Telling apart black holes and naked singularities with trajectories of massive particles

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Angelos Karakonstantakis[1], Włodek Kluźniak[1], Maciek Wielgus[2]

[1]Nicolaus Copernicus Astronomical Center Warsaw, Poland
[2]Instituto de Astrofísica de Andalucía-CSIC, Glorieta de la Astronomía s/n, E-18008 Granada, Spain

Introduction

Naked singularities are another possible final state of gravitational collapse. Distinguishing between black holes and naked singularities is crucial for understanding the nature of spacetime singularities.

Recent gravitational wave detections and black hole imaging have enabled the study of strong gravity regions in the universe. In addition, strong gravity could also impact the observable signatures of tidal disruption events (TDEs), luminous transient phenomenon when a star orbiting a supermassive compact object and is ripped apart by tidal forces. For a sufficiently heavy central compact object the associated tidal radius may be very small in the mass units of the central object, hence TDEs can be used to probe the strong gravity regime (Stone et al.; 2019).

Reissner-Nordström metric

Reissner-Nordström spacetime is a static, spherically symmetric solution to Einstein's field equations. It is described by two physical parameters: *mass* M, and *charge* Q. The metric function is

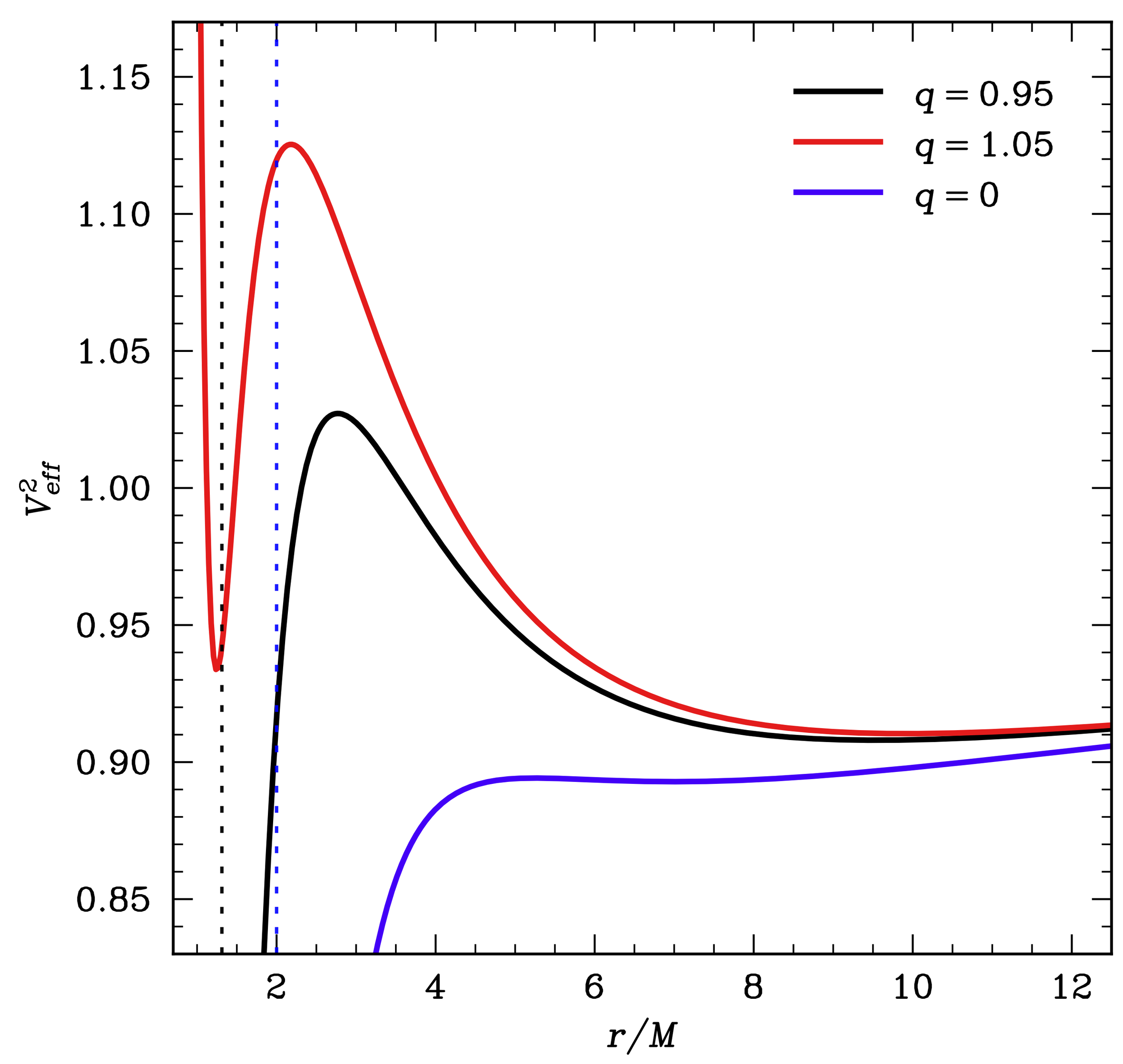
$$f(r) = -g_{tt} = 1 - 2M/r + Q^2/r^2 \, .$$

The radial equation of motion for a massive, neutral (uncharged) particle is $\dot{r}^2 + V^2_{\mathrm{eff}}(r) = \epsilon^2$. With the effective potential being $V^2_{\mathrm{eff}}(r) = f(r)\left[1 + (\ell/r)^2\right]$. The specific angular momentum ℓ and the specific energy ϵ are constant of motion.

The main objective / key point

We investigate the differences in the orbital dynamics of massive particles around black holes and naked singularities in the Reissner-Nordström geometry. We found that **particles reflected by a black hole follow a stream-like family of orbits** within a narrow range of deflection angles, whereas in case of **naked singularities particles are scattered in all directions** on the plain of motion. For debris originated by a TDE, our results showcase differences that would affect the self intersection of the fallback stream, which is often considered necessary to form a circular accretion disk and is expected to also impact the observable signatures of such events.

The effective potential

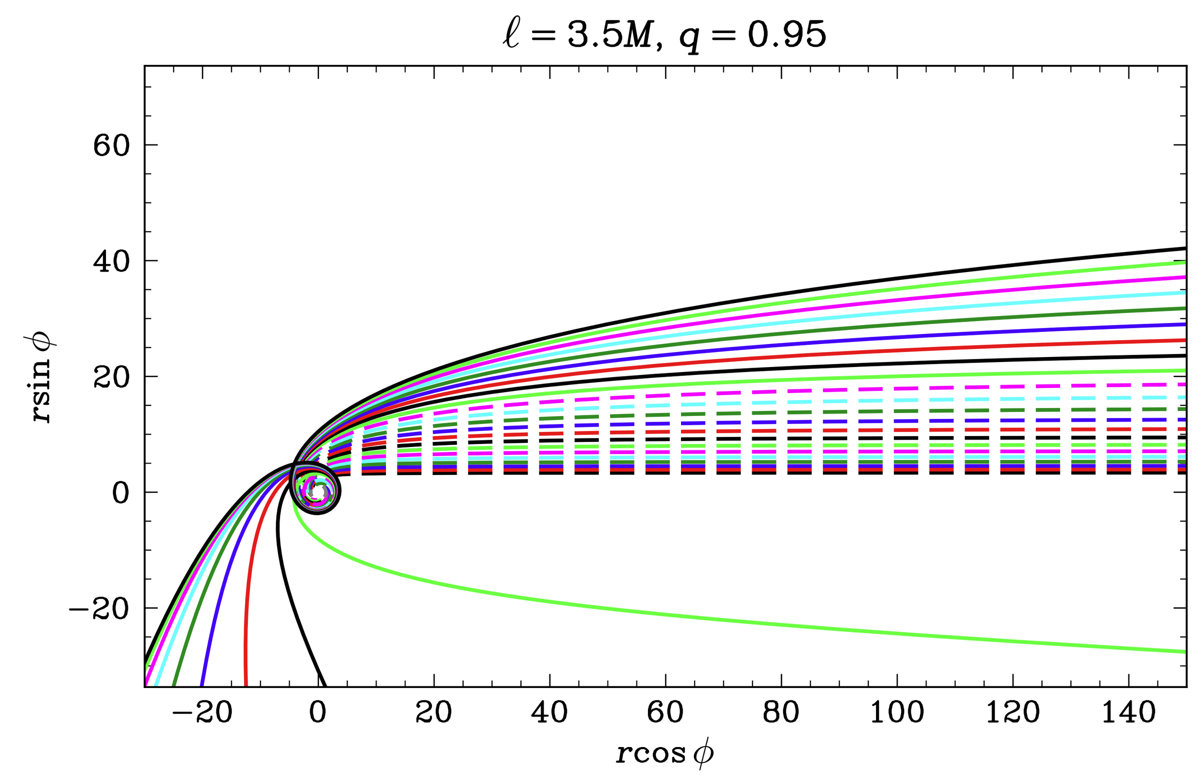


Initial condition

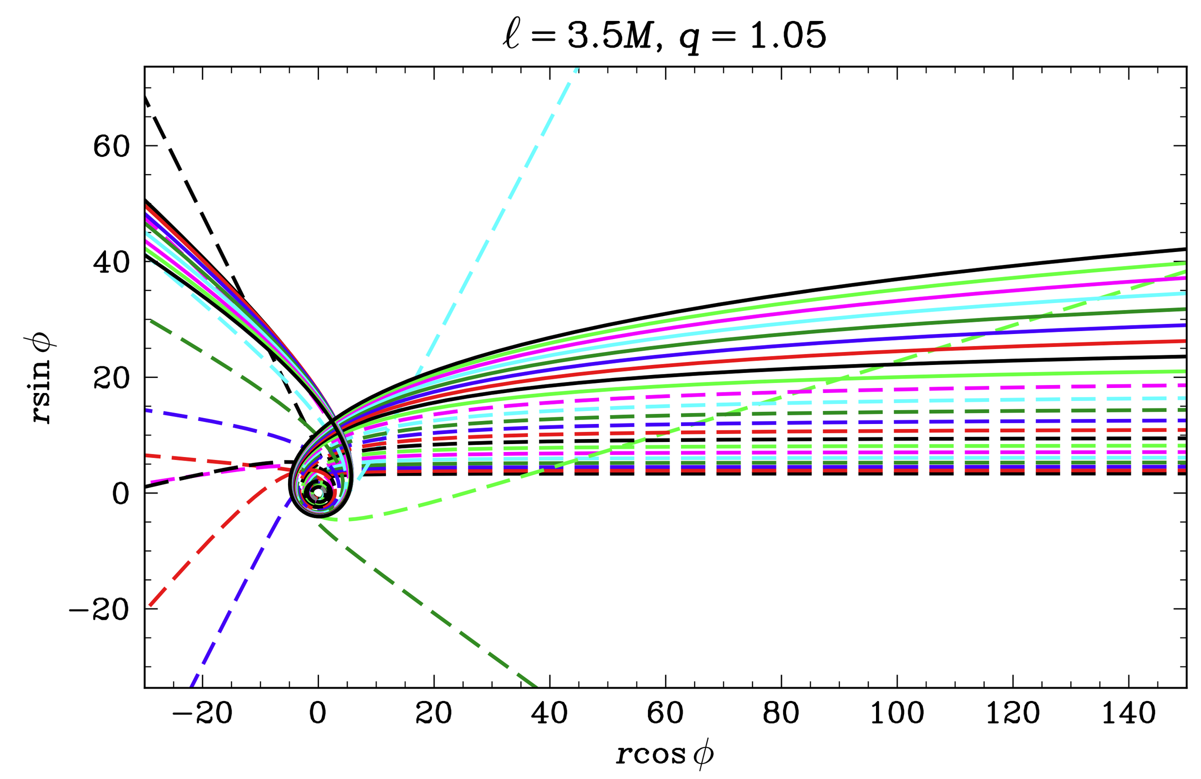
We consider time-like geodesics initiated at a large distance from the compact object. The particles have a fixed specific angular momentum value ℓ and an initial velocity v_0 that is inversely proportional to the impact parameter, y_0.

$$y_0 = \frac{\ell}{v_0} = \frac{\ell}{\sqrt{\epsilon^2 - 1}}$$

Black Hole (BH) case



Naked Signularity (NkS) case



dashed orbits are captured at the event horizon only in the BH case

Conclusions

Our results show that the stream of particles is reflected by the BH roughly in the same direction while, for NkS it is scattered in all directions.

There are certain caveats in our analysis: we assumed unbound particles with a fixed angular momentum moving toward the central singularity. In the accepted picture of TDEs, post-disruption stellar debris acquire a spread of specific energy and angular momentum. Roughly half of the stellar debris is unbound and escape the system and the other half remain bound to the central object. However, we have explained that the differences in BHs and NkS comes from the interplay between the presence of a centrifugal barrier and the reflective zero-gravity sphere, rather than the orbital characteristics.

For future work, we aim to address the above limitations by performing GR-SPH simulations of a TDE in Reissner-Nordström spacetime. The preliminary simulation results support the conclusion of this work:

SPH simulations (animation)