Parameter Estimation of Compact Objects through QPO Data Using MCMC Analysis.

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Outline

Motivation

- QPOs are observed in the X-ray emission from accreting compact objects, such as black holes and neutron stars. Analyzing the properties of QPOs can provide valuable insights into the physics of these astrophysical systems and their parameters (mass, spin).
- Compact objects are characterized by parameters such as mass, spin, and accretion rate. MCMC analysis, coupled with QPO data, offers a Bayesian framework for estimating these parameters. It allows for a probabilistic exploration of parameter space.

Accretion disk

Figure: Black hole Accretion disck. Credit from Internet.

QPO models for understanding twin-peak QPOs in X-ray emission.

1. Relativistic precession (RP0: $\nu_U = \nu_\phi$, $\nu_L = \nu_\phi - \nu_r$) model and its variants[1].
   - RP1: $\nu_U = \nu_\theta$, $\nu_L = \nu_\phi - \nu_r$
   - RP2: $\nu_U = \nu_\phi$, $\nu_L = \nu_\theta - \nu_r$
2. Epicyclic resonance (ER0: $\nu_U = \nu_\theta$,$\nu_L = \nu_r$) model and its variants[1].
   - ER1: $\nu_U = \nu_\theta$, $\nu_L = \nu_\theta - \nu_r$
   - ER2: $\nu_U = \nu_\theta - \nu_r$, $\nu_L = \nu_r$
3. tidal disruption (TD) model: $\nu_U = \nu_\phi + \nu_r$, $\nu_L = \nu_\phi$ [1].
4. warped disc (WD) model: $\nu_U = 2\nu_\phi - \nu_r$, $\nu_L = 2(\nu_\phi - \nu_r)$ [1].

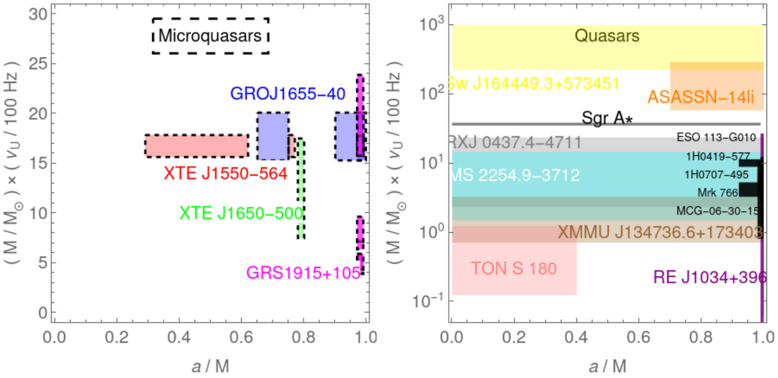


Figure: Credit from Martin Kološ [1]

Bayesian Inference

Bayes' theorem of conditional probability, and the posterior density $P(\vec{\Theta}|\vec{Y})$ is given [2] :

$$P(\vec{\Theta}|\vec{Y}) = \frac{P(\vec{\Theta})P(\vec{Y}|\vec{\Theta})}{P(\vec{Y})} \tag{1}$$

here $P(\vec{\Theta})$ and $P(\vec{Y}|\vec{\Theta})$ denote the prior density and likelihood,respectively. $\Theta_i = [M, a, r]$ parameter space, and observation $(\vec{Y})$. $P(\vec{Y})$ is a normalizing constant.

MCMC Algorithm

How does this algorithm work?. Acceptance ratio $A = P(\Theta_{i+1}|Y)/P(\Theta_i|Y) > 1$

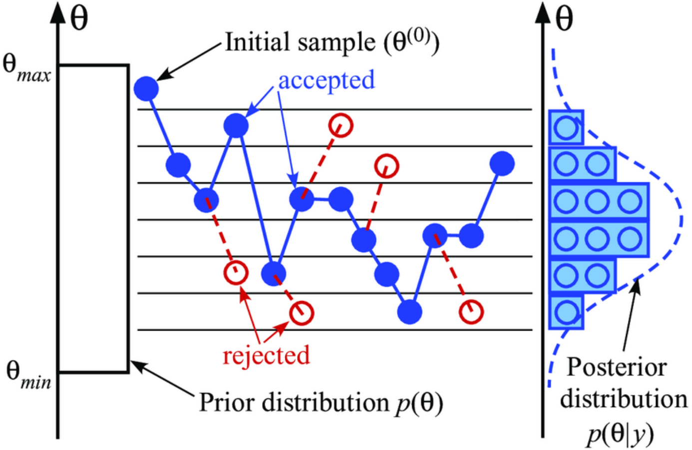


Figure: Credit from Seung-Seop Jin [2]

Application to Parameter Estimation

The priors are set to be Gaussian priors within boundaries, i.e.,

$$P(\Theta_i) \sim \exp\left(\frac{1}{2}\left(\frac{\Theta_i - \Theta_{m,i}}{\sigma_i}\right)^2\right) \quad (2)$$

where $\Theta_{\text{low},i} < \Theta_i < \Theta_{\text{high},i}$ for the parameters $\Theta_i = [M, a, r]$ and σ_i are their corresponding sigmas.

The central component of this analysis is the likelihood function, denoted as $\mathcal{L}$, and can be expressed as follows:

$$\log \mathcal{L} = \log \mathcal{L}_{\text{U}} + \log \mathcal{L}_{\text{L}}, \quad (3)$$

Wherein $\log \mathcal{L}_{\text{U}}$,and $\log \mathcal{L}_{\text{L}}$ characterizes the likelihood associated with the upper and lower-frequency data, respectively and given by:

$$\log \mathcal{L}_{\text{U}} = -\frac{1}{2}\sum_i \frac{(\nu^i_{\text{U, obs}} - \nu^i_{\text{U, th}})^2}{(\sigma^i_{\text{U, obs}})^2}, \quad \log \mathcal{L}_{\text{L}} = -\frac{1}{2}\sum_i \frac{(\nu^i_{\text{L, obs}} - \nu^i_{\text{L, th}})^2}{(\sigma^i_{\text{L, obs}})^2} \quad (4)$$

Application to Parameter Estimation

Table: Mass and spin measurements of XTE J1550-564 obtained from previous observations: Fe $K\alpha$ reflection spectroscopy (r) or continuum fitting (c). The last two columns present the QPO data, which will be utilized in our analysis.

Object	M ($M_\odot$)	a	ν_U	ν_L
XTE J1550-564	9.1 ± 0.6	$0.29 < a < 0.62^{r,c}$ [1]	276 ± 3	184 ± 5 [3]

Table: Gaussian priors of the Kerr black hole for X-ray Binary

Object	μ_M	σ_M	μ_a	σ_a
XTE J1550-564	9.1	0.3	0.445	0.165

Results and Conclusion (RP0)

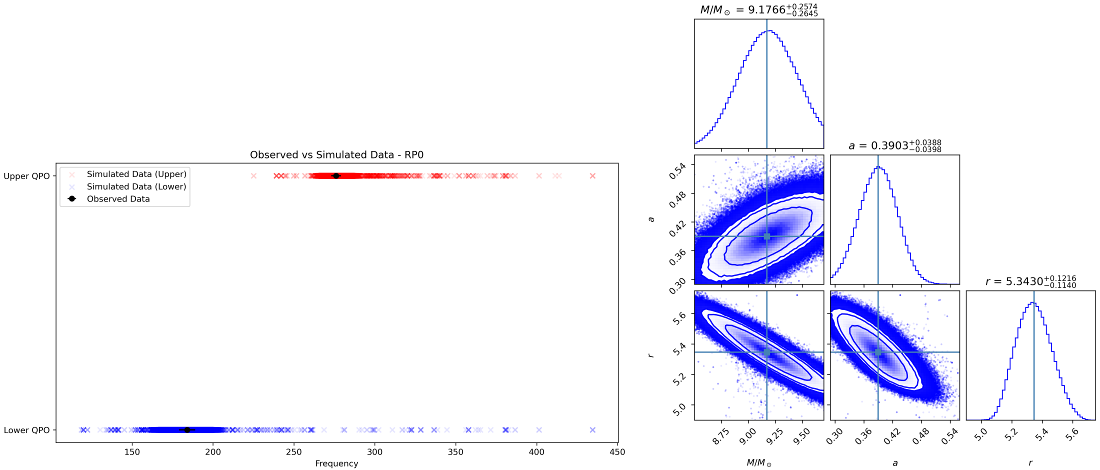
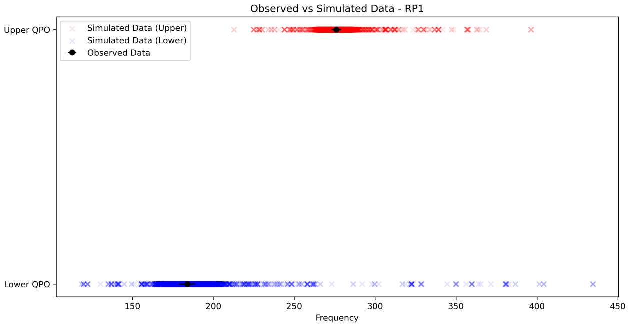


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (RP1)



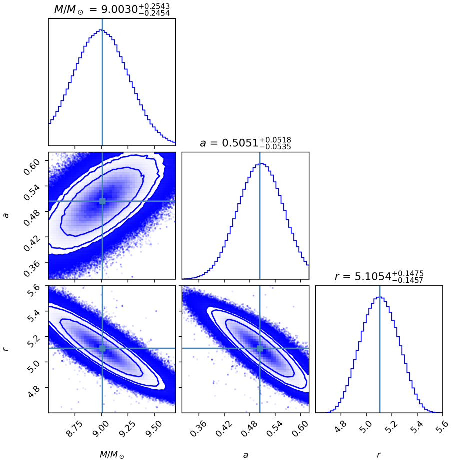


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (RP2)

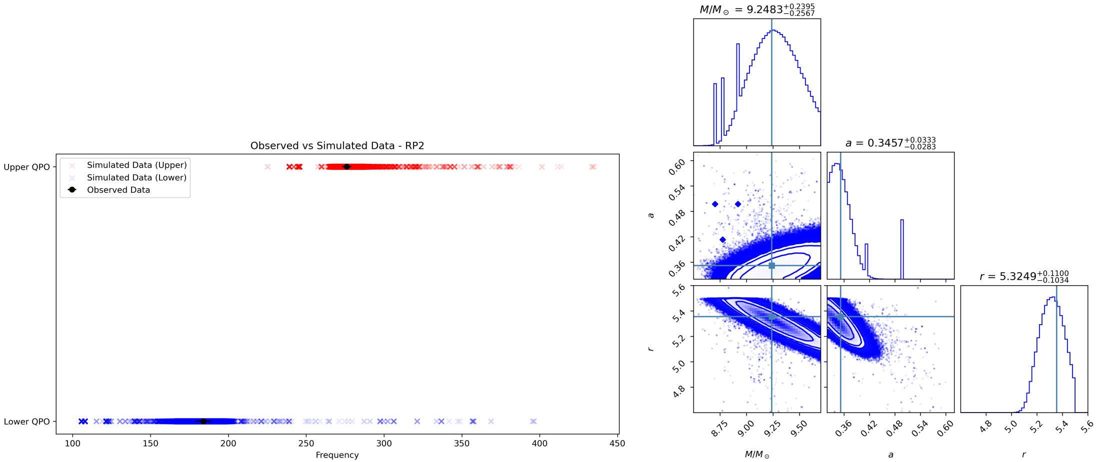


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (ER0)

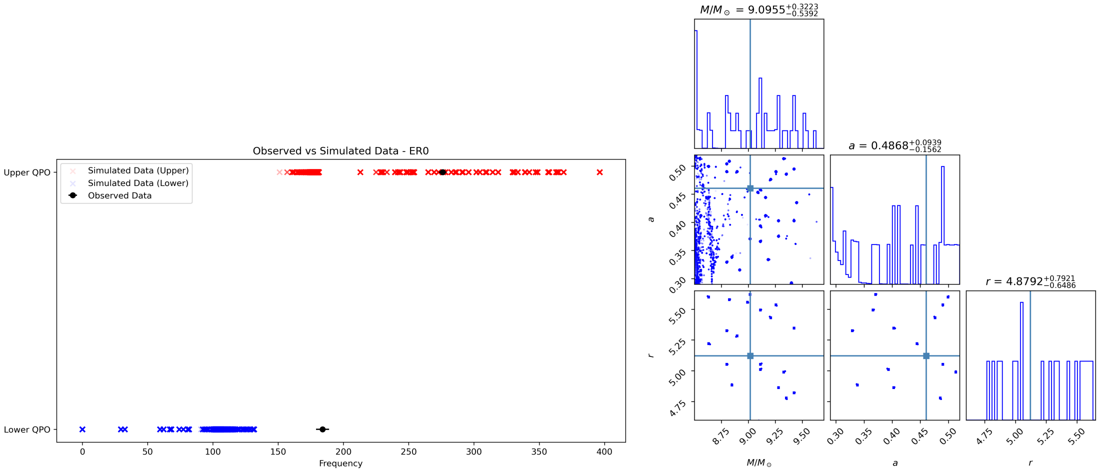


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (ER1)

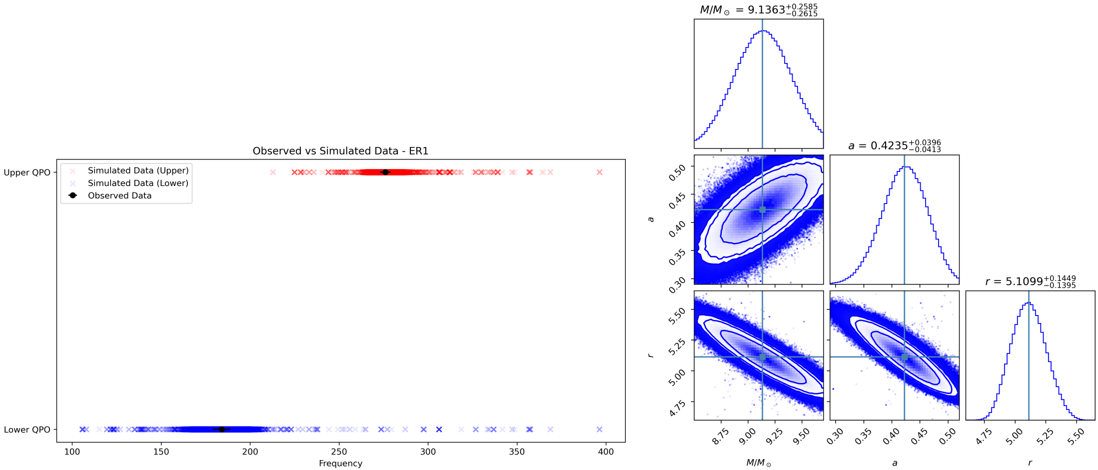
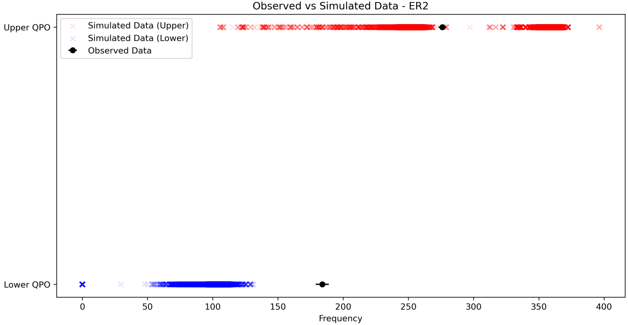


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (ER2)



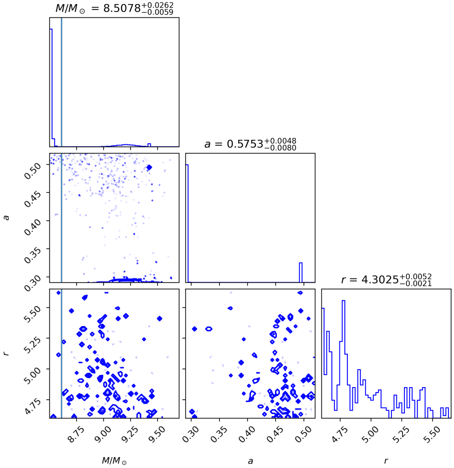
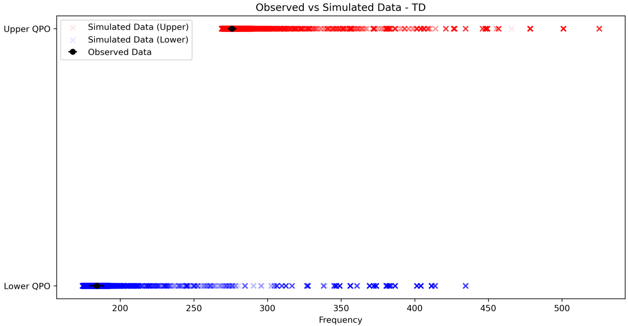


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (TD)



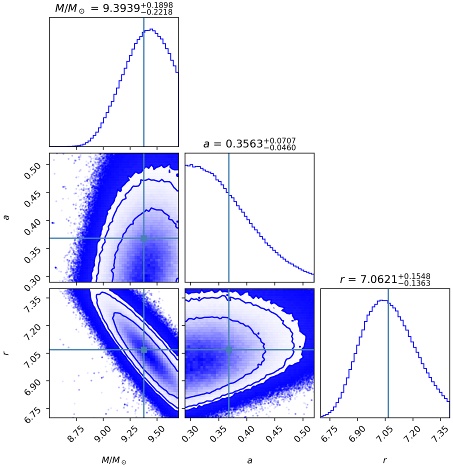


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Results and Conclusion (WD)

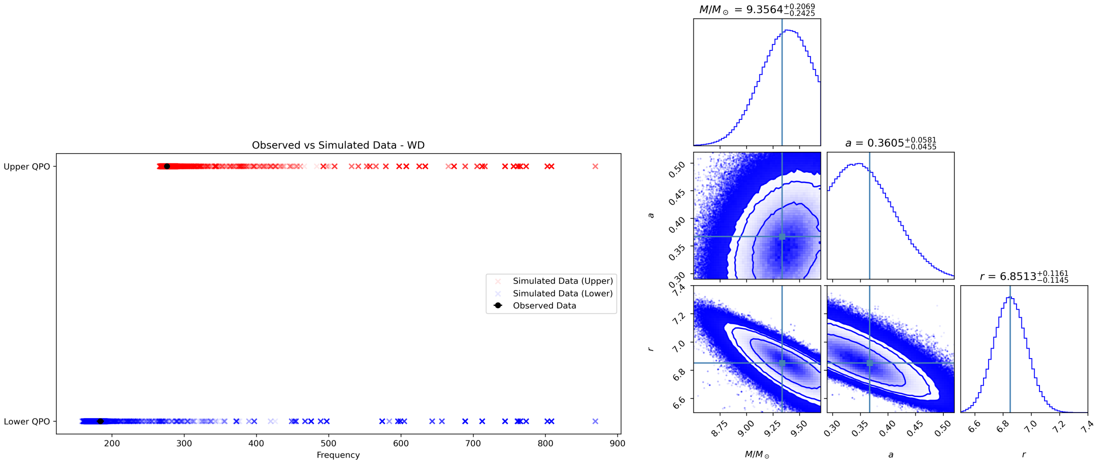


Figure: QPO points for the sample values of the MCMC code with real data (left), Estimated parameter results and posterior density (right).

Conclusion

The application of MCMC analysis to the study of QPOs demonstrates its effectiveness in exploring the parameter space associated with the characteristics of our spacetime. Through this analysis, we have successfully estimated the posterior distribution of the relevant parameters, contributing valuable insights into the nature of the spacetime. These estimations are informed by observations obtained through various means. Overall, MCMC proves to be a robust method for understanding and characterizing the underlying parameters governing QPOs in our observed spacetime.

References

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THANKS FOR YOUR ATTENTION!