# Electromagnetic radiation reaction near black holes: Orbital widening and the role of the tail

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RAGtime 27 | Opava | 10.11.2025 - 14.11.2025

RAGTIME 27TH RELATIVISTIC ASTROPHYSICS GROUP MEETING

## Abstract

We examine the orbital motion of a classical charged particle around a Schwarzschild black hole immersed in a uniform magnetic field, accounting for both the **local radiation-reaction force and the nonlocal curvature-induced tail term**. Using backward-in-time integration of the third-order **DeWitt–Brehme** equation together with the reduced second-order **Landau–Lifshitz** formulation, we show that both equations produce identical trajectories. We also report the novel **orbital widening effect** and demonstrate that it persists even when the tail term is included in the equations of motion. Finally, we present the energy evolution of a particle on a circular orbit, highlighting the influences of radiation reaction and the tail term.

## Background setup

In astrophysical environments, the magnetic field near a black hole is sufficiently weak that it does not alter the spacetime geometry. This requirement is expressed by the condition [1]

$$B \ll B_G = \frac{c^4}{G^{3/2}M_\odot}\left(\frac{M_\odot}{M}\right) \approx 10^{19}\frac{M_\odot}{M}\text{Gauss}.$$

## Metric and magnetic field

The Schwarzschild spacetime metric is given by

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

$$f(r) = 1 - \frac{2M}{r}.$$

Let us consider a Schwarzschild black hole placed in an external, asymptotically uniform magnetic field. When the magnetic field is sufficiently weak—so that it does not modify the spacetime geometry—the Maxwell equations admit a simple solution for the electromagnetic four-potential in this background [2], which can be written as

$$A_\phi = \frac{B}{2}g_{\phi\phi} = \frac{B}{2}r^2\sin^2\theta.$$

The energy and angular momentum of a test charged particle are

$$\mathcal{E} = \frac{E}{m} = f(r)\frac{dt}{d\tau},$$

$$\mathcal{L} = \frac{L}{m} = r^2\sin^2\theta\left(\frac{d\phi}{d\tau} + \frac{qB}{2m}\right).$$

Note, that these quantities are not conserved when the radiation-reaction is included into the equations of motion as shown further.

## DeWitt-Brehme vs Landau-Lifshitz equations

The motion of the particle in the curved spacetime undergoing the electromagnetic radiation reaction is governed by the **DeWitt-Brehme equation** [2]

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + \frac{2q^2}{3m}\left(\frac{D^2u^\mu}{d\tau^2} - \frac{Du^\nu}{d\tau}\frac{Du_\nu}{d\tau}u^\mu\right) + \mathcal{F}^\mu_{tail}.$$

The reduced form of this equation, known as the **Landau-Lifshitz equation** is given by [3,4]

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + k\frac{q}{m}\Big[F^\mu{}_{\nu;\alpha}u^\nu u^\alpha + \frac{q}{m}\left(F^\mu{}_\nu F^\nu{}_\alpha u^\alpha + F_{\alpha\beta}F^\beta{}_\sigma u^\sigma u^\mu u^\alpha\right)\Big] + \mathcal{F}^\mu_{tail}.$$

For convenience, we introduce the magnetic interaction parameter $\mathcal{B}$ and radiation reaction parameter k

$$\boxed{\mathcal{B} = \frac{qB}{2m}, \quad k = \frac{2q^2}{3m}.}$$

**As illustrated in the figure below, both formulations produce identical trajectories under the conditions considered, confirming the consistency of the covariant Landau-Lifshitz equation in curved spacetime.**

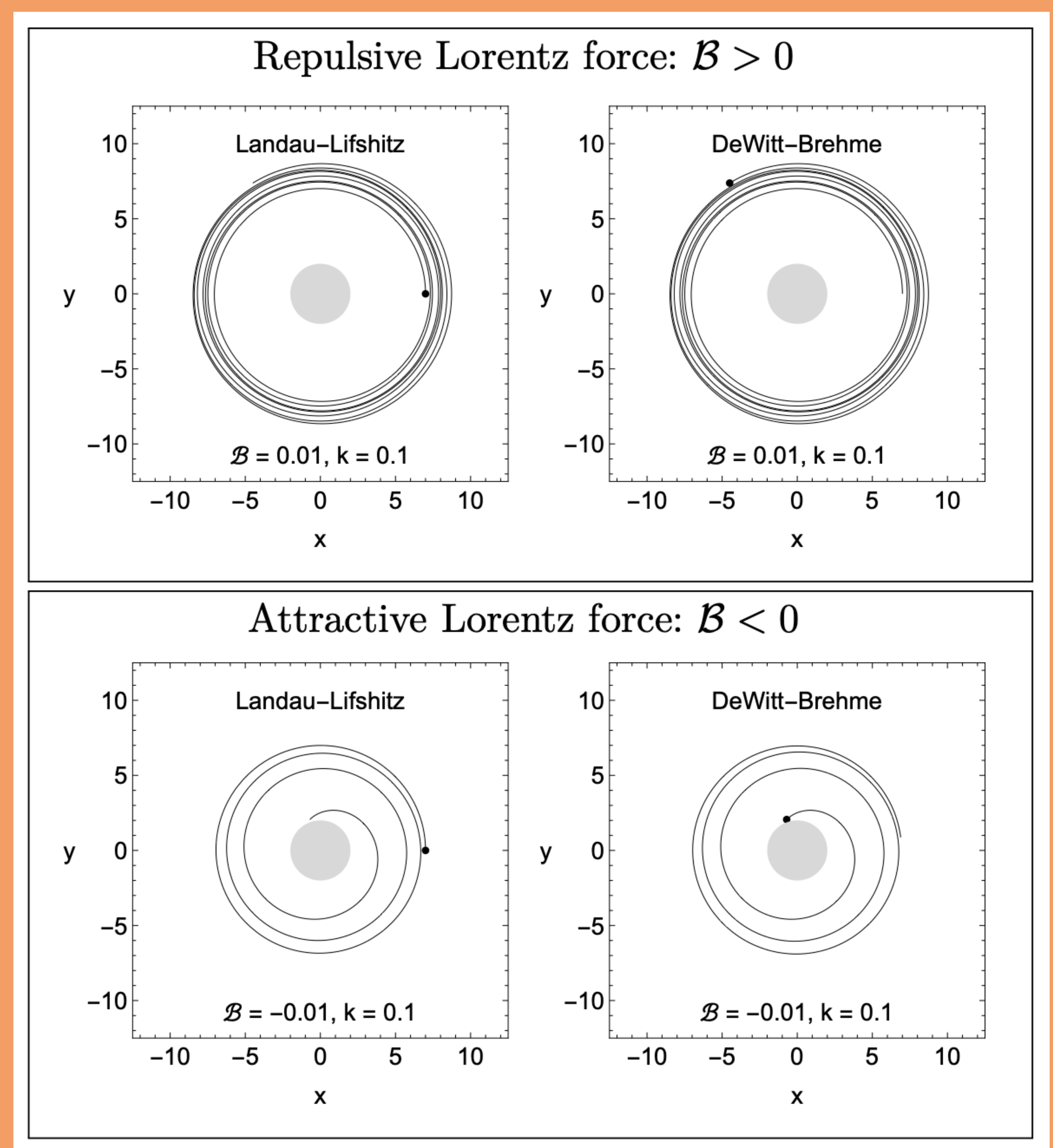


Fig. 1. Comparison of the trajectories of radiating charged particle obtained by integrating the DeWitt-Brehme and Landau-Lifshitz equations.

## Orbital widening effect with tail term

When radiation reaction is included, the particle's orbit may either shrink (**inspiral**) or expand (**orbital widening**). The tail term generally drives inspiral, but for sufficiently strong magnetic fields $\mathcal{B} > 0$, the Lorentz and radiation-reaction forces dominate over the tail contribution.

**As a result, orbital widening persists even when the full self-force — including the nonlocal tail term — is retained.**

This confirms that the orbital widening effect is robust and not an artifact of neglecting the tail term.
Below we show trajectories of charged particle for different values of the radiation and magnetic parameters.

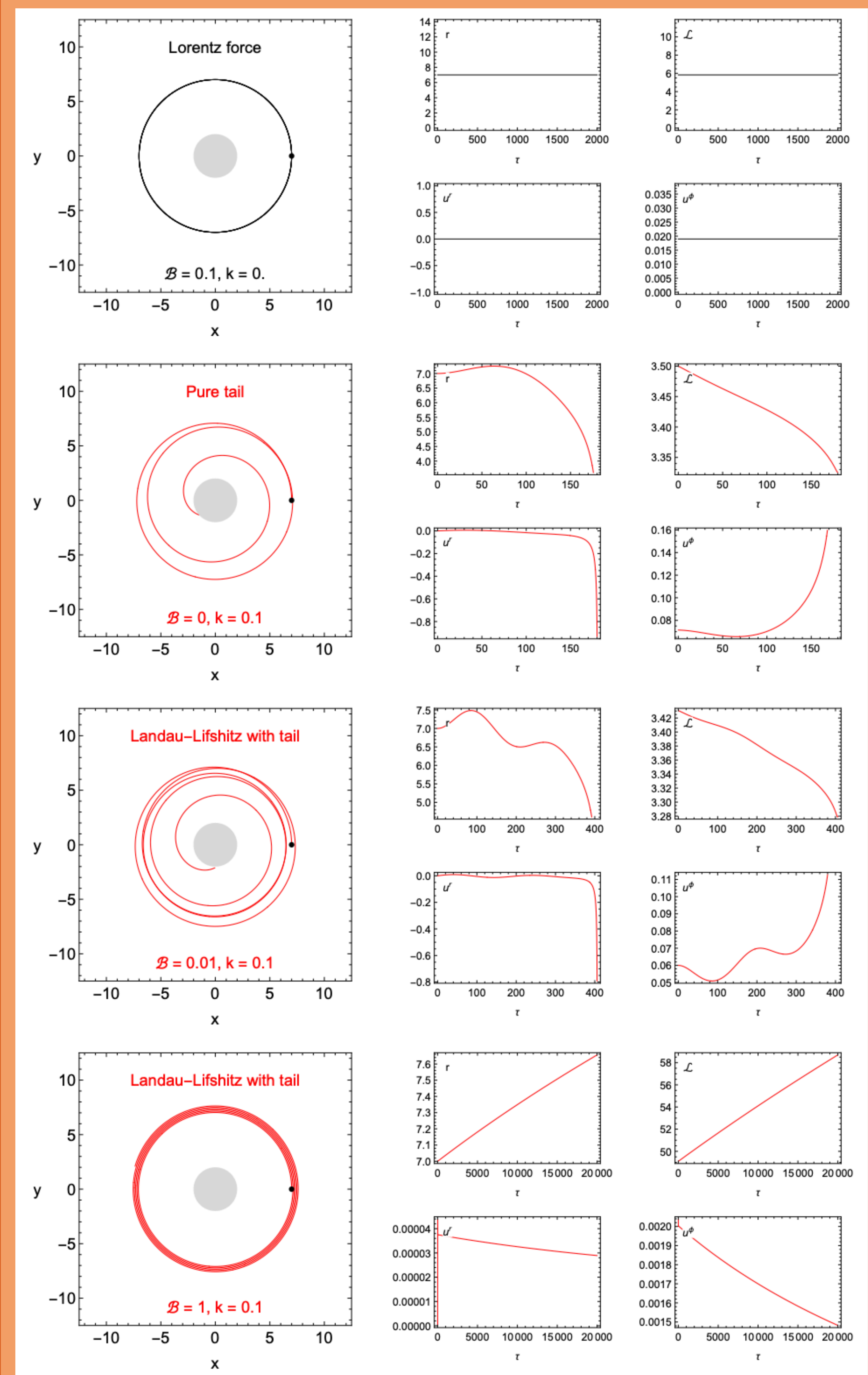


Fig. 2. The trajectories of the particle when the tail term is taken into account for the different values of $\mathcal{B}$. The starting point is marked by dot. Orbital widening effect is demonstrated in the last pannel.

## Energy evolution

Energy evolution of a radiating charged particle in a circular orbit is given by [4]

$$\frac{d\mathcal{E}}{d\tau} = -4k\mathcal{B}^2\mathcal{E}^3_{c.o.} + 2k\mathcal{B}\,\mathcal{E}_{c.o.}\left(2\mathcal{B}f + \frac{u^\phi}{r}\right) + f\mathcal{F}^t_{tail}.$$

In most scenarios, only the first term is taken into account, leading to a standard energy loss due to synchrotron radiation. However, the full form of the equation at circular orbit leads to the dominance of the second term, which is positive, thus leading to the increase of energy.

Below we plot the energy evolution of a radiating charged particle, from which it is clear that for most relevant values of the positive magnetic parameter, the energy is increasing, leading to the orbital widening effect.

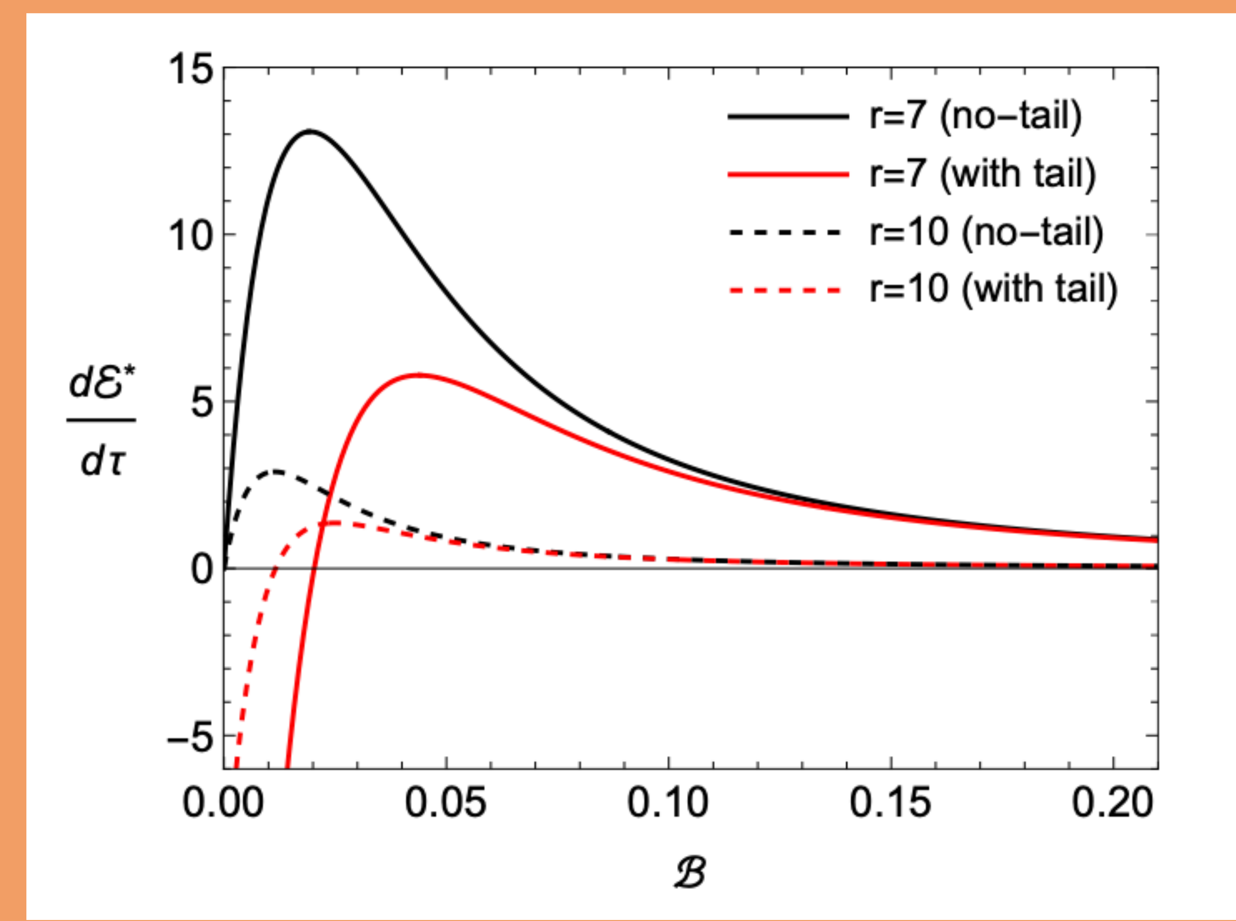


Fig. 3. Energy evolution of a radiating charged particle on a circular orbit, shown for different orbital radii (r = 7, r = 10) and in the presence (red) or absence (black) of the tail contribution. The quantity $\mathcal{E}^* = (10^5/k)\mathcal{E}$ denotes the rescaled energy.

## References

[1] Gal'tsov D.V., et al., *J. Exp. Theor. Phys.* 47, 419 (1978).
[2] Wald R.M., *General Relativity* (1984).
[3] Tursunov A., et al., *ApJ* 861, 2 (2018).
[4] Juraev B., et al., under review in *PRD* (2025).