

Charged particle dynamic in radiation belts around black hole

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RAGTIME

25TH RELATIVISTIC ASTROPHYSICS GROUP MEETING

Silesian University in Opava (28.11.2023)

Earth's magnetosphere and Van Allen radiation belts

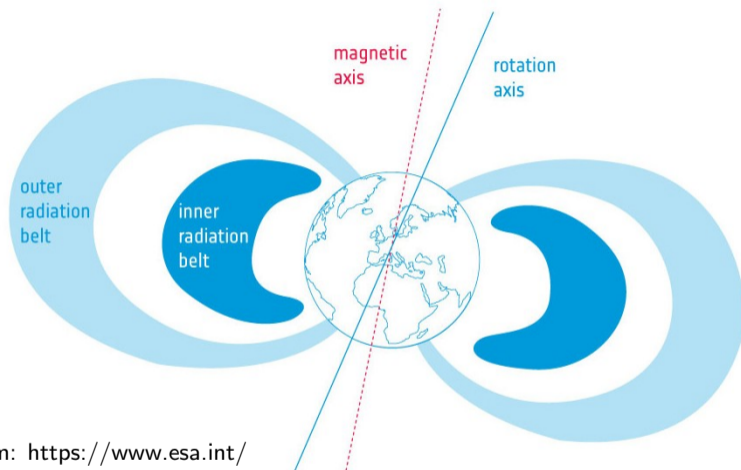
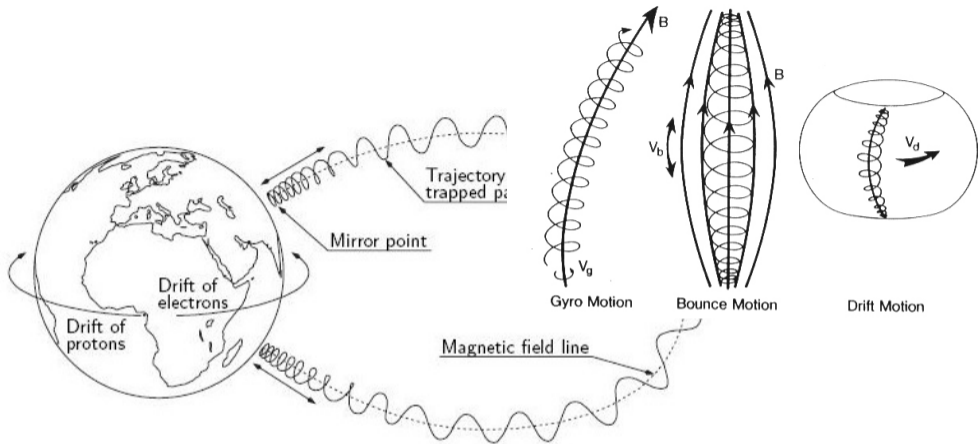


figure from: <https://www.esa.int/>

Radiation belts - populations of trapped p^+ and e^- : 5 particles / cm^3

- inner belt: mostly ~ 100 MeV protons, L-shell 1.2-2.0;

- outer belt: mostly ~ 10 MeV electrons, L-shell 3.0-8.0



adiabatic invariants & timescales
 $(e^-, 100 \text{ keV}, L = 4,$
 $5 \text{ nT} = 5 \cdot 10^{-5} \text{ Gs})$

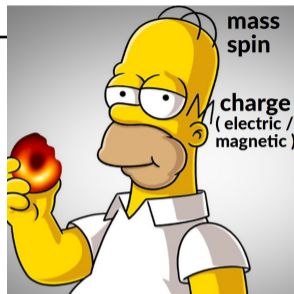
gyration	$\mu = \frac{p^2 \sin^2 \alpha}{2mB}$	0.1 ms
bounce	$J = \oint v_{ } ds$	0.5 s
drift	$\phi = \oint \mathbf{A} d\mathbf{l}$	2 h

• task: explore the dynamic in radiative belts, but now around black hole

Black hole magnetosphere

A) Black hole alone - BH own EM field

- no-hair theorem - black hole have only three hairs: mass, spin, **charge** (electric / magnetic)
⇒ monopole character of EM field around BH
- \nexists of magnetic monopole, but plasma accretion
⇒ BH will have **split monopole** magnetic field
- no dipole magnetic field on BH! only monopole



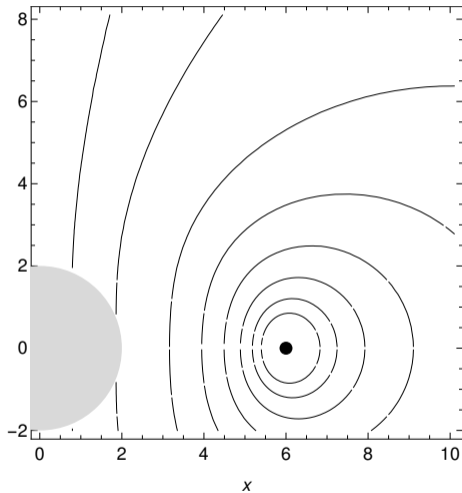
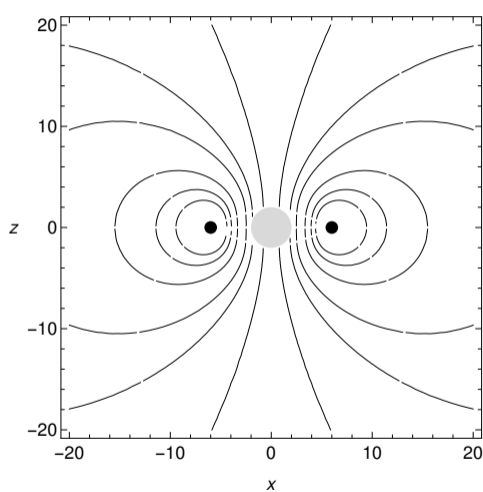
B) Black hole in plasma

electromagnetic field around BH generated by accretion disk

- 0) Vacuum Maxwell Equations, Wald (1974) || 1) Force Free Electrodynamics, Blandford-Znajek (1977); || 2) Magneto-hydrodynamics, Gammie+(2003); || 3) Particle-In-Cell, Crinquand+(2020); || 4) Physics-informed neural networks (202?)

- **magnetic field generated by current loop in equatorial plane (Petterson loop)**

Current loop in equatorial plane of Schwarzschild black hole



Current loop around BH - historical context

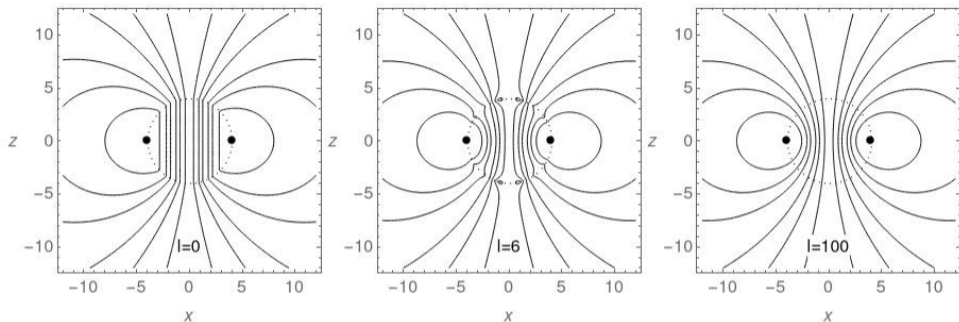
multipole expansion:

- D. M. Chitre and C. V. Vishveshwara. Electromagnetic field of a current loop around a Kerr black hole. *Phys. Rev. D* 12, 1538 (1975).
- J. A. Petterson. Stationary axisymmetric electromagnetic fields around a rotating black hole. *Phys. Rev. D* 12, 2218 (1975).
- J. Bičákák and L. Dvořák. Stationary electromagnetic fields around black holes. II. general solutions and the field of some special sources near a Kerr black hole. *Gen. Relativ. Gravit.* 7, 959 (1976).
- R. L. Znajek. Charged current loops around Kerr holes. *MNRAS* 182, 639 (1978).
- I. G. Moss. Black holes with current loops revisited. *Phys. Rev. D* 83, 123046 (2011).

full analytic solution:

- David Kofroň and Petr Kotlařík: Debye superpotential for charged rings or circular currents around Kerr black holes, *Phys. Rev. D* 106, 104022 (2022)

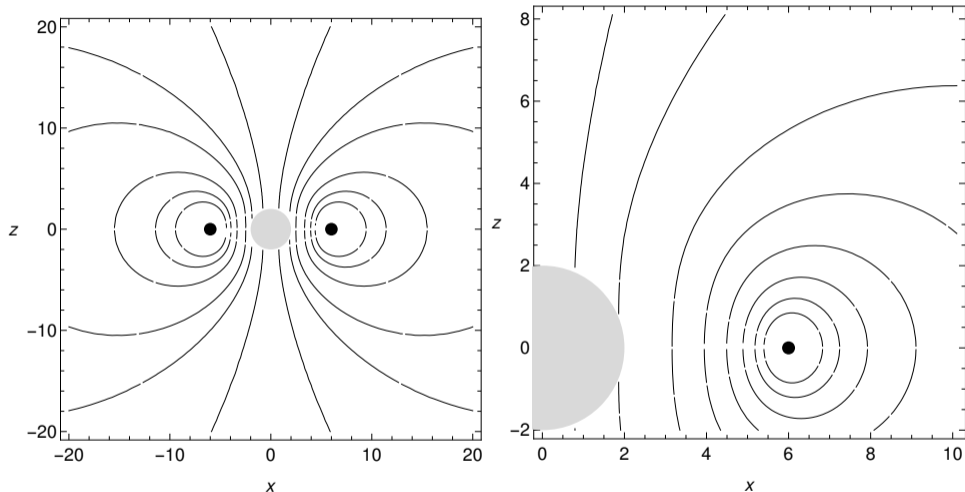
Current loop in multipole expansion - Petterson (1974)



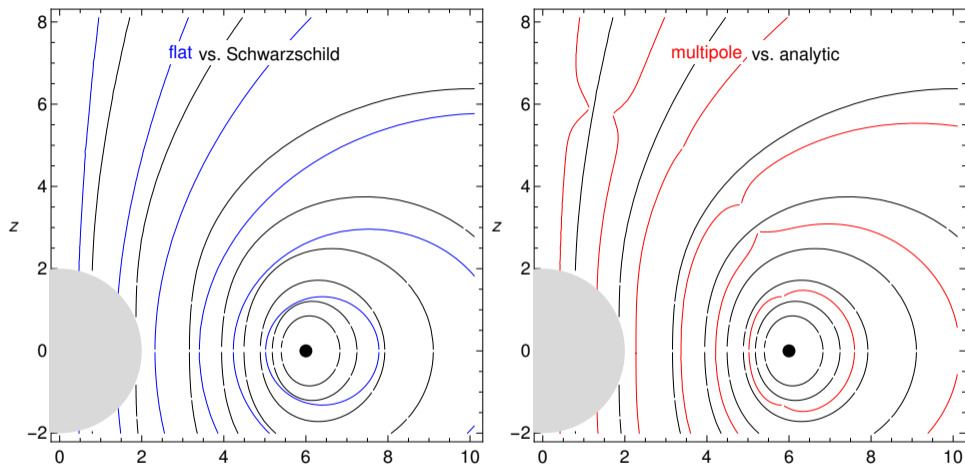
$$A_\phi = \mu_0 J \sum_{n=0,2}^{\infty} \frac{\sqrt{\pi} \sin \theta P_{n+1}^1(0) Y_{n+1}^1(\theta, 0)}{\sqrt{(n+1)(n+2)(2n+3)}} \cdot \begin{cases} r^{n+2} a^{-n-1} & \text{for } r \in (0, a) \\ a^{n+2} r^{-n-1} & \text{for } r \in (a, \infty) \end{cases}$$

inner $r < a$ and outer $r > a$ solutions, the matching at $r = a$ discontinuity

$$r \in (0, a) \quad A_{\hat{\phi}} = \frac{\mu_0 J r \sin \theta}{4a} + \dots \quad r \in (a, \infty) \quad A_{\hat{\phi}} = \frac{\mu_0 J a^2 \sin \theta}{4r^2} + \dots \quad (1)$$



$$\left(\text{flat spc. limit } A^{\hat{\phi}}(r, \theta) = \mu_0 I a \frac{(2 - k^2)K(k^2) - 2E(k^2)}{\pi k^2 \sqrt{a^2 + 2ar \sin \theta + r^2}}, \quad k^2 = \frac{4ar \sin \theta}{a^2 + r^2 + 2ar \sin \theta} \right)$$



$$\left(\text{flat spc. limit } A^{\hat{\phi}}(r, \theta) = \mu_0 I a \frac{(2 - k^2)K(k^2) - 2E(k^2)}{\pi k^2 \sqrt{a^2 + 2ar \sin \theta + r^2}}, \quad k^2 = \frac{4ar \sin \theta}{a^2 + r^2 + 2ar \sin \theta} \right)$$

”Knowledge of the charged particle motion in the magnetic field is essential in studies of radiation belts.”

Lorentz equation: gravity and magnetic field

$$\frac{du^\mu}{d\tau} + \Gamma_{\alpha\beta}^\mu u^\alpha u^\beta = \frac{q}{m} g^{\mu\rho} F_{\rho\sigma} u^\sigma + \dots$$

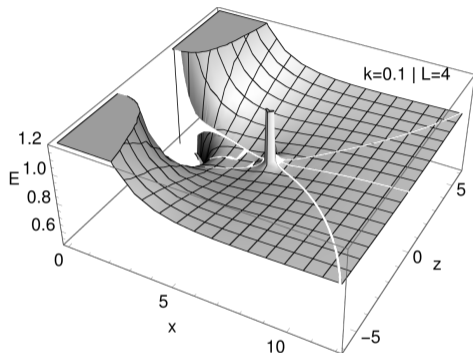
gravity ~ 1 Lorentz force up to $\sim 10^{11}$

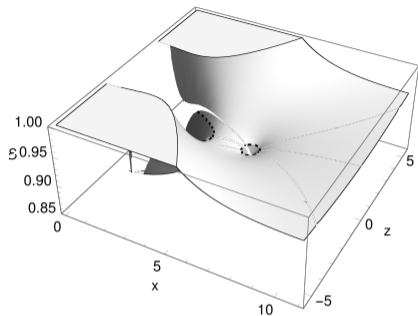
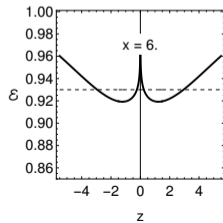
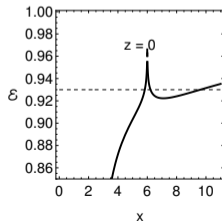
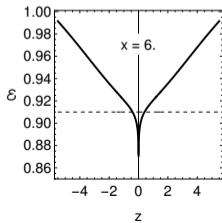
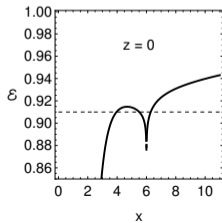
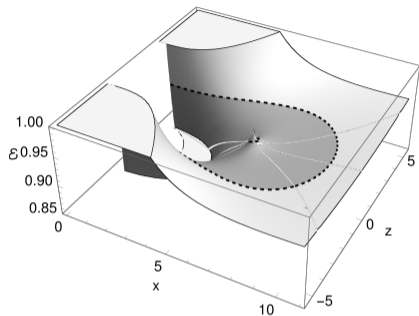
- symmetries \rightarrow conserved quantities:
energy E and angular momentum L

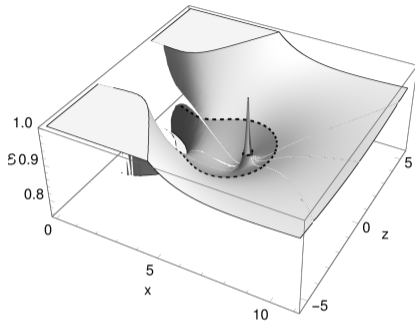
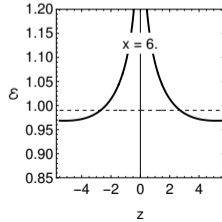
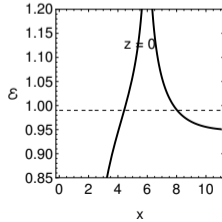
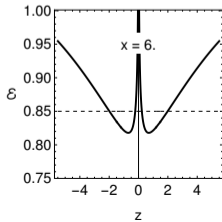
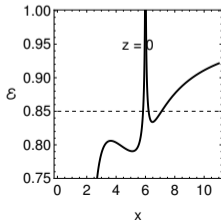
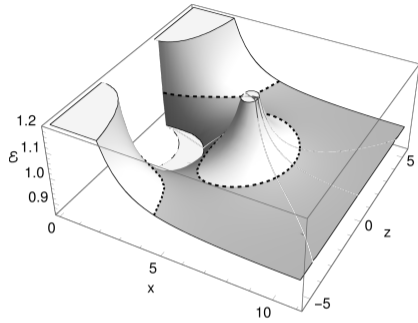
test charged particle is moving in 2D effective potential $V_{\text{eff}}(x, z)$

region close th BH horizon \downarrow || peak at current loop \uparrow - particles repelled by loop

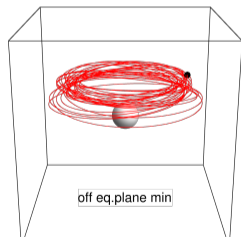
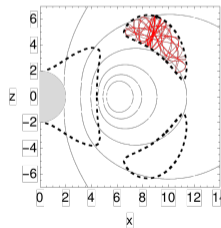
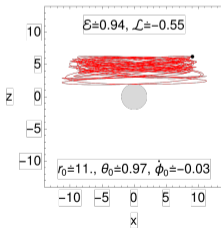
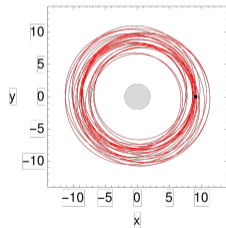
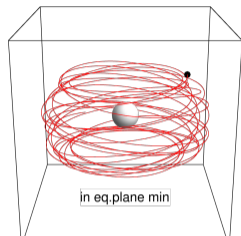
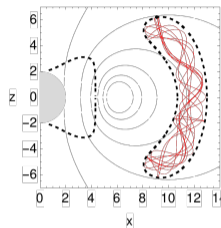
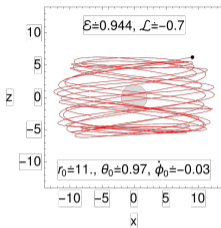
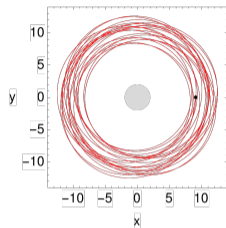
code in Mathematica for charged particle motion: github.com/XyhwX/particle



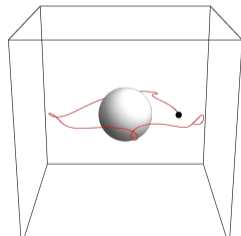
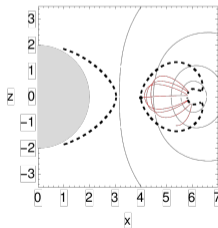
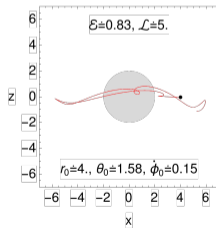
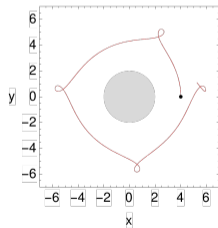
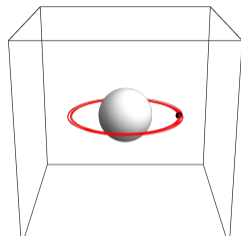
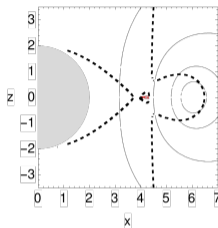
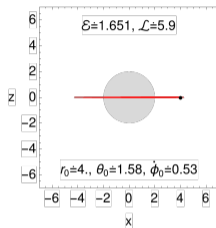
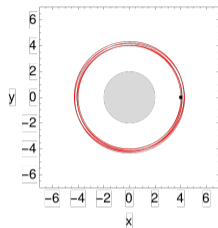
$k = 0.01$  $k = -0.01$ 

$k = 0.1$  $k = -0.1$ 

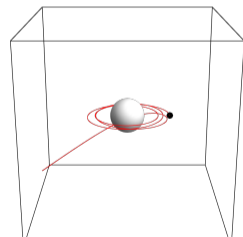
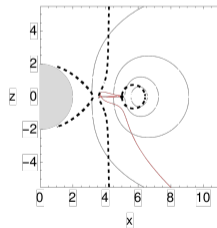
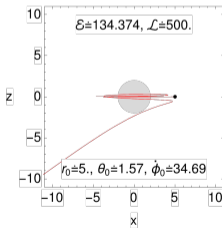
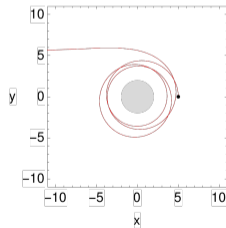
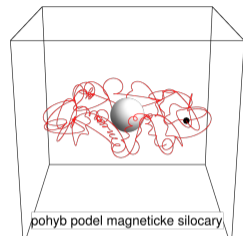
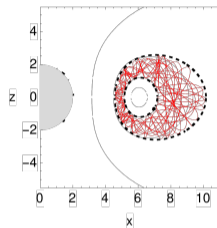
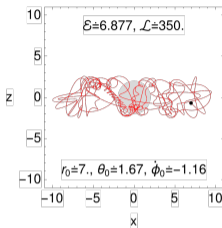
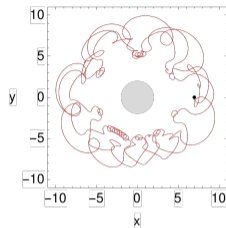
In and Off equatorial plane minima - like in dipole



Attractive and repulsive Lorentz force - like in uniform

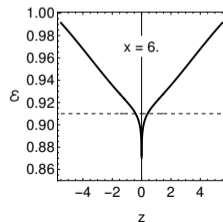
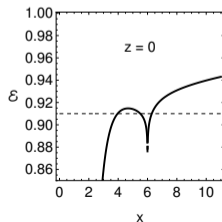
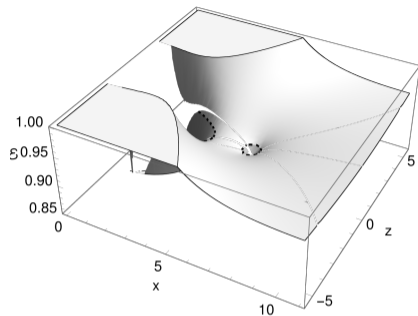
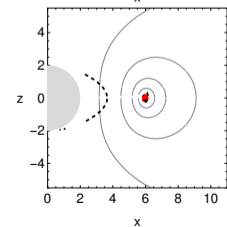
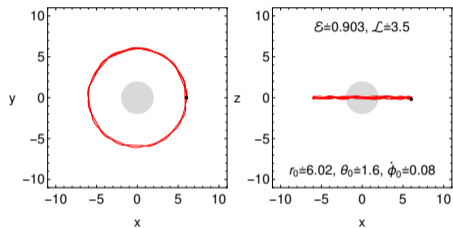


Motion close to the current loop - repulsion from the loop

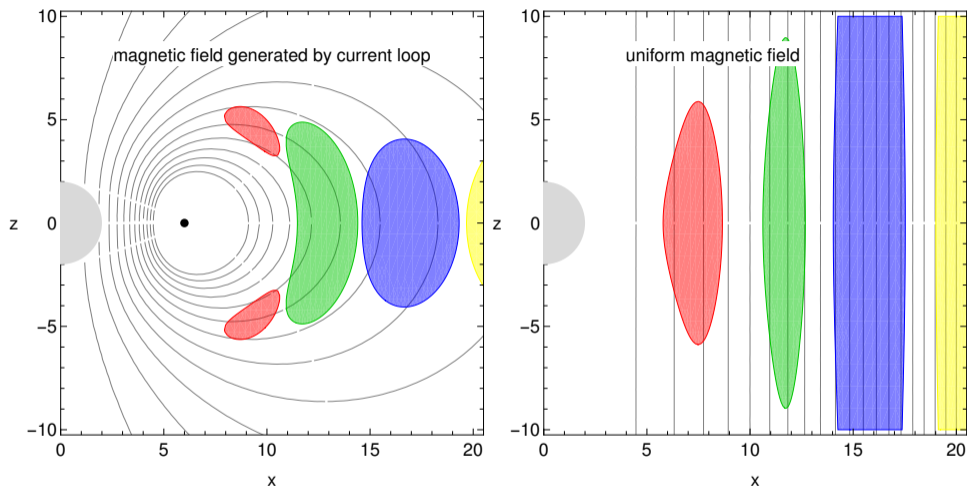


Can particles generate the loop current?

$k = 0.01$



Radiation belts around black hole



for electrons in $B = 10^{-4}$ Gs around $M = 10M_{\odot}$ black hole
belt inner edge - ISCO; outer edge - trapping boundary?

BH magnetosphere & charged particles & radiation belts

- Maxwell equations are linear even in curved spacetime - from one current loop, other configurations can be integrated \rightarrow disk, sphere, . . .
- Charged particle dynamic helps you to understand forces in the system
 - ▶ full analytic solution for current loop \sim uniform inside, dipole outside
 - ▶ charged particles are repelled from the current loop position
 - ▶ can we construct a self-supporting solution where the charged particles are generating the current loop?
- One test charged particle dynamic \sim low-density collisionless plasma model only. Charged particle motion is essential in studies of radiation belts. Belts around black holes can also be constructed for uniform, parabolic, . . . mag. configurations.

Thank you for your attention

more info: <https://github.com/XyhwX> martin.kolos@physics.slu.cz

- ongoing work, started at last RAGtime, article in preparation