27th RAGtime

Gravitational decoupling and regular hairy black holes: Geodesic stability and quasinormal modes

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Abstract

- We investigate the **dynamic stability** of a **Regular Hairy Black Hole** (BH) metric, obtained via Gravitational Decoupling (GD), focusing on the influence of the **hairy parameter** (β).
- Stability is probed using **Lyapunov exponents** for geodesic orbits and **quasinormal modes** (QNMs) for linear field perturbations.
- **Key Findings:** The β parameter significantly contracts the ISCO and Photon Sphere radii. This non-rotating solution's dynamics mimetize the effects of the Kerr spin parameter a. The spacetime is confirmed to be **linearly stable**.
- **Conclusion:** The β-induced orbital deviations provide crucial new observational signatures for the strong-field regime. Our results contribute directly to the ongoing establishment of observational constraints for **modified gravity theories**.

Introduction

- **Context:** The groundbreaking detection of **gravitational waves** (GWs) has validated General Relativity in the strong-field regime, simultaneously driving the theoretical search for **regular (singularity-free)** Black Hole (BH) solutions within modified gravity.
- **Model Background: Regular Hairy BHs**, characterized by the **hairy parameter** (β), offer singularity-free models via the gravitational decoupling (GD) technique.
- **Objective:** To quantify β's influence on orbital and dynamic stability and characterize its potential to **mimic** the effects of rotation.

Methodology

- **Metric Adoption:** The regular hairy BH metric is adopted from the literature, having been derived by applying the gravitational decoupling (GD):

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2,$$

$$f(r) = 1 - \frac{2}{r} + \frac{2e^{-\frac{r}{\beta}}}{r}\left(1 + \frac{r}{\beta} + \frac{r^2}{2\beta^2}\right).$$

This metric is asymptotically flat and reduces to the Schwarzschild solution in the limit $\beta \to 0$.

- **Geodesic Analysis:** Orbital stability is evaluated by calculating Lyapunov Exponents to determine shifts in the ISCO and Photon Sphere radius as a function of β.

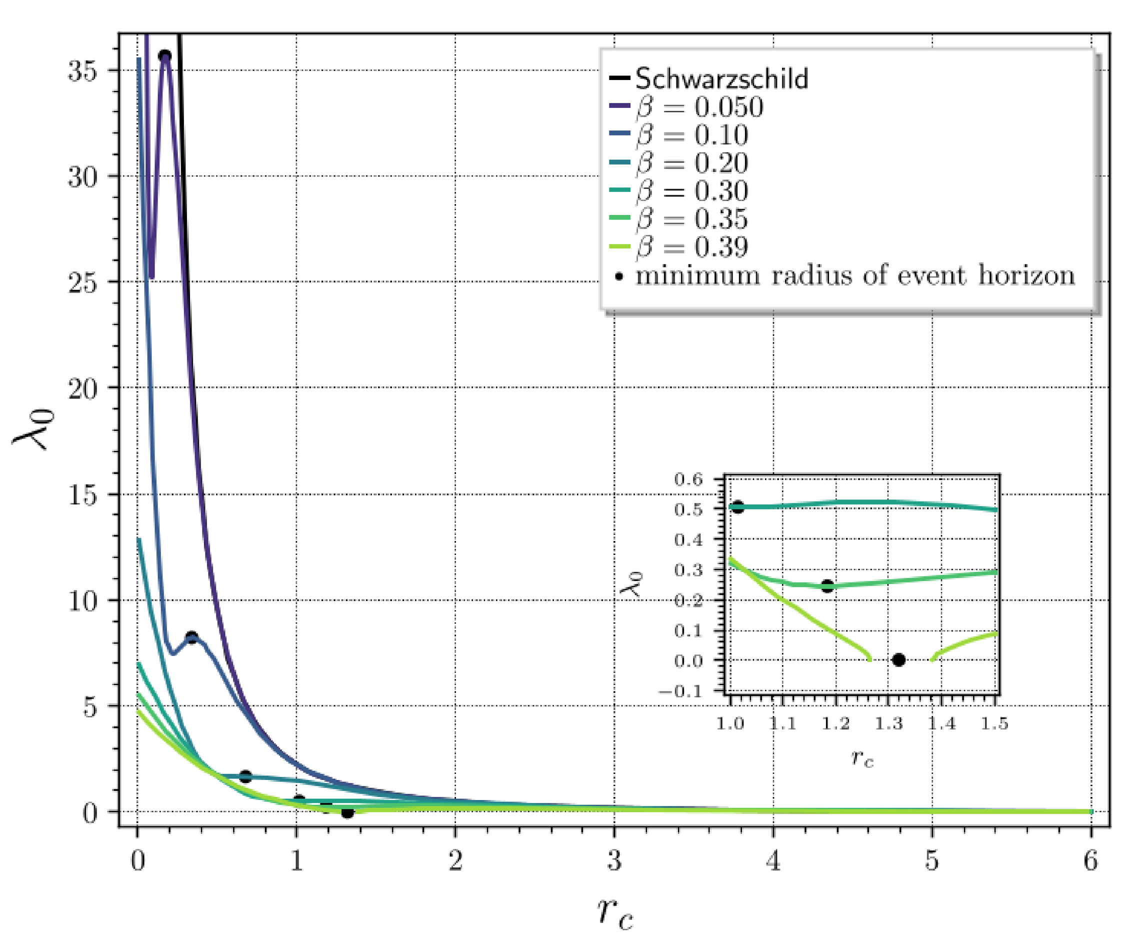

Figure 1. Lyapunov exponents λ_0 for a circular time-like geodesic around the regular hairy black hole .

- **Perturbation Study:** Quasinormal Modes (QNMs) are computed for various fields (scalar, electromagnetic, gravitational) using the **WKB approximation method** to assess the linear stability of the spacetime.

- **Eikonal Limit:** In the eikonal limit, the QNMs are directly related to the angular velocity Ω_c and the Lyapunov exponents λ_0 of the unstable null geodesics:

$$\omega = l\Omega_c - i\left(n + \frac{1}{2}\right)|\lambda_0|.$$

The QNM frequency is governed by the angular orbital frequency of the null geodesics, while the decay rate is determined by the Lyapunov exponent of the unstable null geodesics.

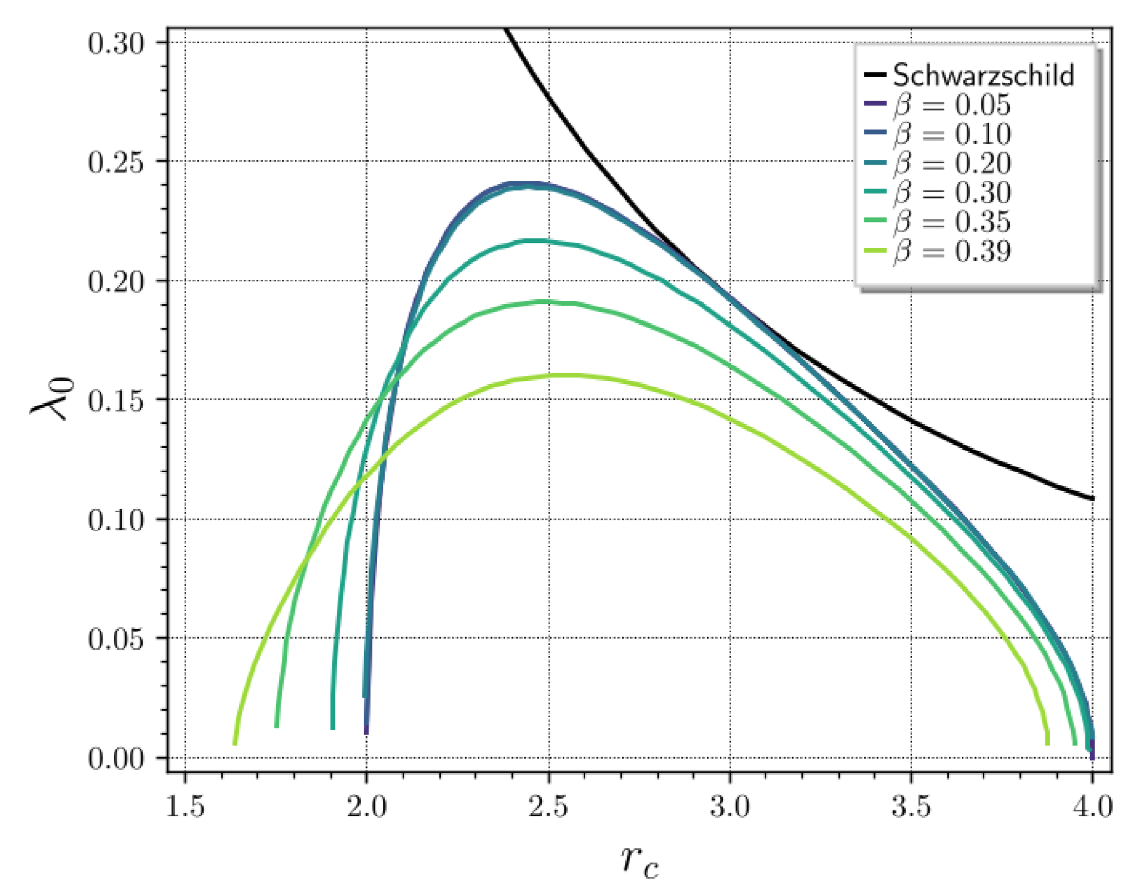

Figure 2. Lyapunov exponents λ_0 for a circular null geodesic around the regular hairy black hole .

Results

- **Geodesic Dynamics:** ↑ β significantly **reduces the radius** of both the ISCO and the Photon Sphere, providing testable dynamic signatures. The orbital modifications caused byβexhibit a dynamic behavior analogous to the effects of the rotation parameter a in the Kerr solution.

- **Perturbative Stability (QNMs):** The imaginary component of all calculated QNM frequencies remains negative, confirming the linear stability of the spacetime. The **effective potential** barrier for perturbations is directly amplified with increasing values of β.

Conclusion

- The hairy parameter (β) is the **primary factor** dictating the observed **dynamic deviations** in the spacetime structure.
- The β parameter effectively **mimics** the dynamic impact of the Kerr spin parameter a for specific values.
- The β -induced deviations provide crucial new observational signatures for **testing the nature of compact objects in the strong-field regime**.

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