Pions within the Hartree approximation

Kamil Sokołowski

University of Wroclaw

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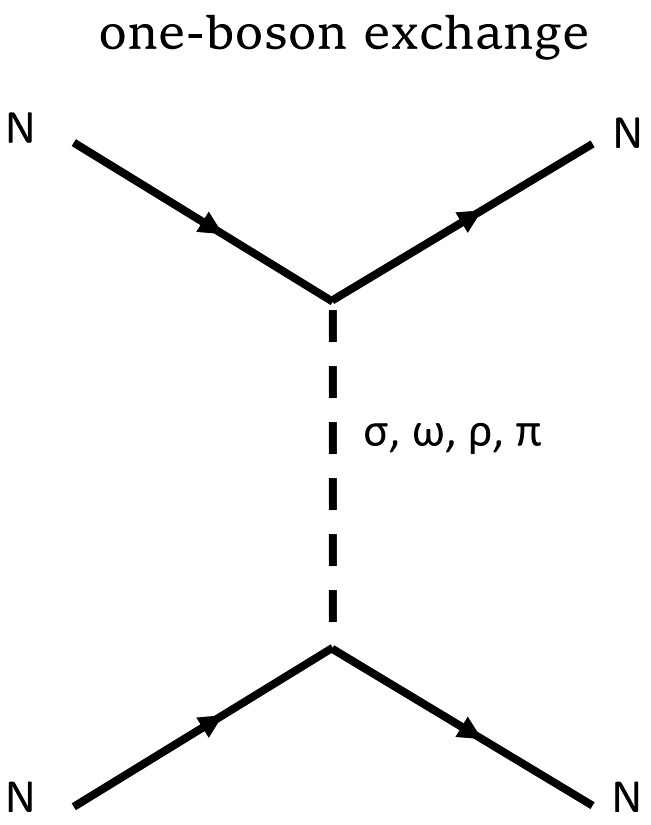
INFINITE NUCLEAR MATTER



SATURATION DENSITY n_0	0.155 ± 0.05 fm^{-3} ($\approx 2 \times 10^{14}$ g/cm^3)
BINDING ENERGY $\frac{B}{A}$	-16 ± 1.0 MeV
COMPRESSION MODULUS $K = 9n_B \frac{\partial p}{\partial n_B}$	250 ± 50 MeV
SYMMETRY ENERGY e_{sym}	32.0 ± 2.0 MeV

N. K. Glendenning. Special and General Relativity. With Applications to White Dwarfs, Neutron Stars and Black Holes. *Springer*, 2007
L.-J. Guo, et al.. Insights into Neutron Star Equation of State by Machine Learning. *The Astrophysical Journal*, 2024.

HARTREE APPROX.

$$\underbrace{\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi}_{\mathcal{L}_{Dirac}} + \underbrace{\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) + g_\sigma\sigma\bar{\psi}\psi}_{\mathcal{L}_\sigma} \underbrace{- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\omega\omega_\mu\bar{\psi}\gamma^\mu\psi}_{\mathcal{L}_\omega}$$

$$\underbrace{-\frac{1}{4}\rho^a_{\mu\nu}\rho^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu} - g_\rho\bar{\psi}\gamma^\mu\tau^a\rho^a_\mu\psi}_{\mathcal{L}_\rho} \underbrace{- \frac{1}{3}g_3\sigma^3 - \frac{1}{4}g_4\sigma^4 + \frac{1}{4}c_4(\omega_\mu\omega^\mu)^2}_{\mathcal{L}_{non-lin}}$$

!

Infinite # of d.o.f →

Mean-field approx.:

1. $\sigma \to \langle\sigma\rangle \equiv \sigma_0$
2. $\omega_\mu \to \langle\omega_\mu\rangle \equiv \delta_{\mu 0}\omega_0$
3. $\rho^a_\mu \to \langle\rho^a_\mu\rangle \equiv \delta^{a3}\delta_{\mu 0}\rho^3_0$

$$\begin{cases} m^* \equiv m - g_\sigma\sigma_0 \\ E^*(\boldsymbol{p}) \equiv E(\boldsymbol{p}) - g_\omega\omega_0 \\ \mu_n^* \equiv \mu_n - g_\omega\omega_0 + g_\rho\rho_0 \\ \mu_p^* \equiv \mu_p - g_\omega\omega_0 - g_\rho\rho_0 \end{cases}$$

d.o.f:
g_σ, g_3, g_4, g_ω, c_4, g_ρ

EOS: $P(\varepsilon)$

energy-momentum tensor →

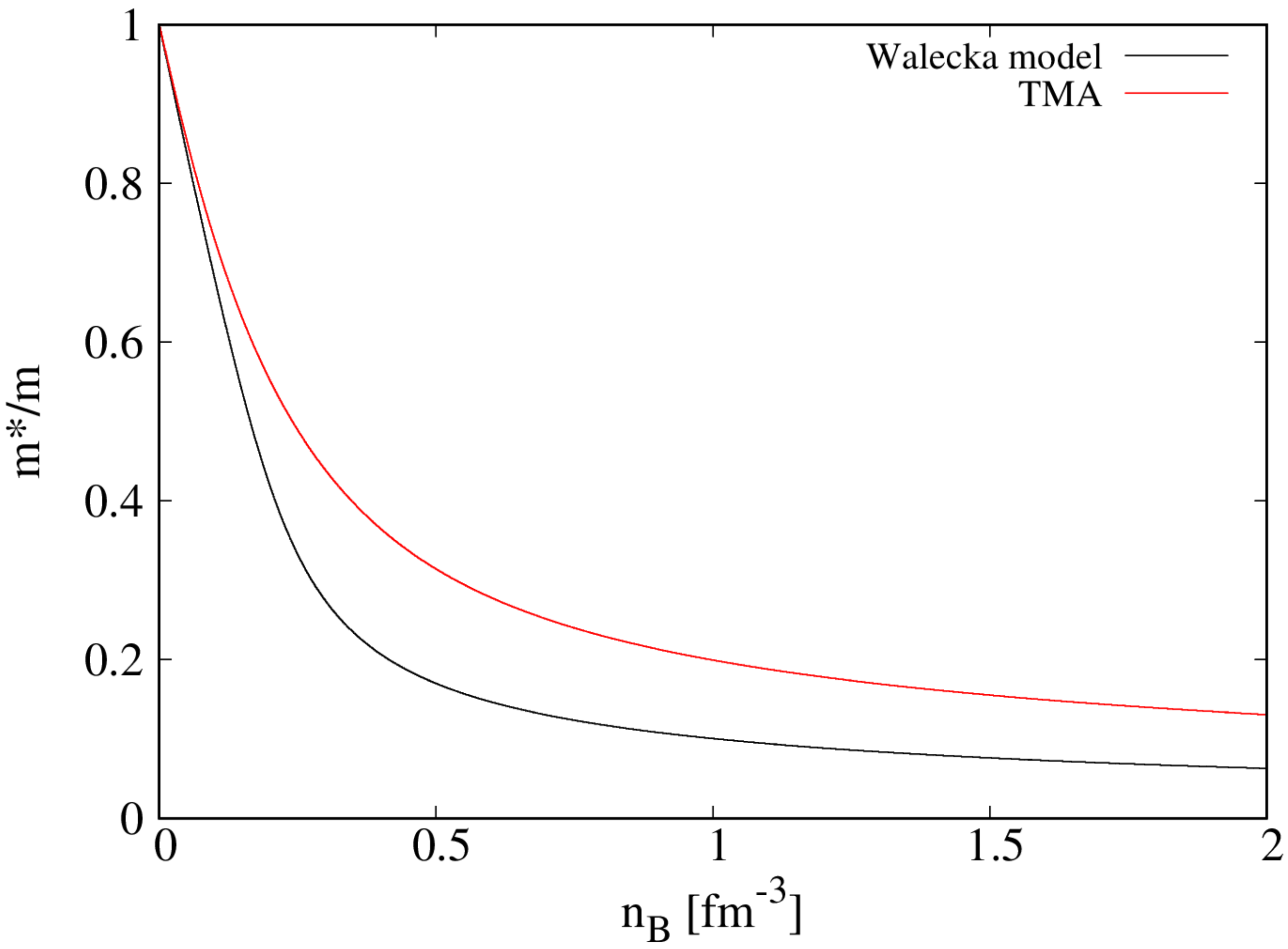
$$T^{\mu\nu}_{MFA} = i\bar{\psi}\gamma^\mu\partial^\nu\psi - g^{\mu\nu}\left(\frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\sigma^2\sigma_0^2\right)$$

perfect fluid →

$$\langle T^{\mu\nu}\rangle = diag(\varepsilon, P, P, P)$$

HARTREE APPROX., PROPERTIES 1

	Walecka	TMA
n_0 [fm^{-3}]	0.148	0.147
$E/\mathrm{n_B} - m$ [MeV]	−15.76	−16.03
K [MeV]	544.84	317.12
E_{sym} [MeV]	20.35	31.61

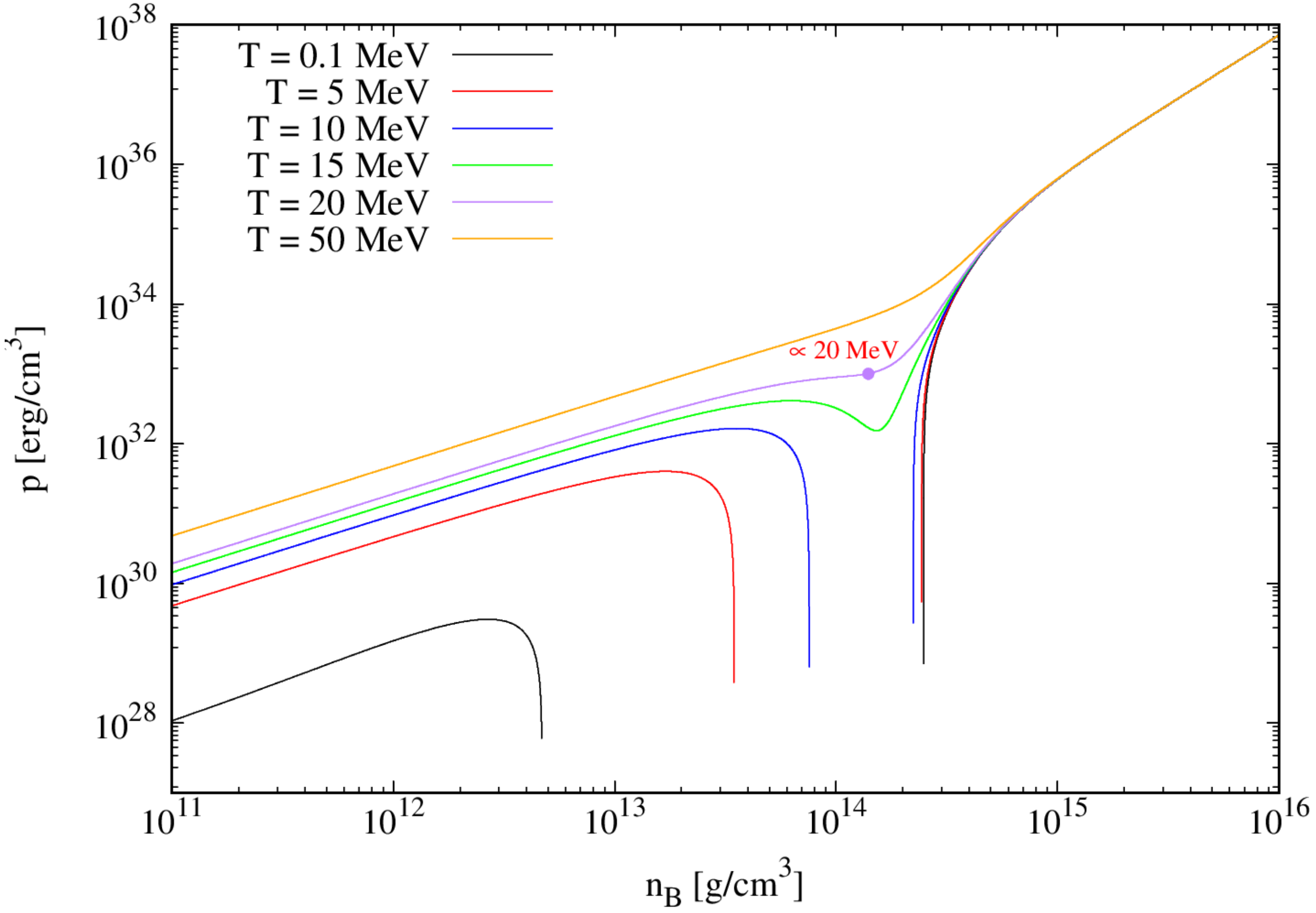


B. D. Serot, J. D. Walecka, The Relativistic Nuclear Many-Body Problem, *Advances in Nuclear physics*, vol. 16, 1986
Ilona Bednarek. Relativistic mean field models of neutron stars. Katowice: Wydawnictwo Uniwersytetu Śląskiego, 2007
H. Toki, D. Hirata, Y. Sugahara, K. Sumiyoshi, and I. Tanihata, Relativistic many body approach for unstable nuclei and supernova, *Nucl. Phys. A*, 1995

HARTREE APPROX., PROPERTIES P.2

$$\Omega = -kT \ln \mathrm{Tr} \exp(-(H - \mu B)/kT) = -P$$

$$f(\boldsymbol{p}; \{\mu, T\}) = \left(\exp\left(\frac{E^*(\boldsymbol{p}) - \mu^*}{kT} \right) + 1 \right)^{-1}$$



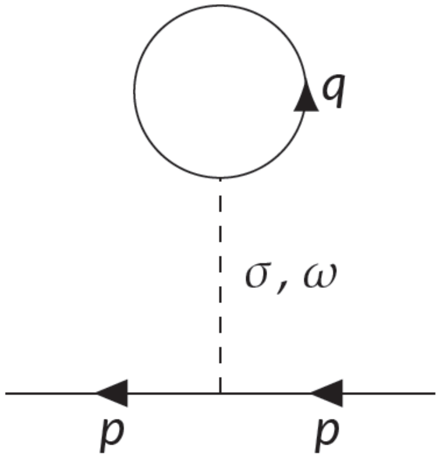
K. Fukushima and C. Sasaki. The phase diagram of nuclear and quark matter at high baryon density. *Progress in Particle and Nuclear Physics*, 2013.

PIONS IN THE HARTREE APPROX.

$$\pi = \{\pi^-, \pi^+, \pi^0\} \qquad \mathcal{L}_\pi = \frac{1}{2}\left(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - m_\pi^2 \boldsymbol{\pi}^2\right) + \mathcal{L}_{\pi NN}$$

$$\mathcal{L}_{\pi NN}^{PV} = -\frac{f_\pi}{\mathrm{m}_\pi} \bar{\psi}\gamma^5\gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau}\psi$$

$$\mathcal{L}_{\pi NN}^{PS} =--ig_\pi \bar{\psi}\gamma^5 \boldsymbol{\pi} \cdot \boldsymbol{\tau}\psi$$



pseudoscalar

$$\begin{cases} \left[\gamma_\mu \partial^\mu - m + g_\sigma \sigma_0 - g_\omega \gamma^0 \omega_0 - \dfrac{f_\pi}{m_{pi}} \gamma^5 \gamma^\mu \partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\tau}\right] \psi(x) = 0 \\ (\Box + m_\sigma^2)\langle \sigma(x)\rangle = g_\sigma \langle \bar{\psi}(x)\psi(x)\rangle \\ (\Box + m_\omega^2)\langle \omega_0(x)\rangle = g_\omega \langle \psi^\dagger(x)\psi(x)\rangle \\ (\Box + m_\pi^2)\langle \boldsymbol{\pi}(x)\rangle = \dfrac{f_\pi}{m_\pi} \partial_\mu \langle \bar{\psi}(x)\gamma^5\gamma^\mu \boldsymbol{\tau}\psi(x)\rangle \end{cases}$$

Parity transformation

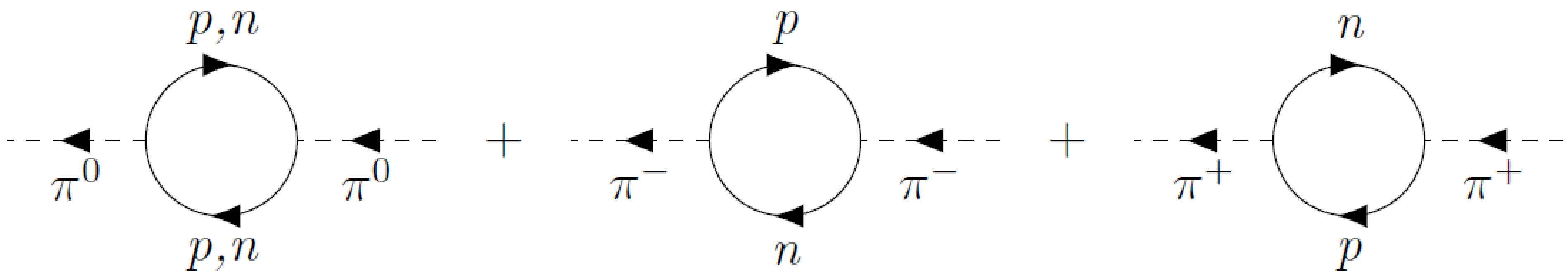
$P: \quad \boldsymbol{x} \to -\boldsymbol{x}$

$$\langle \boldsymbol{\pi}(x)\rangle \to -\langle \boldsymbol{\pi}(-x)\rangle$$

$$\langle \boldsymbol{\pi}(x)\rangle = 0$$

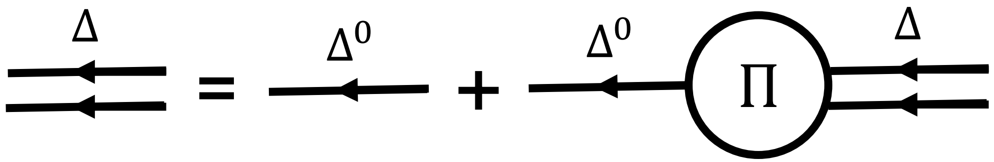
B. Banerjee, N. K. Glendenning, M. Gyulassy, Pion Condensation in a Relativistic Field Theory Consistent with Bulk Properties of Nuclear Matter, *Nucl. Phys. A,* 1981
T. Matsui, B. D. Serot, The Pion Propagator in Relativistic Quantum Field Theories of the Nuclear Many-Body Problem, *Ann. of Phys.*, 1982

PIONS - PROPAGATOR APPROACH



Dyson's equation:

$$i\Delta(p) = i\Delta^0(p) + i\Delta^0(p)\Pi(\mathrm{p})\Delta(\mathrm{p})$$



$$i\Delta^0_{ab}(p) = \frac{\delta_{ab}}{p^2 - m_\pi^2 + i\epsilon}$$

$$i\Delta^0_{ab}(p) = \frac{\delta_{ab}}{p^2 - m_\pi^2 - \mathrm{Re}\Pi(p)}$$

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(p), \boldsymbol{p})$$

$\rightarrow$

$$f(\boldsymbol{p}; \{\mu, T\}) = \exp\left(-\frac{E(\boldsymbol{p}) - \mu}{kT}\right)$$

VIRIAL EXPANSION

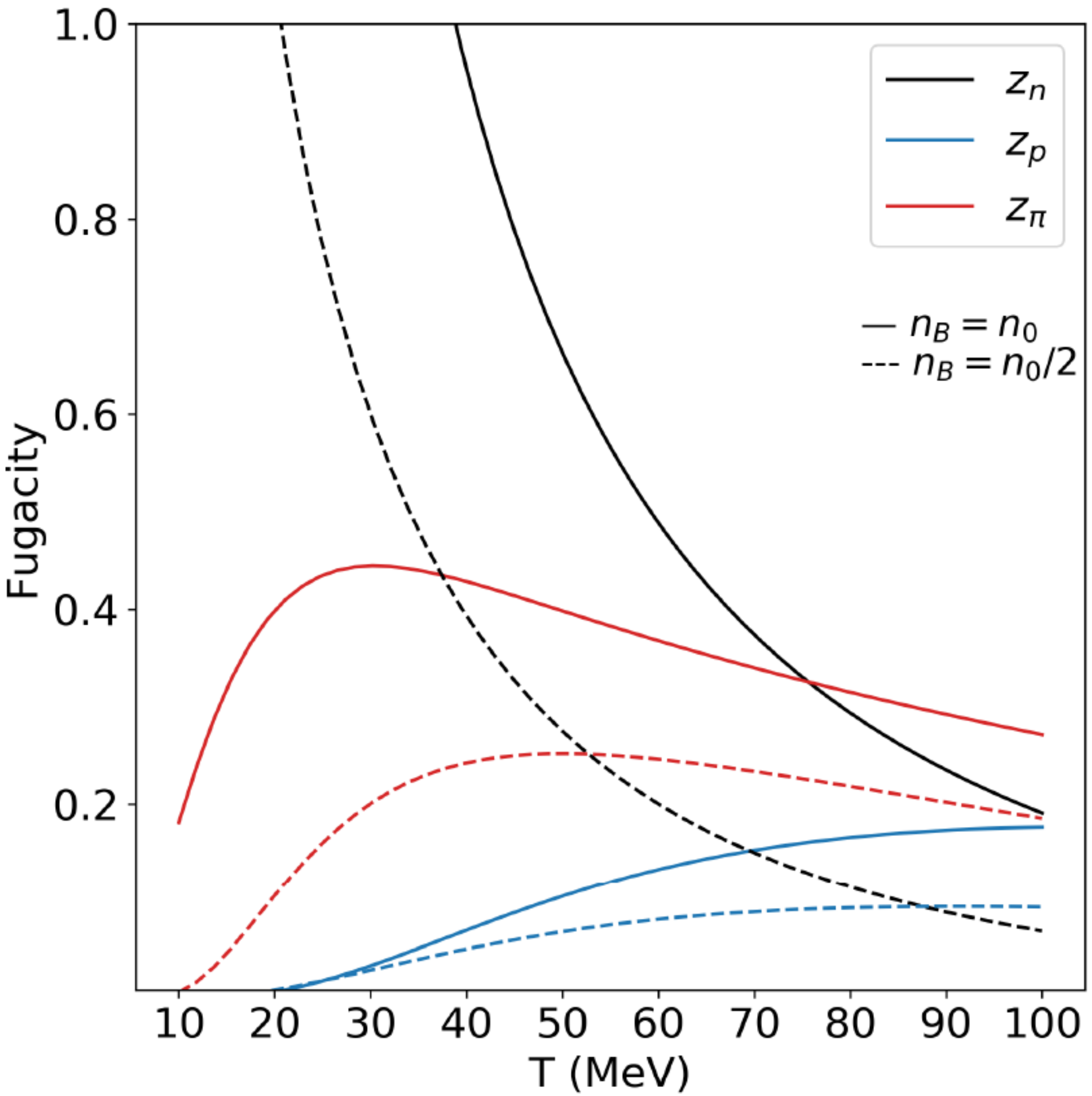
$$\frac{\Omega(T,\{\mu_i\})}{V} = -T\left(\sum_i \frac{b_i}{\lambda_i^3} z_i + \sum_{i,j} \frac{b_{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} z_i z_j + \dots\right)$$

$$z_i = \exp\left(\frac{\mu_i - m_i}{kT}\right) \quad , \quad \lambda_i = \sqrt{\frac{2\pi}{m_i kT}} \quad , \quad b_{ij} \propto \delta_l^{ij}$$

$$n_{\pi^-} = \int \frac{d^3p}{(2\pi)^3} \exp\left(\frac{-\left(\sqrt{p^2 + m_\pi^2} - \mu_{\pi^-}\right)}{kT}\right) + n_{\pi^-}^{int}$$

$$n_{\pi^-}^{int} = \sum_{N=n,p} z_N z_{\pi^-} b_2^{N\pi^-}$$

$$\lambda_i = \sqrt{\frac{2\pi}{m_i kT}}$$



B. Fore, S. Reddy, Pions in Hot Dense Matter and Their Astrophysical Implications, *Phys. Rev. C*, 2020

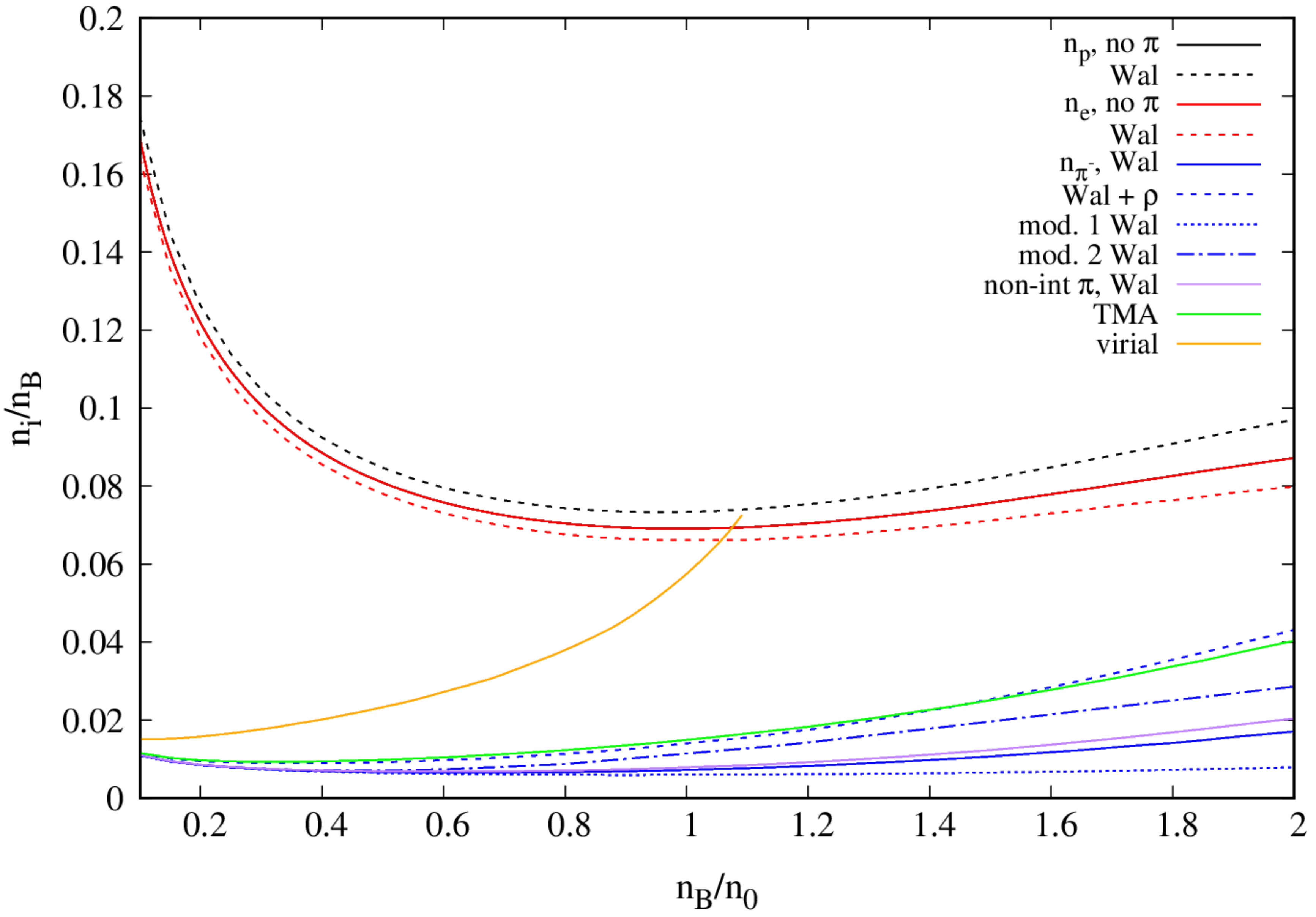
PION ABUNDANCE IN β-EQ.

$$n_\pi = \int \frac{d^3p}{(2\pi)^3} f(p)$$

$$\begin{cases} n_B = n_p + n_n \\ 0 = n_p + n_{\pi^+} - n_e - n_{\pi^-} \\ \mu_e = \mu_n - \mu_p \end{cases}$$

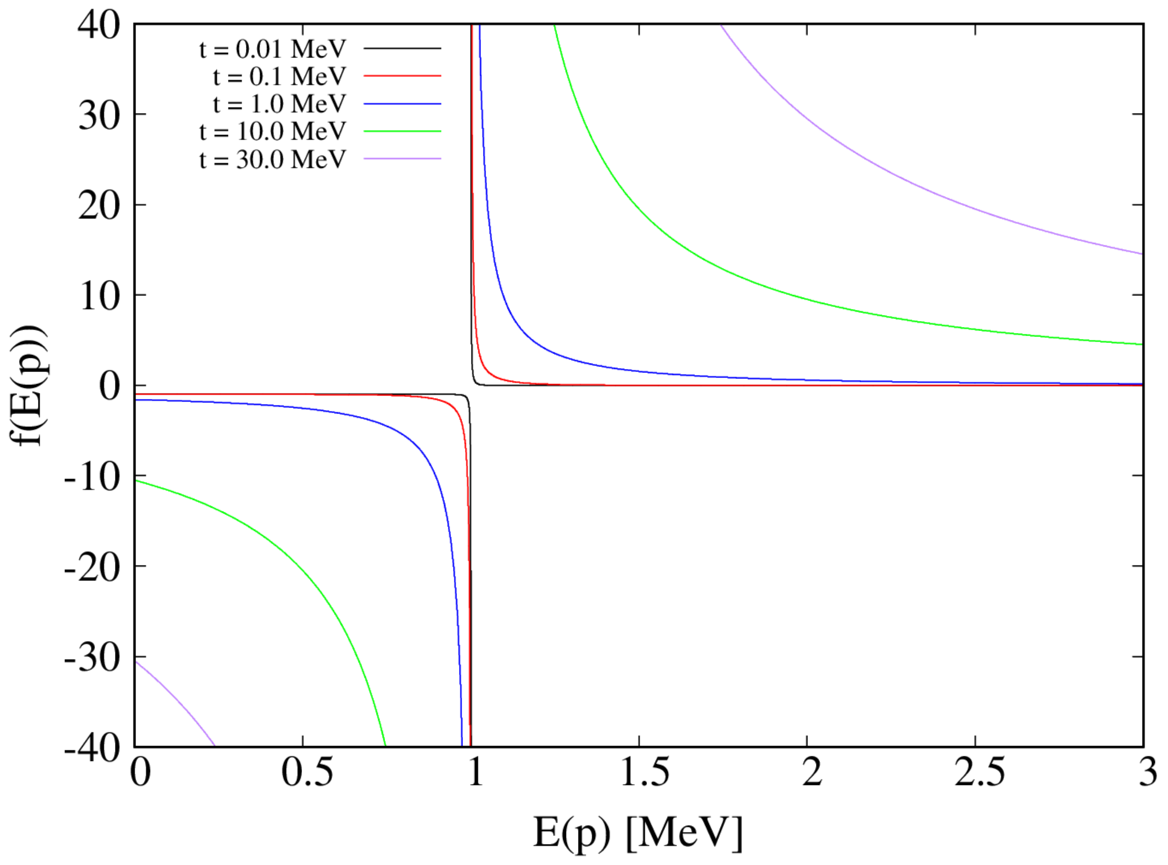
$$n, p \to n, p + \pi^0$$
$$n \to p + \pi^-$$
$$p \to n + \pi^+$$

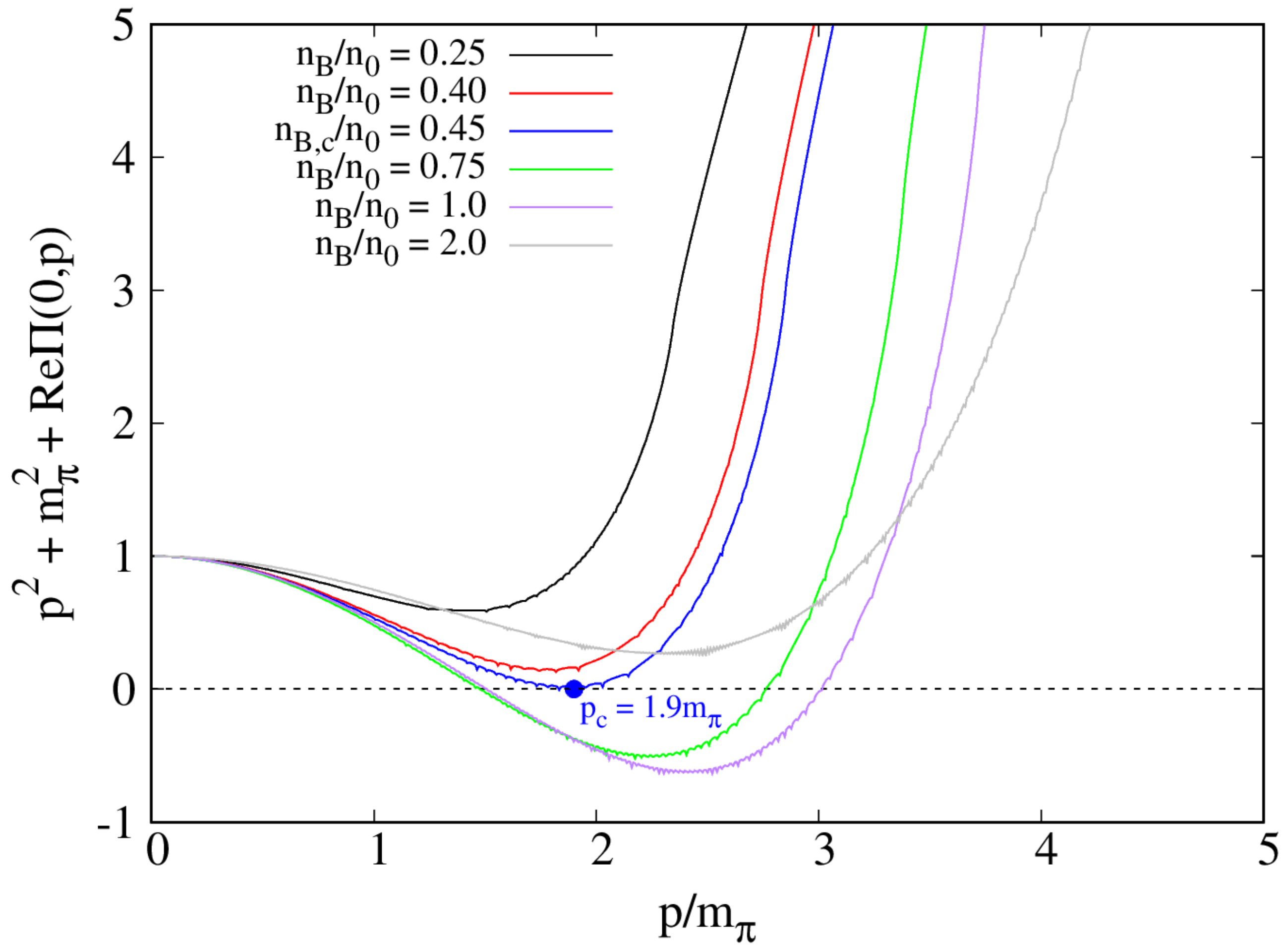
$$\begin{cases} \mu_{\pi^-} = \mu_n - \mu_p = \mu_e \\ \mu_{\pi^+} = -(\mu_n - \mu_p) \\ \mu_{\pi^0} = 0 \end{cases}$$



PION CONDENSATE

$$f(\boldsymbol{p};\{\mu,T\}) = \left(\exp\left(\frac{E(\boldsymbol{p})-\mu}{kT}\right)-1\right)^{-1}$$

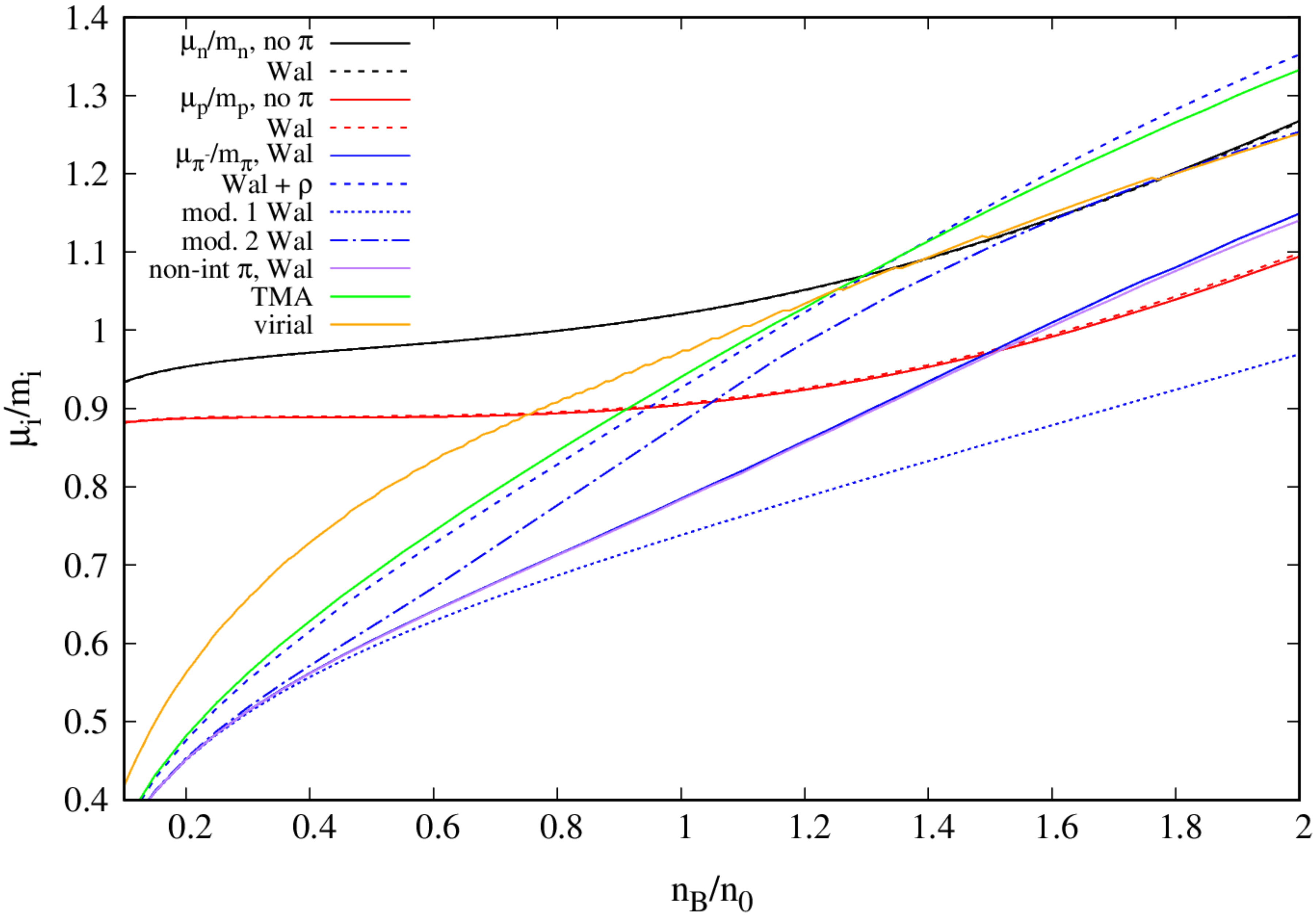




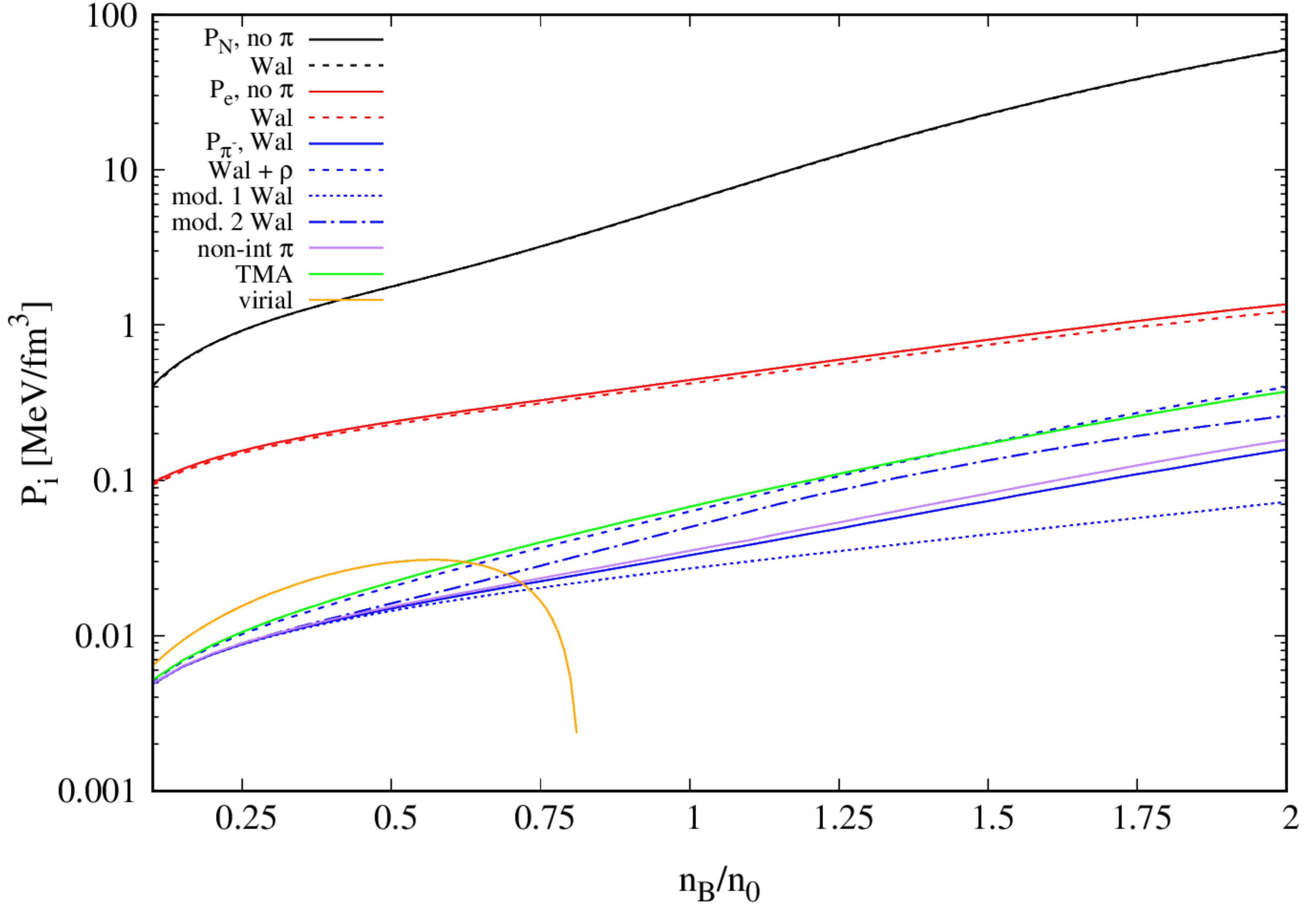
$$n \geq 0 \quad \Rightarrow \quad E(\boldsymbol{p}) - \mu \geq 0$$

$$E(\boldsymbol{p}) = \boldsymbol{p}^2 + m_\pi^2 + \mathrm{Re}\Pi(E(p)=0,\boldsymbol{p}) = 0$$

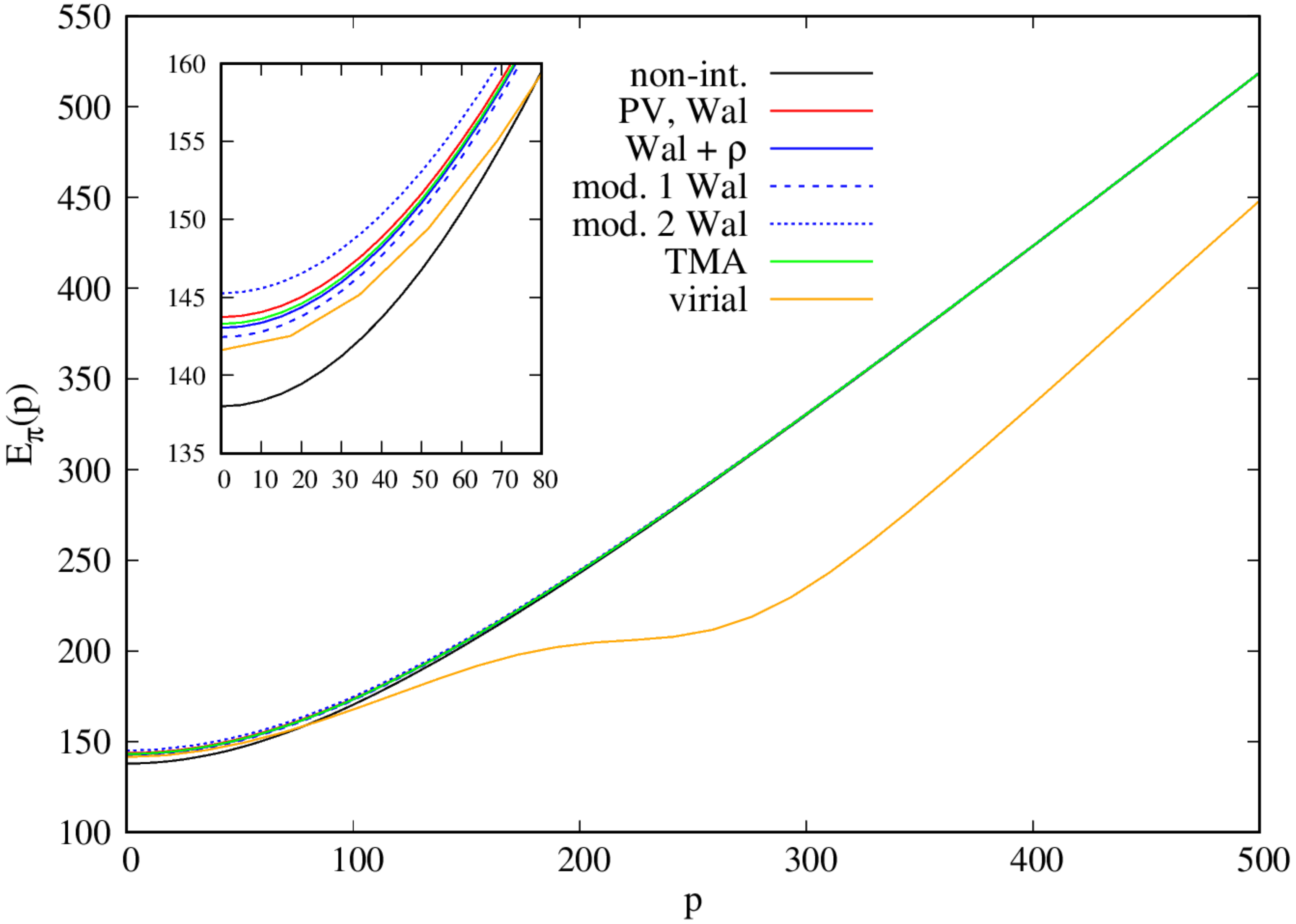
CHEM. POTENTIALS IN β-EQ.



PRESSURES IN β-EQ.

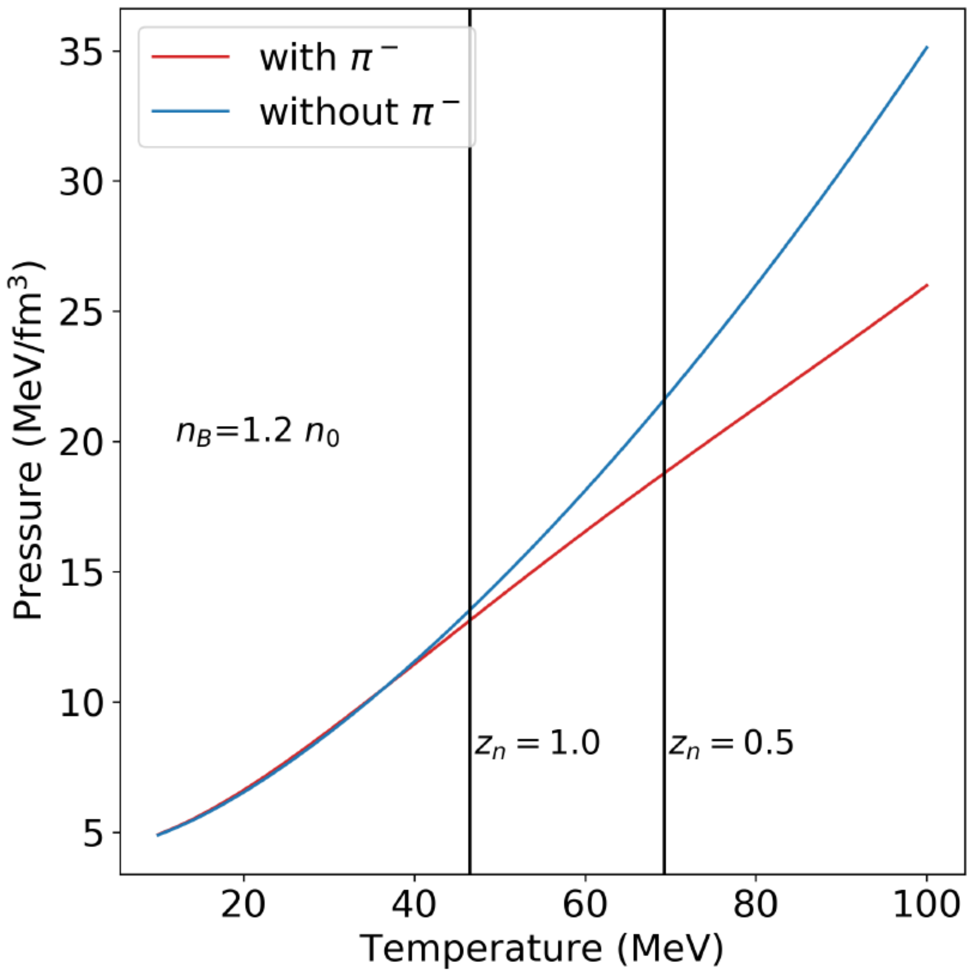


DISPERSION RELATIONS IN β-EQ.



SUMMARY

- Using the pion polarization loops, one can potentially get a better description at higher densities
- Virial expansion is closely related to experimental data but at low densities and high temperatures
- Both the pion polarization and virial expansion do not handle the appearance of the pion condensate
- The potential approach for obtaining a better agreement with the virial exp. is the application of the HF approx.
- The inclusion of thermal pions becomes crucial at high temperatures beyond the scope of NS but within the supernova explosions



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