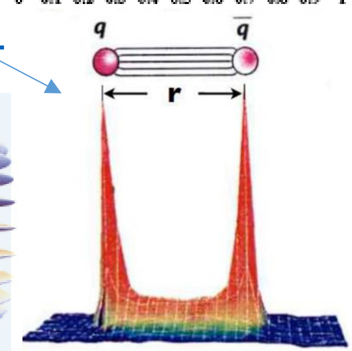
twists for clock

B

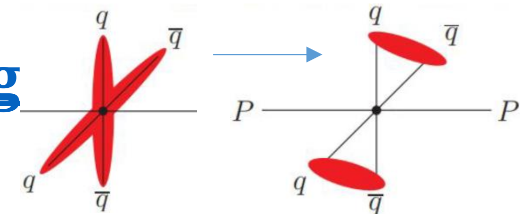


~1GeV/fm

\+ $V \propto -1/r$ QCD ρ en.

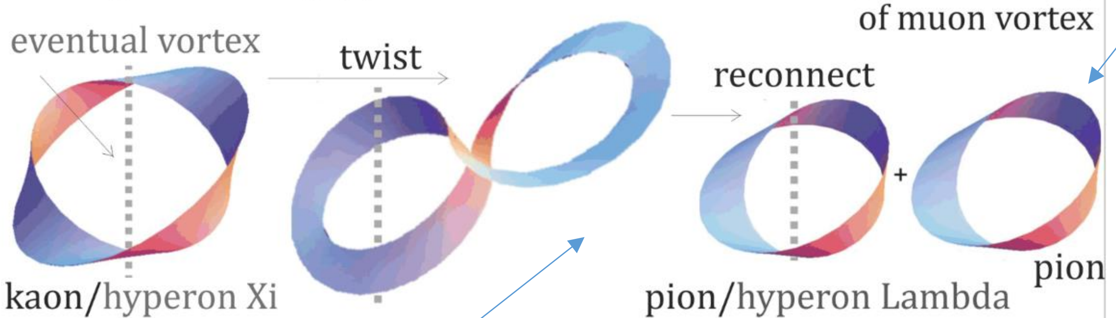


Hadronizarion: high energy **quark string reconnecting**

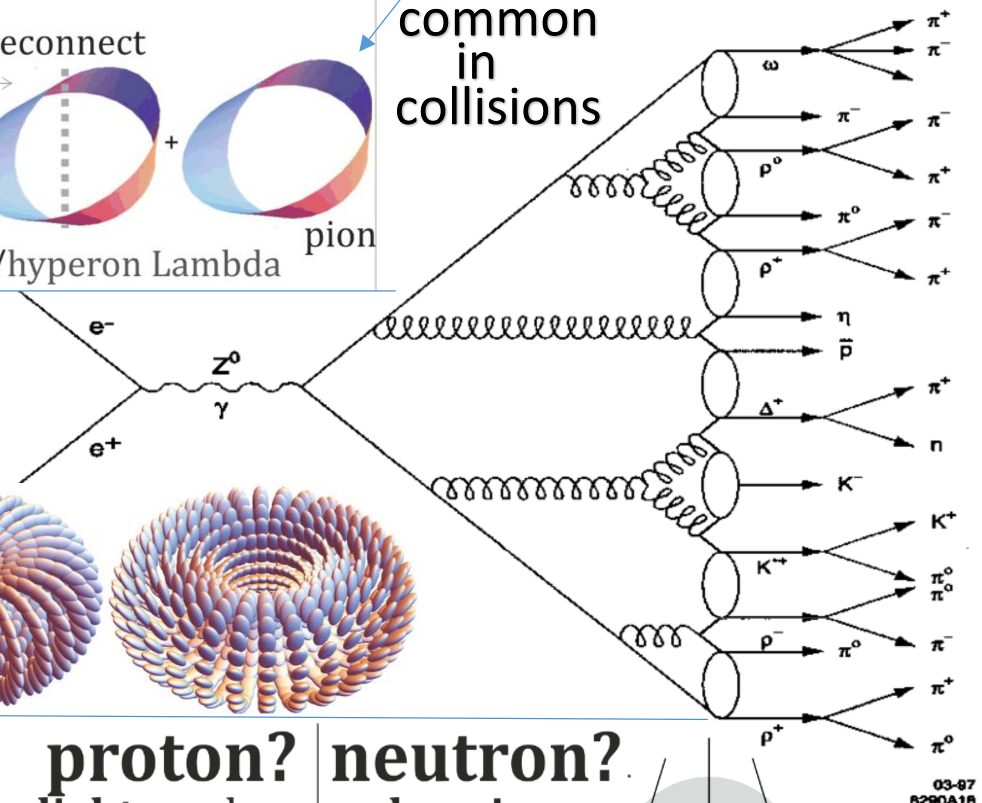


→ **simple loops: 3 neutrinos**, if **twisted: pions, kaons**,
inward/outward **rotation: quark→ hedgehog ≡ charge**, **knots: baryons**
... nuclei

strangeness decay (?):



π, K common in collisions



Decays e.g.

$\pi^+ \to \mu^+ + \nu_\mu$

$\pi^0 \to \gamma + \gamma$

$\Sigma^0 \to \Lambda^0 + \gamma$

$\pm s$: left/right twists?

$n^0 \to p^+ + e^- + \bar{\nu}_e$ 0s

$\Lambda^0 \to p^+ + \pi^-, n^0 + \pi^0$

$\Xi^0 \to \Lambda^0 + \pi^0$ 123s

$\Xi^- \to \Lambda^0 + \pi^-$

$\Omega^- \to \Xi + \pi, \Lambda^0 + K^-$

$K^+ \to \mu^+ + \nu_\mu, 2 \vee 3\pi$

$K_S^0 \to 2\pi$

$K_L^0 \to 3\pi, \ldots$

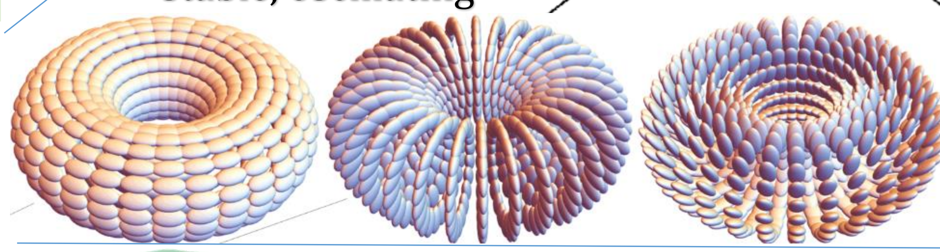
$\omega \to 3\pi, \ldots$

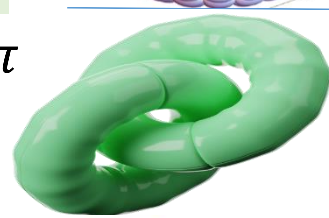
$\eta \to \gamma + \gamma, 3\pi^0$

$\phi(1020) \to 2K$

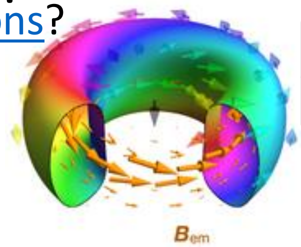
$\Delta^{++} \to p^+ + \pi^+$ (2 hedgehogs)

3 neutrinos
stable, oscillating ↔





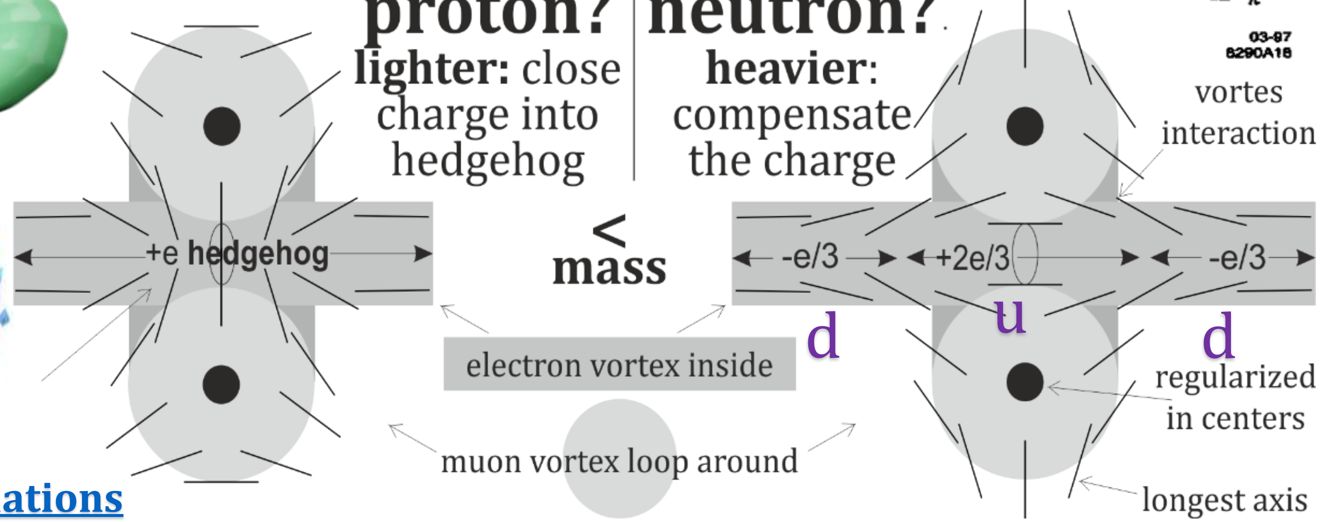
? Hopfions?



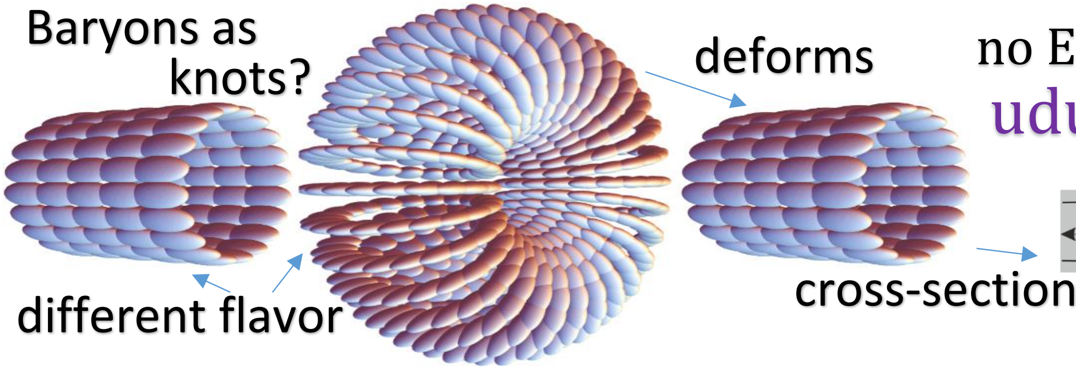
proton? **lighter:** close charge into hedgehog

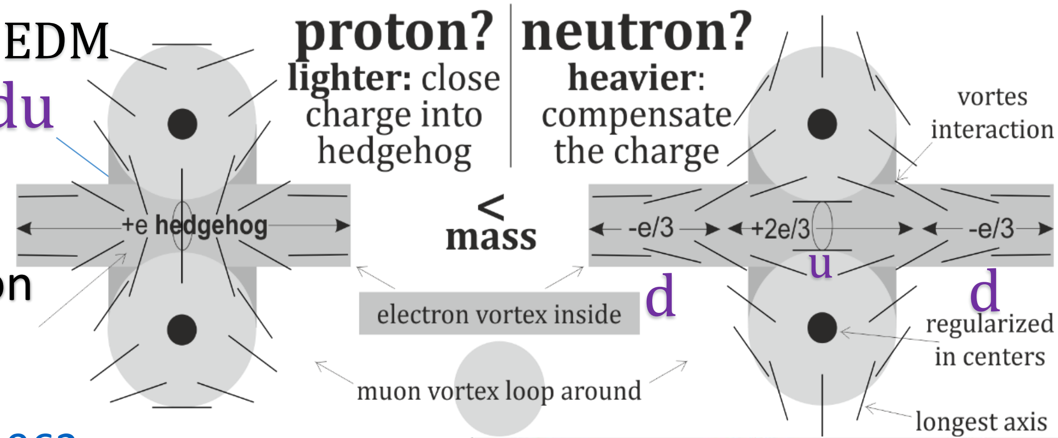
neutron? **heavier:** compensate the charge

< **mass**



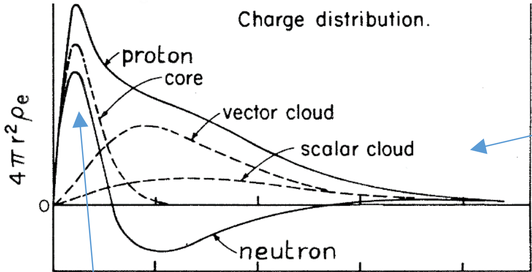
Electron, neutrino ... neutral particle **oscillations**





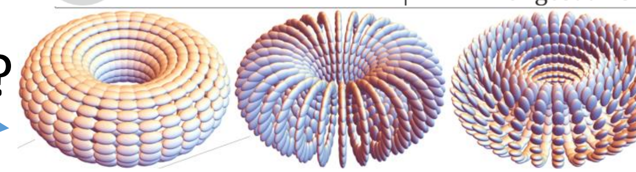
u, c, t: $+\frac{2}{3}\pi$

due to loop around?

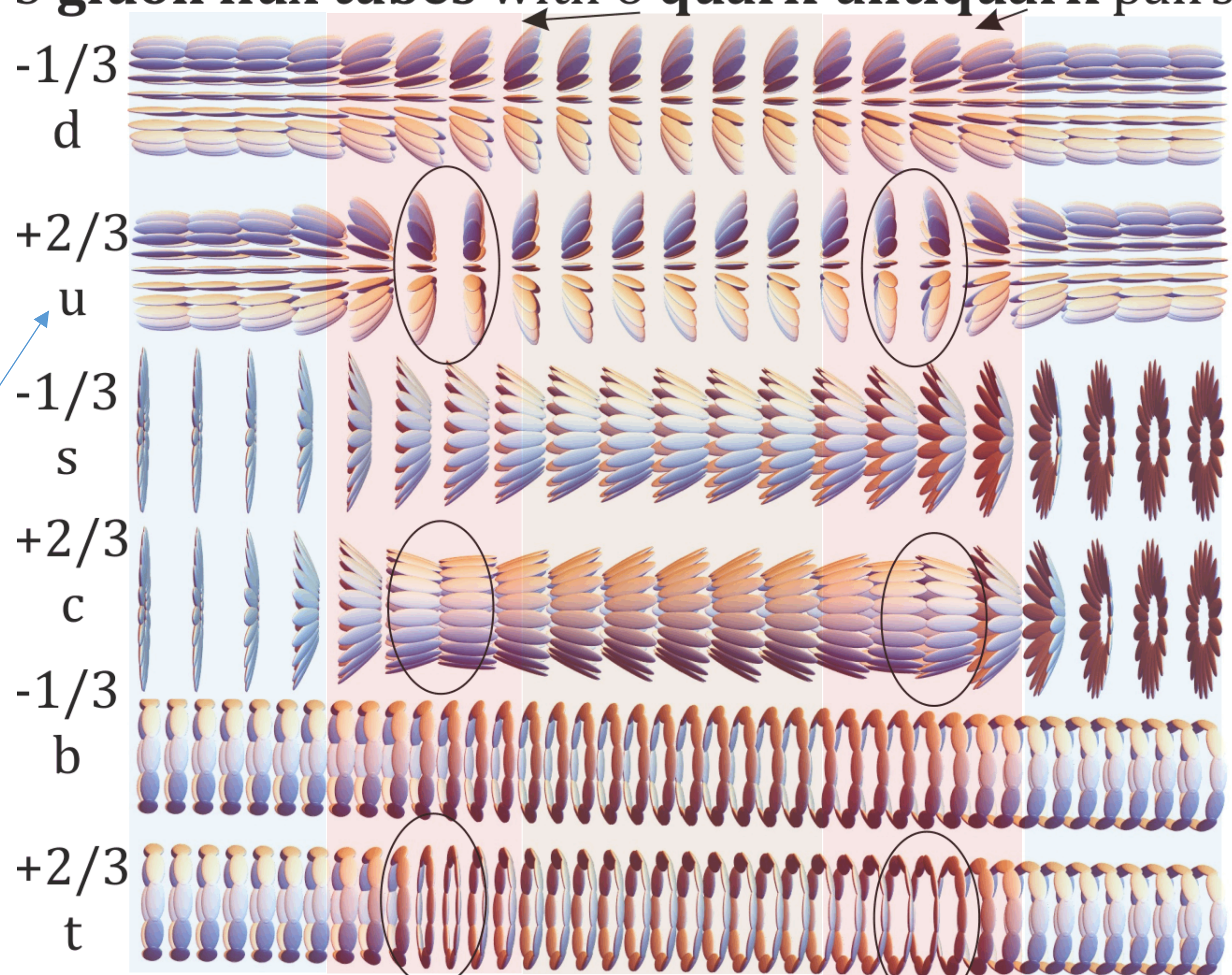


Wilson 1962

neutrinos?



3 **gluon flux tubes** with 6 **quark-antiquark** pairs



udu merged in proton? No EDM

Gluons SU(3) – rotate with phase

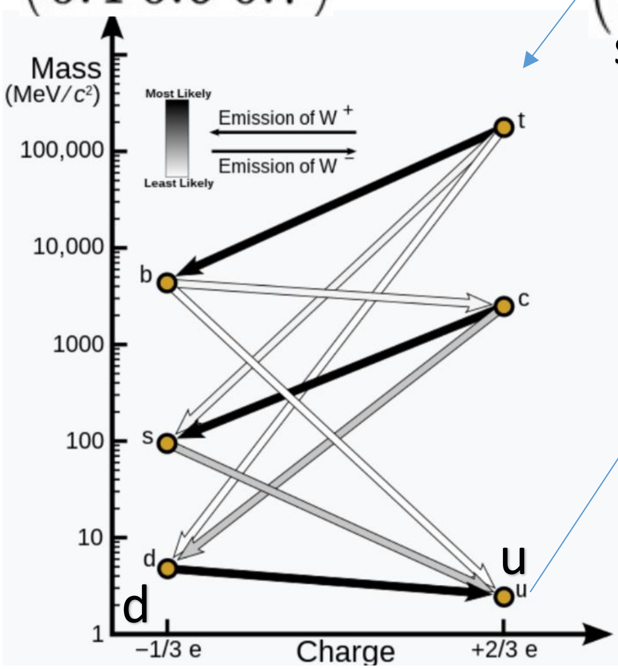
$$\begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

PNMS

CKM

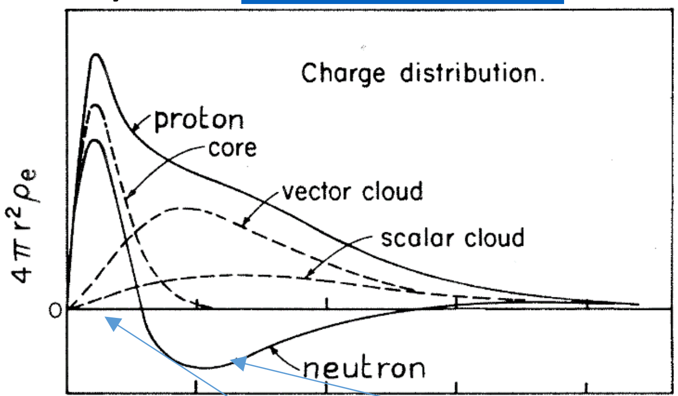
$$\begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

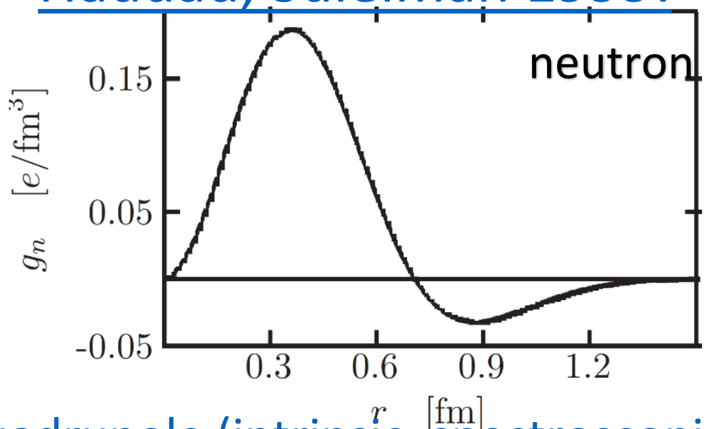
stabilized by loop

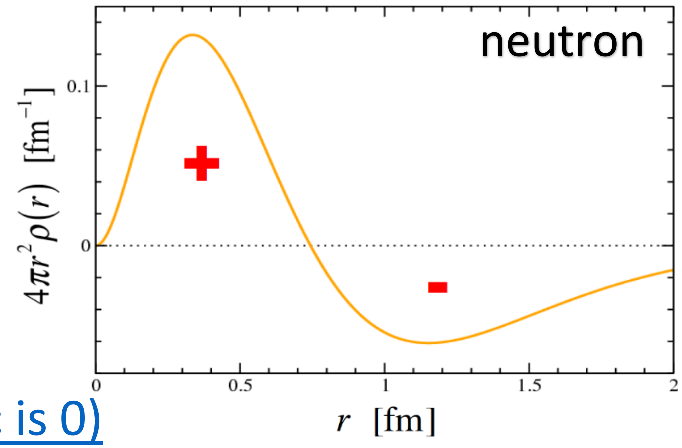


reduced by loop around

Baryons: Wilson 1962 Haddad, Suleiman 1999: Greene 2015







Neutron: "+" core, **"-" shell**, quadrupole (intrinsic, spectroscopic is 0)

g-factors: $g_e \approx -2, g_\mu \approx -2, g_p \approx +5.6, \boldsymbol{g_n \approx -3.8}$

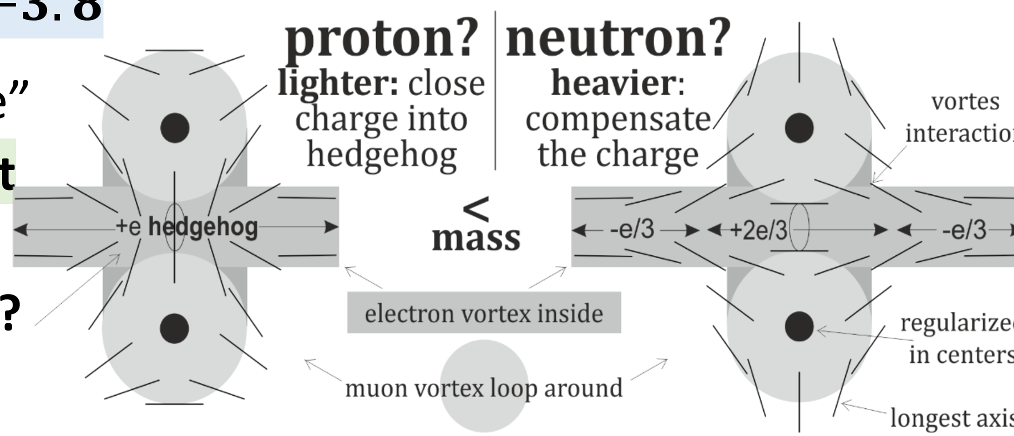
Deuteron: "two baryons share required charge"

0.2859 e·fm² - **large electric quadrupole moment**

"**+ - +**" (EDM zero), how to get it for p+n? **EMC**

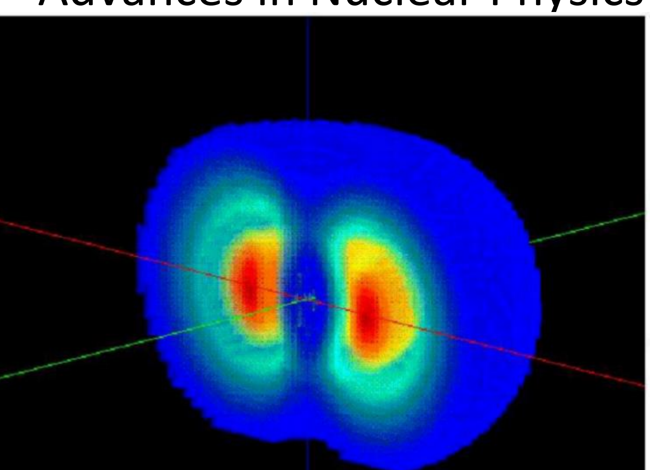
$\mu_d \approx \mu_p + \mu_n$ – **aligned spins/angular mom.?**

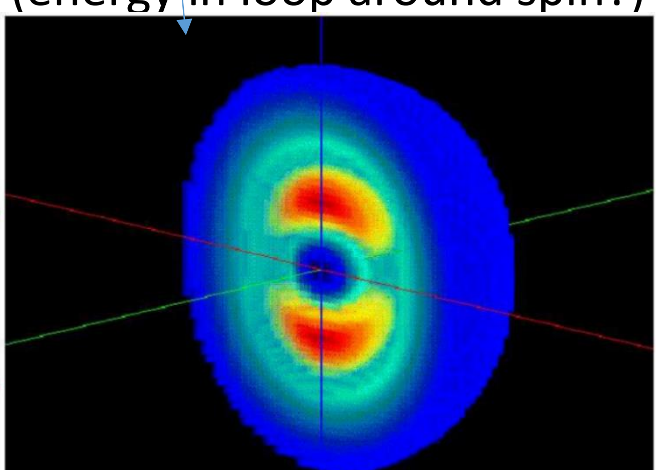
0.857 vs 0.879 μ_N magnetic dipole moments

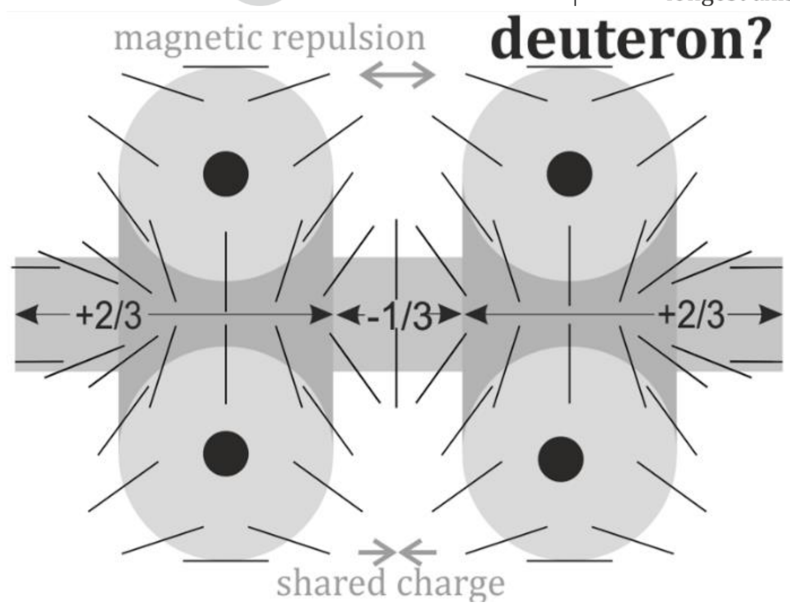


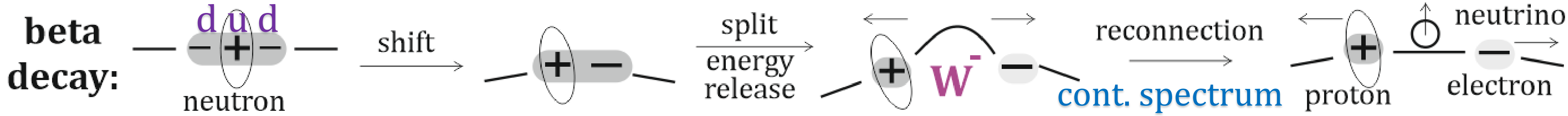
The deuteron: structure and form factors (2001)

Advances in Nuclear Physics (energy in loop around spin?)









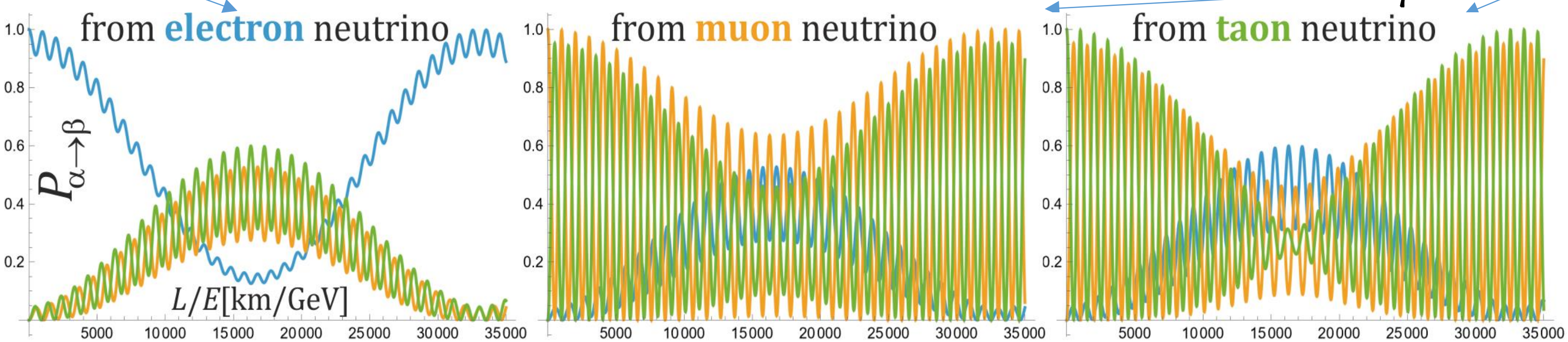
3 **neutrinos** as loops: from **beta decay**, very light and stable,

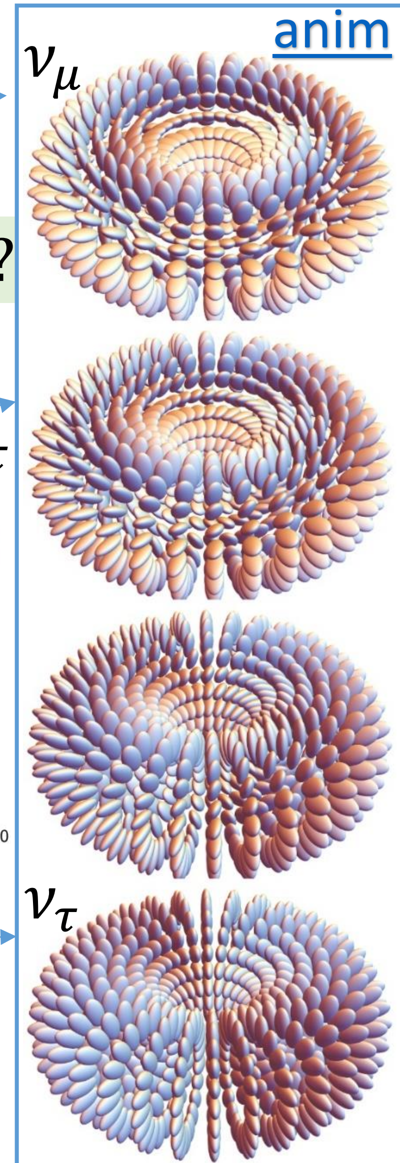
oscillating: $|\nu_j(t)\rangle = e^{-\frac{i}{\hbar}(E_j t - \vec{p}_j \cdot \vec{x})} |\nu_j(0)\rangle$ ... electron clock +0.28%?

mass ≠ flavor eigenbase, "**chiral**" ν_e $\bar{\nu}_e$ same gravity $(\pm\Gamma_\nu^0)\tilde{\Gamma}_\mu$?

Energy conservation – density/length, vary loop length?

From ν_e different, favored low energy phase twisting: $\nu_\mu \leftrightarrow \nu_\tau$

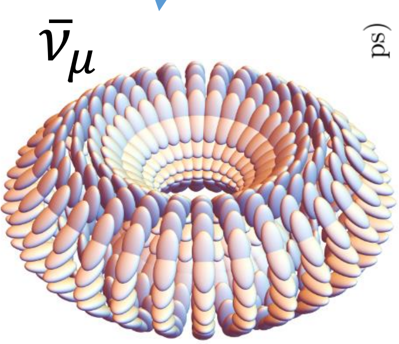


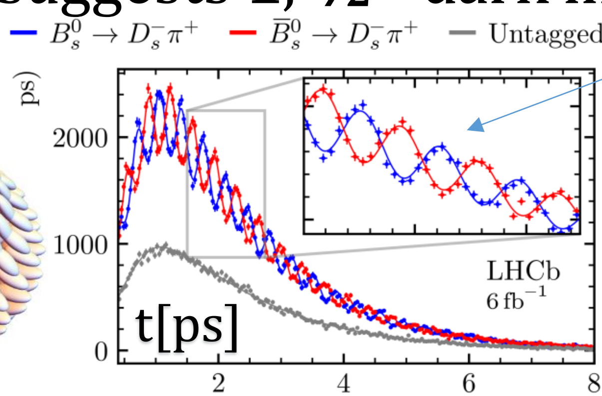


Gauge invariance U(1) rotate cross-section: same energy, differ. interact. sterile?

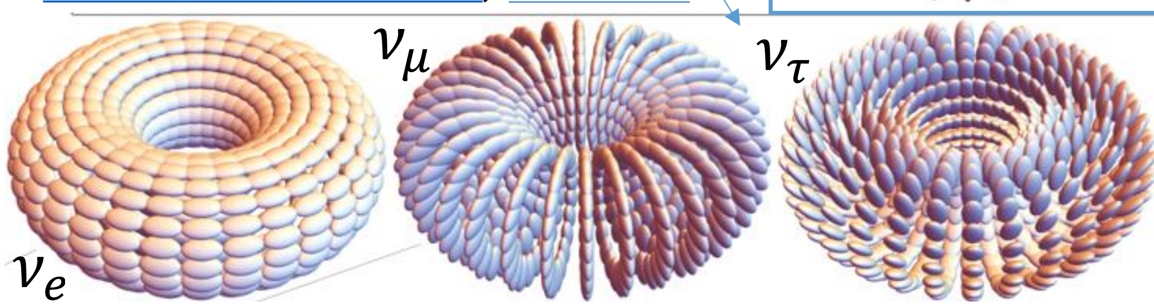
Antineutrino – opposite charge in cross-section ... **1** vs **½**?

Beta decay suggests **1**, **½** - dark matter contribution?





B − $\overline{\text{B}}$ oscillations, neutral



Quarks: ~1% of $m_p < m_n$ (?) mass. Clock like electron?

Some issues below, **charge distribution** more **objective**

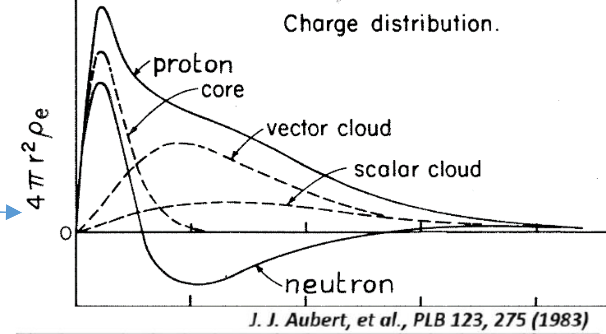
spin crisis: ~40% + ~40% gluons + ~20% angular moment.?

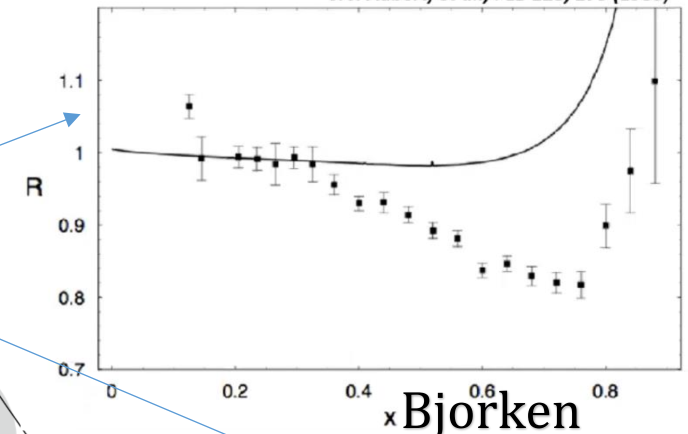
proton radius puzzle: **+electron**: 877, **+muon**: 842am

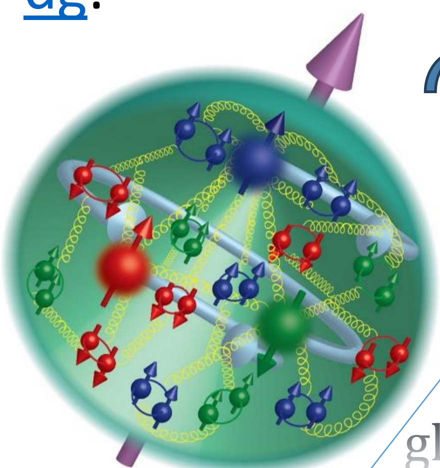
- **neutron lifetime puzzle** 878s **bottle** vs 887s **beam**
- **EMC effect**: quark distribution modified inside nuclei

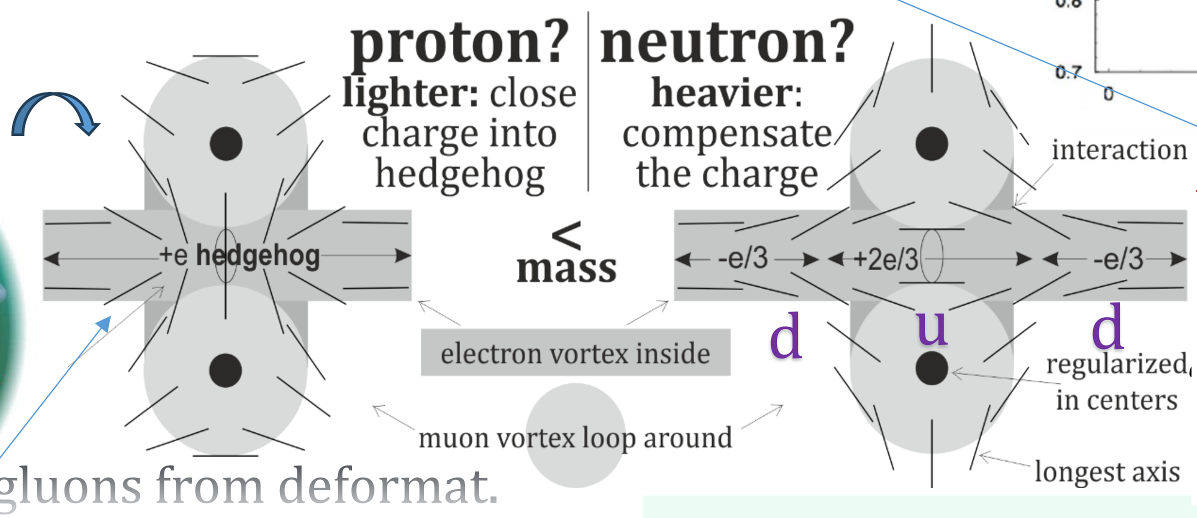
three-nucleon force? **deuteron quadrupole**, **halo nuclei**?

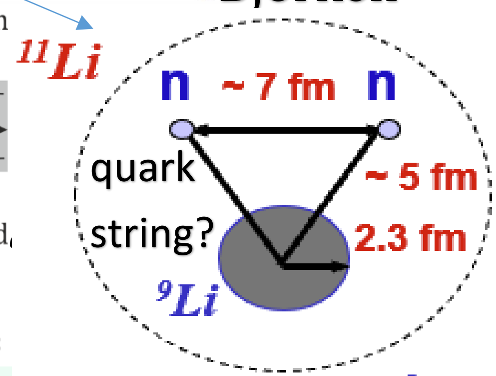
dg:











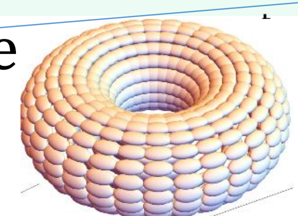
Quarks: 3 · 2 vortex intrinsic twists

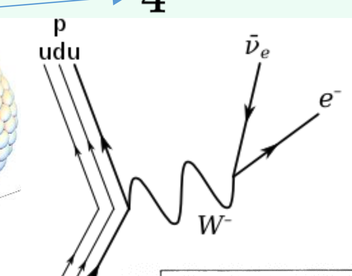
$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Gluons: $G = \Gamma_{\text{potential}} \times \Gamma_i$ **curvatures**, short distance

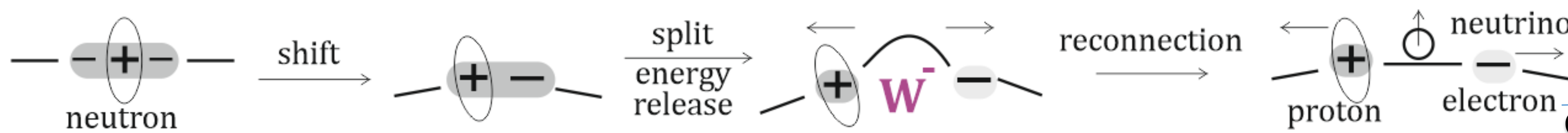
$W^\pm$ **boson**: vortex inward/outward “tilt-wave”?

Z^0 **boson**: vacuum perturbation toward pair creation

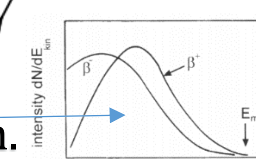




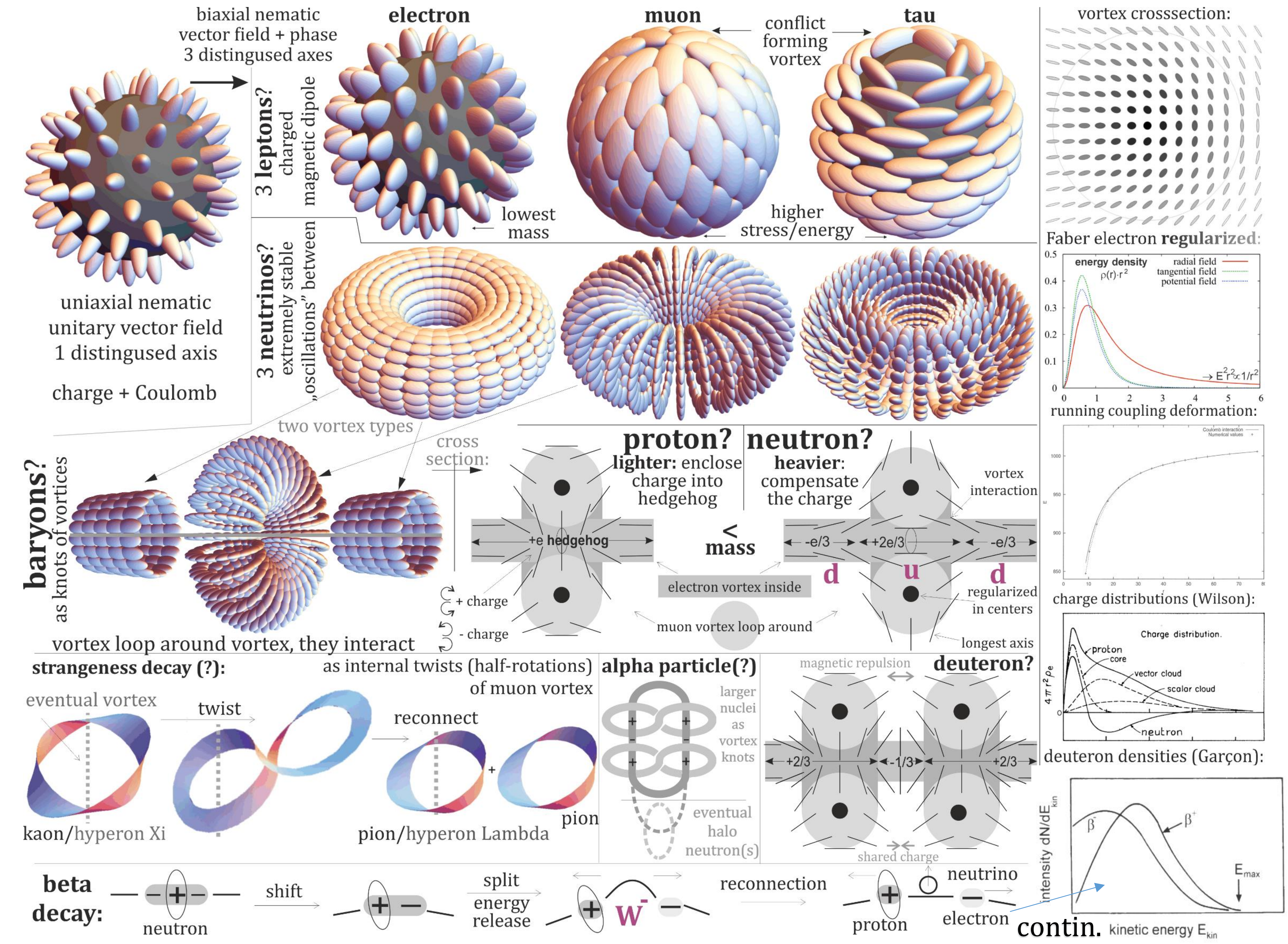
beta decay:



contin.



gluon flux tube/color/quark string



Intermediate step: transform unitary "**director**" field → **rotation** matrix field O

Affine **connection**: $O \to O(I + \epsilon\Gamma_\mu)$ for $\boldsymbol{\Gamma_\mu = O^T O_\mu} = O^T \partial_\mu O$ anti-symmetric

Denote its coordinates with 2 vectors: (boosts +)

EM: $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$

GEM: $\vec{\Gamma}^g_\mu := (\Gamma_{\mu,01}, \Gamma_{\mu,02}, \Gamma_{\mu,03})$ tiny boosts of 0th axis

$$\Gamma_\mu = O^T O_\mu = \begin{pmatrix} 0 & \vec{\Gamma}^g_{\mu,1} & \vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}^g_{\mu,3} \\ -\vec{\Gamma}^g_{\mu,1} & 0 & -\vec{\Gamma}_{\mu,3} & \vec{\Gamma}_{\mu,2} \\ -\vec{\Gamma}^g_{\mu,2} & \vec{\Gamma}_{\mu,3} & 0 & -\vec{\Gamma}_{\mu,1} \\ -\vec{\Gamma}^g_{\mu,3} & -\vec{\Gamma}_{\mu,2} & \vec{\Gamma}_{\mu,1} & 0 \end{pmatrix}$$

3D 1st axis **curvature**: $\vec{R}_{\mu\nu} \equiv \vec{R}^{ee}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ - **electromagnetism** we will focus on

4D 0th axis curvature: $\vec{R}^{gg}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}^g_\nu$ - **GEM** approximation of general relativity

4D EM-GEM interaction: $\vec{R}^{eg}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}^g_\nu$, $\vec{R}^{ge}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}_\nu = -\vec{R}^{eg}_{\nu\mu}$

Fermat principle: Sun's light bend
e.g. **GEM slowing EM**: time dilation

$F_{\mu\nu} = [\Gamma_\mu, \Gamma_\nu]$ in place of curvature for EM with topologically quantized charge?

$$[\Gamma_\mu, \Gamma_\nu] = \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu} + \vec{R}^{eg}_{\nu\mu} \\ \vec{R}^{eg}_{\mu\nu} - \vec{R}^{eg}_{\nu\mu} & \vec{R}^{ee}_{\mu\nu} + \vec{R}^{gg}_{\mu\nu} \end{pmatrix} := \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu,1} + \vec{R}^{eg}_{\nu\mu,1} & -\vec{R}^{eg}_{\mu\nu,2} + \vec{R}^{eg}_{\nu\mu,2} & -\vec{R}^{eg}_{\mu\nu,3} + \vec{R}^{eg}_{\nu\mu,3} \\ \vec{R}^{eg}_{\mu\nu,1} - \vec{R}^{eg}_{\nu\mu,1} & 0 & -\vec{R}^{ee}_{\mu\nu,3} - \vec{R}^{gg}_{\mu\nu,3} & \vec{R}^{ee}_{\mu\nu,2} + \vec{R}^{gg}_{\mu\nu,2} \\ \vec{R}^{eg}_{\mu\nu,2} - \vec{R}^{eg}_{\nu\mu,2} & \vec{R}^{ee}_{\mu\nu,3} + \vec{R}^{gg}_{\mu\nu,3} & 0 & -\vec{R}^{ee}_{\mu\nu,1} - \vec{R}^{gg}_{\mu\nu,1} \\ \vec{R}^{eg}_{\mu\nu,3} - \vec{R}^{eg}_{\nu\mu,3} & -\vec{R}^{ee}_{\mu\nu,2} - \vec{R}^{gg}_{\mu\nu,2} & \vec{R}^{ee}_{\mu\nu,1} + \vec{R}^{gg}_{\mu\nu,1} & 0 \end{pmatrix}$$

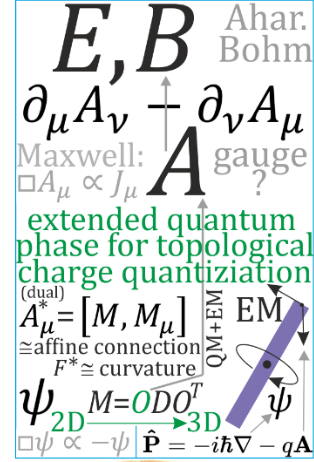
GEM – confirmed by Gravity Probe B approximation of **general relativity:** https://en.wikipedia.org/wiki/Gravitoelectromagnetism:

GEM equations	Maxwell's equations
$\nabla \cdot \mathbf{E}_g = -4\pi G \rho_g$	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{J}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$

$[\Gamma_\mu, \Gamma_\nu]$ vanishes in flat spacetime

GEM causes spatial curvature:

$$0 = \partial_\mu \partial_\nu O - \partial_\nu \partial_\mu O = \partial_\mu(O\Gamma_\nu) - \partial_\nu(O\Gamma_\mu) = O[\Gamma_\mu, \Gamma_\nu]$$

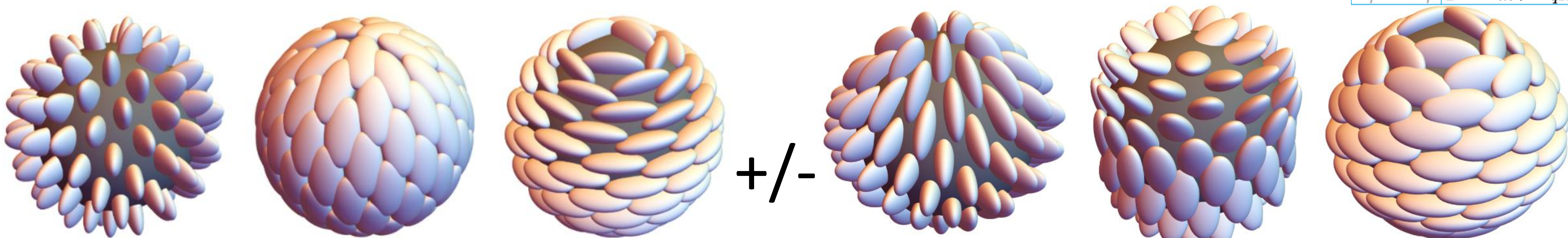


Let O matrix **rotate some objects**, e.g. **ellipsoid** in biaxial nematic

$M(x) \equiv \boldsymbol{M = ODO^T}$ field for $OO^T = I$ rotation

$D = \mathrm{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ with **Higgs-like e.g.** $\boldsymbol{V(M) = \sum_i (\lambda_i - \Lambda_i)^2}$

For $\Lambda_0 > \Lambda_1 > \Lambda_2 > \Lambda_3$ fixed preferred shape – **vacuum state**



(3D) F tensor containing curvature (so Gauss law gives topological charge)

Let us postulate: $\boldsymbol{F_{\mu\nu} = [M_\mu, M_\nu]} = \partial_\mu M\, \partial_\nu M - \partial_\nu M\, \partial_\mu M$, getting **vacuum**:

Curvatures:

Gauss law ∝ topological charge

$\Lambda_1 \gg \Lambda_2 \approx \Lambda_3$

High energy: **EM**

Low energy: **QM**

QED, ψ Lorentz group

$$O^T F_{\mu\nu} O = O^T [M_\mu, M_\nu] O \approx [\Gamma_\mu D - D\Gamma_\mu, \Gamma_\nu D - D\Gamma_\nu] =$$

$$(\Lambda_1 - \Lambda_2)(\Lambda_3 - \Lambda_1)(\Lambda_2 - \Lambda_3) \begin{pmatrix} 0 & \frac{-\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & \frac{\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} \\ \frac{\vec{R}_{\mu\nu,3}}{\Lambda_1 - \Lambda_2} & 0 & \frac{-\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} \\ \frac{-\vec{R}_{\mu\nu,2}}{\Lambda_3 - \Lambda_1} & \frac{\vec{R}_{\mu\nu,1}}{\Lambda_2 - \Lambda_3} & 0 \end{pmatrix}$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

for $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ and $\boldsymbol{\Gamma_\mu = O^T O_\mu}$, $\vec{\Gamma}_\mu := (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$ as previously

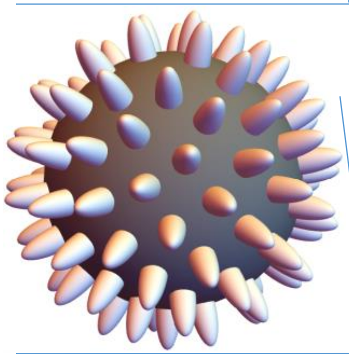
Postulate **Lagrangian** as EM: $\mathcal{L} = \sum_{\mu=1}^{3} \|F_{\mu 0}\|_F^2 - \sum_{1\leq\mu<\nu\leq 3} \|F_{\mu\nu}\|_F^2 - V$

and **four-potential** $\boldsymbol{A_\mu}$:

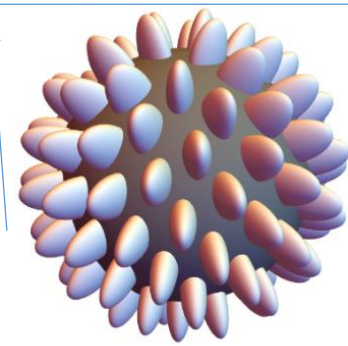
$2F_{\mu\nu} = 2[\boldsymbol{M_\mu}, \boldsymbol{M_\nu}] = \partial_\mu A_\nu - \partial_\nu A_\mu$ for $A_\mu = M\, M_\mu - M_\mu\, M \approx$ (**3D** vacuum)

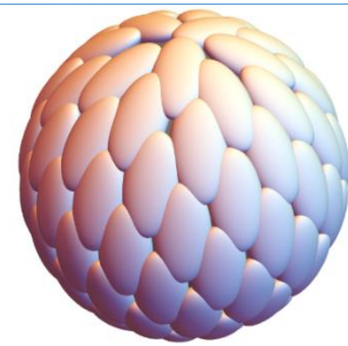
$\Lambda_1 \gg \Lambda_2 \approx \Lambda_3$
High energy: **EM**
Low energy: **QM**
$P = -i\hbar\nabla - qA$

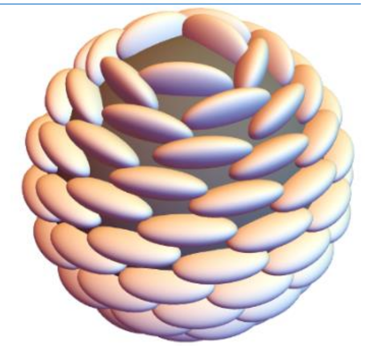
$$\approx O \begin{pmatrix} 0 & \vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & -\vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2 \\ -\vec{\Gamma}_{\mu,3}(\Lambda_1-\Lambda_2)^2 & 0 & \vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2 \\ \vec{\Gamma}_{\mu,2}(\Lambda_1-\Lambda_3)^2 & -\vec{\Gamma}_{\mu,1}(\Lambda_2-\Lambda_3)^2 & 0 \end{pmatrix} O^T$$



shape Λ
dependence
perturbation $\Lambda_2 > 0$







In **uniaxial nematic case** e.g. simplest $(\Lambda_1 = 1, \Lambda_2 = 0, \Lambda_3 = 0)$: $\boldsymbol{M} = \vec{\boldsymbol{n}}\,\vec{\boldsymbol{n}}^T$

$$A_\mu = [M, M_\mu] = \|\vec{n}\|^2 \begin{pmatrix} 0 & (\vec{n}\times\vec{n}_\mu)_3 & -(\vec{n}\times\vec{n}_\mu)_2 \\ -(\vec{n}\times\vec{n}_\mu)_3 & 0 & (\vec{n}\times\vec{n}_\mu)_1 \\ (\vec{n}\times\vec{n}_\mu)_2 & -(\vec{n}\times\vec{n}_\mu)_3 & 0 \end{pmatrix}$$

leading to EM $F_{\mu\nu}$ curvature as Faber: $\partial_\mu(\vec{n}\times\vec{n}_\nu) - \partial_\nu(\vec{n}\times\vec{n}_\mu) = 2\,\vec{n}_\mu \times \vec{n}_\nu$

General: small perturbation $\Lambda_2 > 0$, **shape Λ dependence** in $\Gamma_\mu \xrightarrow{\Lambda} A_\mu,\ R_{\mu\nu} \xrightarrow{\Lambda} F_{\mu\nu}$

EM/Skyrme Landau-de Gennes

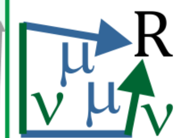
QED $\hbar c$ tiny → unify, also gravity

$$\mathcal{L}_{QED} = -\boxed{\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi} - mc^2\bar{\psi}\psi - \boxed{F_{\mu\nu}F^{\mu\nu}/4}$$

$-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$ gluon QCD $\notin$

$$\mathcal{L} = -\sum_{\alpha\beta\mu\nu} F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V(M)$$

EM-like Higgs-like SO(1,3) minimum



$$F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$$

$G, 1, \hbar$ charge quantizaton

F: shape D **weighted** curvature R

Landau-de Gennes/EM-like
Lorentz-invariant Lagrangian,
vacuum for long-range interactions

***V*=0:** $\boldsymbol{M = O\,D\,O^T \qquad O \in \mathrm{SO}(1,3)}$

stress-energy tensor+Higgs

O: 3D rotation + 4D boost

$\boldsymbol{D = \mathrm{diagonal}(g, 1, \delta, 0)}$

gravity	+ EM	+ QM
$g>>1$ boosts	1	$1>>\delta>0$
$\sim 1/G$	1	Planck $\hbar$

E **B** ← $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ← gauge? **A** ← extended quantum phase for topological charge quantiziation ← (dual) $A^*_\mu = [M, M_\mu]$, $M_\mu = \partial_\mu M$

Aharonov-Bohm

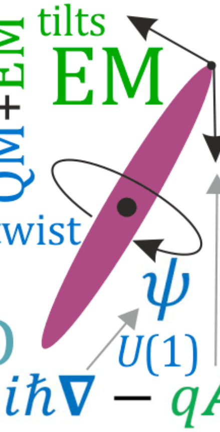
Maxwell: $\Box A_\mu \propto J_\mu$

$\cong$affine connection

$F^* \cong$ curvature

$$\psi_{2D} \xrightarrow{M=ODO^T} 3D$$

$\Box\psi \propto -\psi$ | $\hat{P} = -i\hbar\nabla - qA$



$\bar{M}_\mu$ = {{0, g $\tilde{\Gamma}^1_\mu$, g $\tilde{\Gamma}^2_\mu$, g $\tilde{\Gamma}^3_\mu$}, {-g $\tilde{\Gamma}^1_\mu$, 0, Γ^3_μ, -Γ^2_μ}, {-g $\tilde{\Gamma}^2_\mu$, Γ^3_μ, 0, δ Γ^1_μ}, {-g $\tilde{\Gamma}^3_\mu$, -Γ^2_μ, δ Γ^1_μ, 0}}; (* g >> 1 >> δ > 0 approximation *)

($\bar{F}_{\mu\nu}$ = FullSimplify[coms[$\bar{M}_\mu$, $\bar{M}_\mu$ /. μ → ν], sub]) // MatrixForm

clock propulsion? EM/QM - gravity interactions

$$\begin{pmatrix}
0 & g(\Gamma^3_\nu\tilde{\Gamma}^2_\mu - \Gamma^2_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^2_\nu + \Gamma^2_\mu\tilde{\Gamma}^3_\nu) & g(\Gamma^3_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^3_\nu) & g(-\Gamma^2_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^2_\mu + \Gamma^2_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^2_\nu) \\
g(\Gamma^3_\nu\tilde{\Gamma}^2_\mu - \Gamma^2_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^2_\nu + \Gamma^2_\mu\tilde{\Gamma}^3_\nu) & 0 & \delta R^3_{\{\mu,\nu\}} + g^2\tilde{R}^3_{\{\mu,\nu\}} & \delta R^2_{\{\mu,\nu\}} - g^2\tilde{R}^2_{\{\mu,\nu\}} \\
g(\Gamma^3_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^3_\mu - \Gamma^3_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^3_\nu) & -\delta R^3_{\{\mu,\nu\}} - g^2\tilde{R}^3_{\{\mu,\nu\}} & 0 & R^1_{\{\mu,\nu\}} + g^2\tilde{R}^1_{\{\mu,\nu\}} \\
g(-\Gamma^2_\nu\tilde{\Gamma}^1_\mu + \delta\Gamma^1_\nu\tilde{\Gamma}^2_\mu + \Gamma^2_\mu\tilde{\Gamma}^1_\nu - \delta\Gamma^1_\mu\tilde{\Gamma}^2_\nu) & -\delta R^2_{\{\mu,\nu\}} + g^2\tilde{R}^2_{\{\mu,\nu\}} & -R^1_{\{\mu,\nu\}} - g^2\tilde{R}^1_{\{\mu,\nu\}} & 0
\end{pmatrix}$$

squared +/- energy contributions

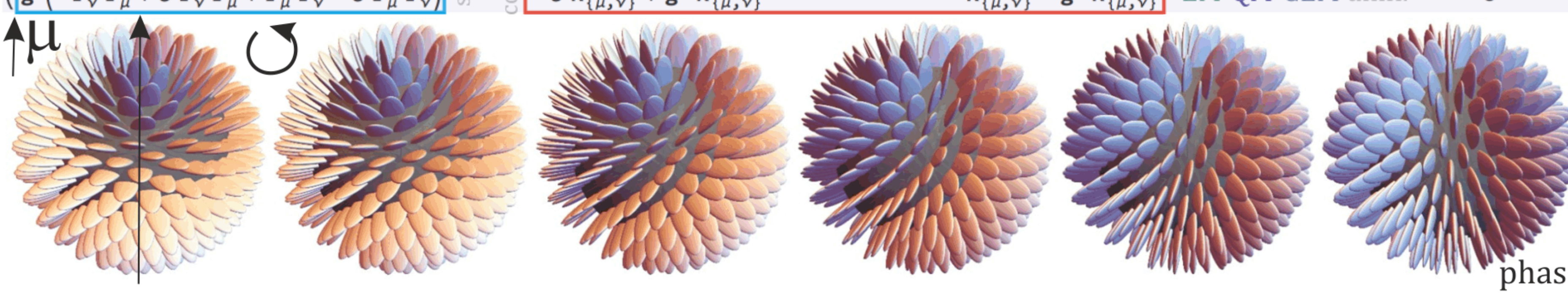
$F_{\mu\nu}$ curvatures

QM: tilt-twist

EM: tilt-tilt

gravity: boosts

EM-QM-GEM unifi.



electron clock(?): phase ψ time evolution

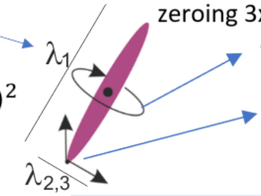
phase spinning

Model: field $M(t,x,y,z) \equiv \boldsymbol{M = ODO^T}$ of real symmetric 3x3 matrices, $OO^T = I$ describes **local rotation**, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$ **shape** of "molecule"
(to be extended to 4x4 tensor field – adding gravitoelectromagnetism)
$(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ is **shape** preferring (Λ_i) shape fixed by model, e.g. $V = \sum_i (\lambda_i - \Lambda_i)^2$

With $\boxed{\textbf{Lagrangian}: \mathcal{L} = \sum_{\mu=1}^{3} ||F_{\mu 0}||_F^2 - \sum_{1\le\mu<\nu\le 3} ||F_{\mu\nu}||_F^2 - V}$

for $F_{\mu\nu} = [\partial_\mu M, \partial_\nu M] = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad A_\mu = [M, \partial_\mu M]$

$(V \approx 0)$ Assume **vacuum case:** $(\lambda_i) \approx (\Lambda_i) = (1, \delta, 0)$ for tiny δ related with Planck $\hbar$
zeroing 3x3 M **variations: 3 rotations**, and 3 axis elongations – zeroing the lowest δ:



twist: ~Klein-Gordon $\quad X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$
tilt1: $X^1 \cdot \Gamma^3 = 0$
tilt2: $X^1 \cdot \Gamma^2 = 0$
all 3 **elongations**: $|B^1| = |E^1|$ (dominated in 4D)

for $X^i := \left(-\nabla \cdot B^i,\ \partial_0 B^i + \nabla \times E^i\right)$ as in **Maxwell equations**

$\Gamma_\mu = O^T O_\mu$
$\vec{\Gamma}_\mu = (\Gamma_{\mu,32}, \Gamma_{\mu,13}, \Gamma_{\mu,21})$
$\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$
$\vec{B}_i = \vec{R}_{0i}$
$\vec{E}_{1,2,3} = (\vec{R}_{32}, \vec{R}_{13}, \vec{R}_{23})$

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

Variating Lagrangian in vacuum ($V = 0$): **Euler-Lagrange** evolution equation:

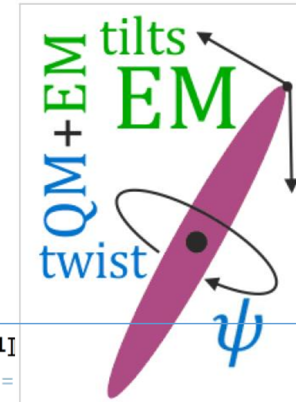
$$\boxed{0 = \sum_{\mu\nu} d_{\mu\nu} \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\Gamma_\nu, [\overline{M}_\mu, G']] - [\Gamma_\mu, [\overline{M}_\nu, G']]\right) + \overline{F}_{\mu\nu,\nu}[\overline{M}_\mu, G'] - \overline{F}_{\mu\nu,\mu}[\overline{M}_\nu, G']\right)}$$

$\overrightarrow{E^i})$

~Maxwell for 2 tilts (high energy) as: $\quad X^1 \cdot \Gamma^3 = 0 = X^1 \cdot \Gamma^2$

~Klein-Gordon for twist (low energy) as: $\quad X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2$

for hedgehog: $\quad 2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|} \cdot \nabla\right)^2\right)\psi$




```
d = DiagonalMatrix[{1, δ, 0}];                        (* ellipsoid shape, δ ~ ℏ *)
Gx = {{0, 0, 0}, {0, 0, -1}, {0, 1, 0}};              (* twist generator *)
Gy = {{0, 0, 1}, {0, 0, 0}, {-1, 0, 0}};              (* tilt1 generator *)
Gz = {{0, -1, 0}, {1, 0, 0}, {0, 0, 0}};              (* tilt2 generator *)
Ga = {{1, 0, 0}, {0, 0, 0}, {0, 0, 0}};               (* 3 elongation generators *)
Gb = {{0, 0, 0}, {0, 1, 0}, {0, 0, 0}};
Gc = {{0, 0, 0}, {0, 0, 0}, {0, 0, 1}};               (* Gpt of G'=Gd + dG^T *)
Gpt = Join[Table[G.d + d.Transpose[G], {G, {Gx, Gy, Gz}}], {Ga, Gb, Gc}];
com[A_, B_] := A.B - B.A;                             (* commutator *)
cd = {{3, 2}, {1, 3}, {2, 1}};                        (* ϵijk*)
vect[m_] := Table[m[[cd[[i, 1]], cd[[i, 2]]]], {i, 3}]; (* → rotation vector *)
Γμ = {{0, -Γ^3_μ, Γ^2_μ}, {Γ^3_μ, 0, -Γ^1_μ}, {-Γ^2_μ, Γ^1_μ, 0}};   (* its matrix form *)
sub = Table[Cross[{Γ^1_μ, Γ^2_μ, Γ^3_μ}, {Γ^1_ν, Γ^2_ν, Γ^3_ν}][[i]] == R^i_{μ,ν}, {i, 3}];
Mμ = com[Γμ, d]; Γν = Γμ /. μ → ν; Mν = com[Γν, d]; Fμν = Simplify[com[Mμ, Mν], sub];
vrip = Table[Simplify[Tr[Fμν.(com[Γν, com[Mμ, Gp]] - com[Γμ, com[Mν, Gp]]) +
      (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,ν}, {i, 3}]).com[Mμ, Gp] - (*integrate by parts*)
      (Fμν /. Table[R^i_{μ,ν} → R^i_{μ,ν,μ}, {i, 3}]).com[Mν, Gp]]], {Gp, Gpt}];
vr = Simplify[Series[vrip / {2 δ^2, 2, 2, -4, 2, 2}, {δ, 0, 0}] // Normal, sub]
```

$\{R^3_{\{\mu,\nu,\nu\}}\Gamma^2_\mu - R^2_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^3_{\{\mu,\nu,\mu\}}\Gamma^2_\nu + R^2_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,$
$R^1_{\{\mu,\nu,\nu\}}\Gamma^3_\mu - R^1_{\{\mu,\nu,\mu\}}\Gamma^3_\nu,\ -R^1_{\{\mu,\nu,\nu\}}\Gamma^2_\mu + R^1_{\{\mu,\nu,\mu\}}\Gamma^2_\nu,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2,\ (R^1_{\{\mu,\nu\}})^2\}$

```
fin = Table[Sum[v /. μ → 0, {v, 1, 3}] - Sum[v /. {μ → cd[[i, 1]]
      {i, 3}], {v, vr}];                                 (* Lagrangian =
sub1 =                                                   (* rename R curvatures as BE fields *)
  Flatten[Table[{R^i_{0,j} → B^i_j, R^i_{cd[[j,1]],cd[[j,2]]} → E^i_j,
    Table[{R^i_{0,j,k} → B^i_{j,k}, R^i_{cd[[j,1]],cd[[j,2]],k} → E^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
Column[FullSimplify[fn = fin /. sub1], Dividers → All]
```

$(B^3_{\{1,1\}} + B^3_{\{2,2\}} + B^3_{\{3,3\}})\Gamma^2_0 - (B^2_{\{1,1\}} + B^2_{\{2,2\}} + B^2_{\{3,3\}})\Gamma^3_0 + \Gamma^3_3(B^2_{\{3,0\}} + E^2_{\{1,2\}} - E^2_{\{2,1\}}) - \Gamma^2_3(B^3_{\{3,0\}} + E^3_{\{1,2\}} - E^3_{\{2,1\}}) + \Gamma^3_2(B^2_{\{2,0\}} - E^2_{\{1,3\}} + E^2_{\{3,1\}}) - \Gamma^2_2(B^3_{\{2,0\}} - E^3_{\{1,3\}} + E^3_{\{3,1\}}) + \Gamma^3_1(B^2_{\{1,0\}} + E^2_{\{2,3\}} - E^2_{\{3,2\}}) - \Gamma^2_1(B^3_{\{1,0\}} + E^3_{\{2,3\}} - E^3_{\{3,2\}})$	**~Klein-Gordon**
$(B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}})\Gamma^3_0 - \Gamma^3_3(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}) - \Gamma^3_2(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}) - \Gamma^3_1(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}})$	**~Maxwell1**
$-((B^1_{\{1,1\}} + B^1_{\{2,2\}} + B^1_{\{3,3\}})\Gamma^2_0) + \Gamma^2_3(B^1_{\{3,0\}} + E^1_{\{1,2\}} - E^1_{\{2,1\}}) + \Gamma^2_2(B^1_{\{2,0\}} - E^1_{\{1,3\}} + E^1_{\{3,1\}}) + \Gamma^2_1(B^1_{\{1,0\}} + E^1_{\{2,3\}} - E^1_{\{3,2\}})$	**~Maxwell2**
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	electric field enforces magnetic (?)
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	-> de Broglie clock/zitterbewegung?
$(B^1_1)^2 + (B^1_2)^2 + (B^1_3)^2 - (E^1_1)^2 - (E^1_2)^2 - (E^1_3)^2$	in 4D dominated by 0-th: gravity?

$E^2_{\{1,3\}}$ - type: 1 - high energy (EM tilt-tilt), 2,3 low energy (QM tilt-twist)
- spatial coordinate (1,2,3), derivative (0,1,2,3)

Hedgehog ansatz for longest axis (± 1 charge), with $\psi(t,x,y,z)$ phase/twist function

In vacuum ($A=0$) leads to **Klein-Gordon-like**: $(\hat{E}-q\phi)^2\psi = (\hat{p}-qA)^2\psi + m^2\psi$

dual formulation ($E \leftrightarrow B$): $A^{hedg} = \frac{1}{r^2}(x,y,z)$ $\qquad (\hat{E} = i\hbar\partial_t \;,\; \hat{p} = -i\hbar\nabla)$

$\Psi = \exp(i\psi)$ $\qquad \hat{p}\Psi = -i\nabla\Psi = \Psi\nabla\psi$ $\qquad$ here from: $X^2\cdot\Gamma^3 = X^3\cdot\Gamma^2$

we get wave-like: $2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

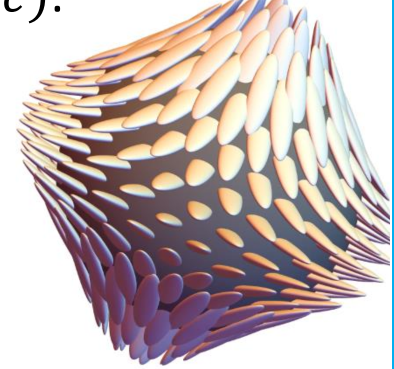
Dirac equation? (also zitterbewegung)

Bispinor for **electron** (up), **positron** (down) with **spin direction** (a,b,c):

$$\frac{1}{4}\begin{bmatrix} 1+c & a-ib & \pm(1+c) & \pm(a-ib) \\ a+ib & 1-c & \pm(a+ib) & \pm(1-c) \\ \pm(1+c) & \pm(a-ib) & 1+c & a-ib \\ \pm(a+ib) & \pm(1-c) & a+ib & 1-c \end{bmatrix}$$

$$S[\Lambda_{\text{boost}}] = \begin{pmatrix} e^{+\chi\cdot\sigma/2} & 0 \\ 0 & e^{-\chi\cdot\sigma/2} \end{pmatrix}$$

$$S[\Lambda_{\text{rot}}] = \begin{pmatrix} e^{+i\phi\cdot\sigma/2} & 0 \\ 0 & e^{+i\phi\cdot\sigma/2} \end{pmatrix}$$



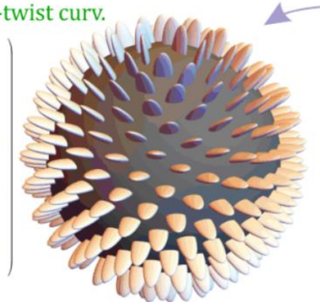
```
sph = {x → r * Cos[θ] * Cos[ϕ], y → r * Cos[θ] * Sin[ϕ], z → r * Sin[θ]}; (*spherical*)
Q0 = FullSimplify[MatrixExp[ϕ * Gz].MatrixExp[θ * Gy].MatrixExp[ψ * Gx] /.
   {ϕ → ArcTan[x, y], θ → -ArcTan[Sqrt[x^2 + y^2], z]}]; (* hedgehog *)
Q = Q0; tQ = Transpose[Q]; M = Simplify[Q.d.tQ];
fBE := Table[Simplify[vect[tQ.com[D[M, c⟦1⟧], D[M, c⟦2⟧]].Q]],
   {c, {{t, x}, {t, y}, {t, z}, {z, y}, {x, z}, {y, x}}}];
M0 = 0.001 * IdentityMatrix[3] + 0.05 * M /. {δ → 0.1, ψ → 0}; (* shape to draw *)
points = SpherePoints[300];
Row[{Column[{"B"1, "B"2, "B"3, "E"1, "E"2, "E"3}], "=", fBE /. ψ → 0 // MatrixForm,
  Graphics3D[{Table[Ellipsoid[p, M0 /. {x → p⟦1⟧, y → p⟦2⟧, z → p⟦3⟧}], {p, points}],
   Gray, Sphere[{0, 0, 0}, 1]}, Boxed → False, ImageSize → Small]}]
```

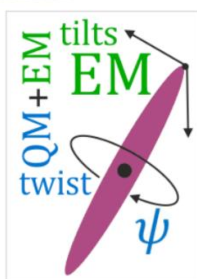
type: EM: high energy tilt-tilt curvature $\qquad$ QM: low energy, tilt-twist curv.

$$\begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ E_1 \\ E_2 \\ E_3 \end{array} = \begin{array}{ccc} 1 & 2 & 3 \end{array}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{x(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{x z \delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \\ -\frac{y(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{y z \delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \\ -\frac{z(-1+\delta)}{(x^2+y^2+z^2)^{3/2}} & 0 & -\frac{z^2 \delta}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)^{3/2}} \end{pmatrix}$$

coordinate






```
Q = Q0 /. {ψ → ψ[t, x, y, z]};                    (* assume phase dependence *)
Γs = Simplify[Table[vect[Transpose[Q].D[Q, v]], {v, {t, x, y, z}}], r > 0];
BE = Simplify[Table[Cross[Γs⟦c⟦1⟧⟧, Γs⟦c⟦2⟧⟧],        (* find BE fields *)
   {c, {{1, 2}, {1, 3}, {1, 4}, {4, 3}, {2, 4}, {3, 2}}}]];
BEd = Simplify[Table[D[BE, v], {v, {t, x, y, z}}]];   (* BE derivatives *)
sub2 = Flatten[Join[Table[Γ^j_(k-1) → Γs⟦k, j⟧, {k, 4}, {j, 3}],
    Table[{B^j_i → BE⟦i, j⟧, E^j_i → BE⟦i + 3, j⟧,
      Table[{B^j_{i,k-1} → BEd⟦k, i, j⟧, E^j_{i,k-1} → BEd⟦k, i + 3, j⟧}, {k, 4}]},
    {i, 3}, {j, 3}]]];
(fne = FullSimplify[fn⟦1 ;; 3⟧ /. sub2] * (x^2 + y^2 + z^2)^2) // Column  (*equations:*)
```

$$-2z\,\psi^{(0,0,0,1)}[t,x,y,z] + (x^2+y^2+2z^2)\,\psi^{(0,0,0,2)}[t,x,y,z] - 2y\,\psi^{(0,0,1,0)}[t,x,y,z] + 2yz\,\psi^{(0,0,1,1)}[t,x,y,z] + x^2\,\psi^{(0,0,2,0)}[t,x,y,z] + 2y^2\,\psi^{(0,0,2,0)}[t,x,y,z] + z^2\,\psi^{(0,0,2,0)}[t,x,y,z] - 2x\,\psi^{(0,1,0,0)}[t,x,y,z] + 2xz\,\psi^{(0,1,0,1)}[t,x,y,z] + 2xy\,\psi^{(0,1,1,0)}[t,x,y,z] + 2x^2\,\psi^{(0,2,0,0)}[t,x,y,z] + y^2\,\psi^{(0,2,0,0)}[t,x,y,z] + z^2\,\psi^{(0,2,0,0)}[t,x,y,z] - 2(x^2+y^2+z^2)\,\psi^{(2,0,0,0)}[t,x,y,z]$$

▸ 0 $\quad$ 0,0 for 2nd, 3rd equation - they are satified, the first equation equalized to 0:

which turns out Klein-Gordon-like: $2\partial_{tt}\psi = \left(\left(\nabla - \vec{A}^{hedg}\right)^2 + \left(\frac{\vec{A}^{hedg}}{|\vec{A}^{hedg}|}\cdot\nabla\right)^2\right)\psi$

for dual: $\vec{A}^{hedg}(x,y,z) = (x,y,z)/r^2$ $\qquad \Psi = \exp(i\psi)$ $\qquad \hat{p}\Psi = -i\nabla\Psi = \Psi\nabla\psi$

```
r = Sqrt[x^2 + y^2 + z^2]; A = {x, y, z} / r^2;
gmA[f_] := Grad[f, {x, y, z}] - A * f; Adg[f_] := (A * r).Grad[f, {x, y, z}];
Simplify[fne⟦1⟧ / r^2 - Sum[gmA[gmA[ψ[t, x, y, z]]]⟦i, i⟧, {i, 3}] -
  Adg[Adg[ψ[t, x, y, z]]]]
```

$$-2\,\psi^{(2,0,0,0)}[t,x,y,z]$$

Derivation of Maxwell-like equations for gravity (GEM), in $E = B = 0$ case

$$\begin{pmatrix} 0 & \tilde{\Gamma}^1_\mu & \tilde{\Gamma}^2_\mu & \tilde{\Gamma}^3_\mu \\ -\tilde{\Gamma}^1_\mu & 0 & -\Gamma^3_\mu & \Gamma^2_\mu \\ -\tilde{\Gamma}^2_\mu & \Gamma^3_\mu & 0 & -\Gamma^1_\mu \\ -\tilde{\Gamma}^3_\mu & -\Gamma^2_\mu & \Gamma^1_\mu & 0 \end{pmatrix}$$

boost+

```
d = DiagonalMatrix[{g, 1, δ, 0}]; cd = {{3, 2}, {1, 3}, {2, 1}}; com[A_, B_] := A.B - B.A;
ξ = DiagonalMatrix[{-1, 1, 1, 1}]; (* signature *)   coms[A_, B_] := A.ξ.B - B.ξ.A;
Γμ = {{0, Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}, {Γ̃^1_μ, 0, -Γ^3_μ, Γ^2_μ}, {Γ̃^2_μ, Γ^3_μ, 0, -Γ^1_μ}, {Γ̃^3_μ, -Γ^2_μ, Γ^1_μ, 0}};   (*3D rot + boost*)
G4 = Table[Coefficient[Γμ, v], {v, {Γ^1_μ, Γ^2_μ, Γ^3_μ, Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}}];   (* rotation + boost generators *)
dg = Table[tm = Table[0, 4, 4]; tm[[i, i]] = 1; tm, {i, 4}];   (* elongation generators *)
Gpt = Join[Table[coms[G, d], {G, G4}], dg];   (* G' size 3+3+4=10 tables *)
sub = Flatten[Table[{Cross[{Γ^1_μ, Γ^2_μ, Γ^3_μ}, {Γ^1_ν, Γ^2_ν, Γ^3_ν}][[i]] == R^i_{μ,ν},   (* EM curvatures *)
      Cross[{Γ̃^1_μ, Γ̃^2_μ, Γ̃^3_μ}, {Γ̃^1_ν, Γ̃^2_ν, Γ̃^3_ν}][[i]] == R̃^i_{μ,ν}}, {i, 3}]];
ds[a_] := Flatten[Table[{R^i_{μ,ν} → R^i_{μ,ν,a}, R̃^i_{μ,ν} → R̃^i_{μ,ν,a}}, {i, 3}]];   (* derivatives *)
cs = Flatten[Table[{Γ^i_μ → 0, Γ^i_ν → 0}, {i, 3}]];   (* assume EM E=B=0 here *)
Mμ = coms[Γμ, d] /. cs; Γν = Γμ /. μ → ν; Mν = coms[Γν, d] /. cs; Fμν = Simplify[coms[Mμ, Mν], sub];
vr = Table[Tr[Fμν.ξ.(coms[Γν, coms[Mμ, Gp]] - coms[Γμ, coms[Mν, Gp]]).ξ +   (* evolution equations *)
      (Fμν /. ds[ν]).ξ.coms[Mμ, Gp].ξ - (Fμν /. ds[μ]).ξ.coms[Mν, Gp].ξ], {Gp, Gpt}] /. cs;
sub1 = Flatten[Table[{R̃^i_{0,j} → B̃^i_j, R̃^i_{cd[[j,1]],cd[[j,2]]} → Ẽ^i_j,   (* GEM EB fields *)
      Table[{R̃^i_{0,j,k} → B̃^i_{j,k}, R̃^i_{cd[[j,1]],cd[[j,2]],k} → Ẽ^i_{j,k}}, {k, 0, 3}]}, {i, 3}, {j, 3}]];
fin = Simplify[Table[Sum[v /. μ → 0, {v, 1, 3}] - Sum[v /. {μ → cd[[i, 1]], ν → cd[[i, 2]]}, {i, 3}], {v, vr}]];
Column[fnl = Limit[(fin[[4 ;; 6]] /. sub1) / 2 / g^4, g → Infinity] // FullSimplify, Dividers → All]
```

$$\begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & E_3 & -E_2 \\ -B_2 & -E_3 & 0 & E_1 \\ -B_3 & E_2 & -E_1 & 0 \end{pmatrix}$$

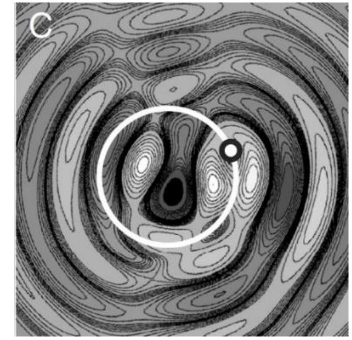
$$\left(\tilde{B}^3_{\{1,1\}} + \tilde{B}^3_{\{2,2\}} + \tilde{B}^3_{\{3,3\}}\right)\tilde{\Gamma}^2_0 - \left(\tilde{B}^2_{\{1,1\}} + \tilde{B}^2_{\{2,2\}} + \tilde{B}^2_{\{3,3\}}\right)\tilde{\Gamma}^3_0 + \tilde{\Gamma}^3_3\left(\tilde{B}^2_{\{3,0\}} + \tilde{E}^2_{\{1,2\}} - \tilde{E}^2_{\{2,1\}}\right) - \tilde{\Gamma}^2_3\left(\tilde{B}^3_{\{3,0\}} + \tilde{E}^3_{\{1,2\}} - \tilde{E}^3_{\{2,1\}}\right) + \tilde{\Gamma}^3_2\left(\tilde{B}^2_{\{2,0\}} - \tilde{E}^2_{\{1,3\}} + \tilde{E}^2_{\{3,1\}}\right) - \tilde{\Gamma}^2_2\left(\tilde{B}^3_{\{2,0\}} - \tilde{E}^3_{\{1,3\}} + \tilde{E}^3_{\{3,1\}}\right) + \tilde{\Gamma}^3_1\left(\tilde{B}^2_{\{1,0\}} + \tilde{E}^2_{\{2,3\}} - \tilde{E}^2_{\{3,2\}}\right) - \tilde{\Gamma}^2_1\left(\tilde{B}^3_{\{1,0\}} + \tilde{E}^3_{\{2,3\}} - \tilde{E}^3_{\{3,2\}}\right)$$

$$-\left(\left(\tilde{B}^3_{\{1,1\}} + \tilde{B}^3_{\{2,2\}} + \tilde{B}^3_{\{3,3\}}\right)\tilde{\Gamma}^1_0\right) + \left(\tilde{B}^1_{\{1,1\}} + \tilde{B}^1_{\{2,2\}} + \tilde{B}^1_{\{3,3\}}\right)\tilde{\Gamma}^3_0 - \tilde{\Gamma}^3_3\left(\tilde{B}^1_{\{3,0\}} + \tilde{E}^1_{\{1,2\}} - \tilde{E}^1_{\{2,1\}}\right) + \tilde{\Gamma}^1_3\left(\tilde{B}^3_{\{3,0\}} + \tilde{E}^3_{\{1,2\}} - \tilde{E}^3_{\{2,1\}}\right) - \tilde{\Gamma}^3_2\left(\tilde{B}^1_{\{2,0\}} - \tilde{E}^1_{\{1,3\}} + \tilde{E}^1_{\{3,1\}}\right) + \tilde{\Gamma}^1_2\left(\tilde{B}^3_{\{2,0\}} - \tilde{E}^3_{\{1,3\}} + \tilde{E}^3_{\{3,1\}}\right) - \tilde{\Gamma}^3_1\left(\tilde{B}^1_{\{1,0\}} + \tilde{E}^1_{\{2,3\}} - \tilde{E}^1_{\{3,2\}}\right) + \tilde{\Gamma}^1_1\left(\tilde{B}^3_{\{1,0\}} + \tilde{E}^3_{\{2,3\}} - \tilde{E}^3_{\{3,2\}}\right)$$

$$\left(\tilde{B}^2_{\{1,1\}} + \tilde{B}^2_{\{2,2\}} + \tilde{B}^2_{\{3,3\}}\right)\tilde{\Gamma}^1_0 - \left(\tilde{B}^1_{\{1,1\}} + \tilde{B}^1_{\{2,2\}} + \tilde{B}^1_{\{3,3\}}\right)\tilde{\Gamma}^2_0 + \tilde{\Gamma}^2_3\left(\tilde{B}^1_{\{3,0\}} + \tilde{E}^1_{\{1,2\}} - \tilde{E}^1_{\{2,1\}}\right) - \tilde{\Gamma}^1_3\left(\tilde{B}^2_{\{3,0\}} + \tilde{E}^2_{\{1,2\}} - \tilde{E}^2_{\{2,1\}}\right) + \tilde{\Gamma}^2_2\left(\tilde{B}^1_{\{2,0\}} - \tilde{E}^1_{\{1,3\}} + \tilde{E}^1_{\{3,1\}}\right) - \tilde{\Gamma}^1_2\left(\tilde{B}^2_{\{2,0\}} - \tilde{E}^2_{\{1,3\}} + \tilde{E}^2_{\{3,1\}}\right) + \tilde{\Gamma}^2_1\left(\tilde{B}^1_{\{1,0\}} + \tilde{E}^1_{\{2,3\}} - \tilde{E}^1_{\{3,2\}}\right) - \tilde{\Gamma}^1_1\left(\tilde{B}^2_{\{1,0\}} + \tilde{E}^2_{\{2,3\}} - \tilde{E}^2_{\{3,2\}}\right)$$

$\tilde{X}^i := \left(-\nabla\cdot\tilde{B}^i,\ \overrightarrow{\partial_0\tilde{B}^i + \nabla\times\tilde{E}^i}\right)$ **Maxwell:** $\tilde{X}^3\cdot\tilde{\Gamma}^2 = \tilde{X}^2\cdot\tilde{\Gamma}^3$, $\tilde{X}^3\cdot\tilde{\Gamma}^1 = \tilde{X}^1\cdot\tilde{\Gamma}^3$, $\tilde{X}^2\cdot\tilde{\Gamma}^1 = \tilde{X}^1\cdot\tilde{\Gamma}^2$

Time crystals: "**oscillation by energy minimization**", **no-go thm**.?



forms coupled "pilot" wave, leading to quantum phenomena

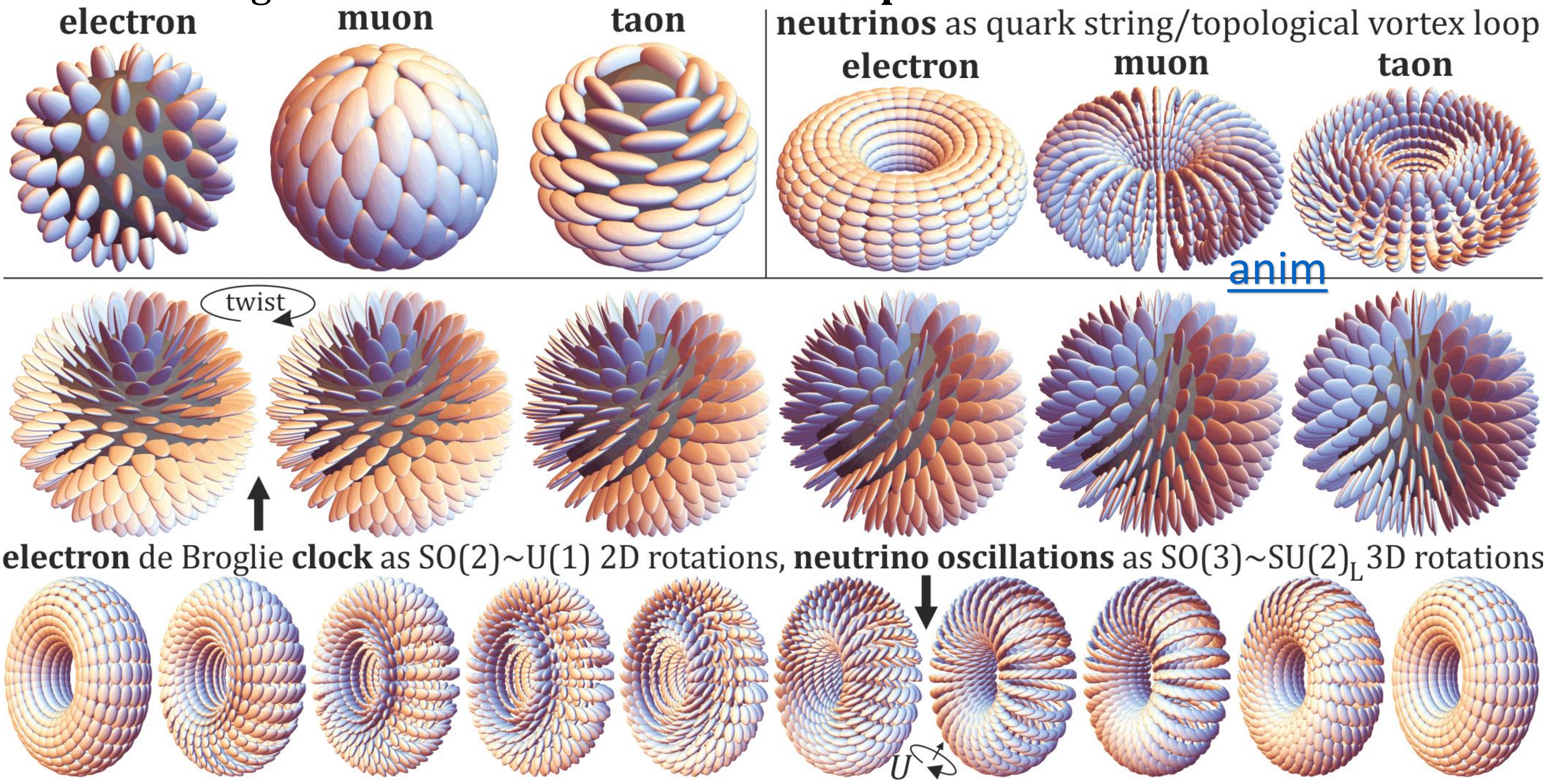
required by relativistic QM: $\boldsymbol{\psi \sim \exp(-iEt/\hbar)}$ **for** $\boldsymbol{E = mc^2}$

e.g. for electron also in angular momentum, and neutrinos

don't radiate!

"**propelled by mass" – structure of particle, like** $\mathcal{H} \propto -(\phi_0\psi_1 - \phi_1\psi_0)^2$

electron angular momentum: **rotation in 2D plane** … **neutrino: 3D rotations**?



Perturbative approx.: **probability** densities, **creation** operators …

Standard model – 26+ parameters, Lagrangian

Noncompatible with **gravity** (“too big infinity”)

“Taylor **expansion” of some simpler model?**

Where to search for it? Non-perturbative: fields

EM-hydro analogy, particles as liquid crystals?

charge quantization + Coulomb-like interaction …

Postulate **skrymion-like Lagrangian** for M:

$$\boldsymbol{\mathcal{L} = -F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} - V(M)} \qquad \text{(Higgs)}$$

(SO(1,3) vacuum)

$$(R\ \sim)\quad F_{\mu\nu\alpha\beta} = \left[\partial_\mu M, \partial_\nu M\right]_{\alpha\beta}$$

field curvature

($\hbar$,G weighted)

EM >> quantum phase >> GEM vacuum dynamics

EM with built-in **charge quantization**, **regularization**

3 leptons: same charge – different mass, $\mu \neq 0$

3 neutrinos: very stable, oscillations, ↑beta decay

3 families of **baryons**: $m_p < m_n, m_d < m_p + m_n$

deuteron with quadrupole moment, $\mu_d \approx \mu_p + \mu_n$

nuclei as knots – also halo neutrons in e.g. ~5 fm

string hadronization reconnections …

effective?

Feynman ensemble of topological defects → QFT

epicycles…?

1
$$-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$

2
$$M^2W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H\partial_\mu H -$$
$$\tfrac{1}{2}m_h^2H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} +$$
$$\tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\tfrac{1}{2}g^2W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2s_wc_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$
$$gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1s_w^2A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$

3
$$\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] +$$
$$\tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 -$$
$$1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$
$$(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_jC^\dagger_{\lambda\kappa}\gamma^\mu(1 +$$
$$\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] -$$

4
$$\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) +$$
$$m_u^\lambda(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1 -$$
$$\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) -$$
$$\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 -$$

5
$$\tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- -$$
$$\partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y -$$
$$\partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ -$$
$$\partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H] +$$
$$\tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \tfrac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] +$$
$$igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]$$