Accretion onto charged compact objects

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No-hair theorem

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Stationary black hole in General Relativity (coupled to Maxwell equations) can be completely characterized by:

- mass m,
- angular momentum a
- and its electric charge Q.

Reissner–Nordström metric

Reissner–Nordström metric can be expressed in Boyer-Lindquist as:

$$g = -f(r) dt \otimes dt + f^{-1}(r) dr \otimes dr + r^2 (d\theta \otimes d\theta + \sin^2 \theta d\phi \otimes d\phi)$$

where

$$f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}.$$

The RN metric:

- is spherically symmetric,
- describes gravitational field of charged and massive compact objects.

Horizons around charged objects

An event horizon should occur when $g_{tt} = 0$ which for Reissner–Nordström metric takes the form

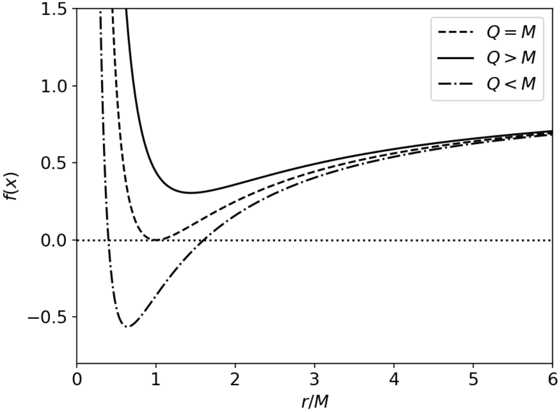
$$f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} = 0$$

The solution of the above equation is given by,

$$r_{\pm} = \frac{1}{2}(2m \pm \sqrt{2m^2 - 4Q^2})$$

When $Q < m$, we have black hole solution with two coordinate singularities and two event horizons. For $Q = m$, we have extreme black hole for which the horizons coincides. When $Q > m$, there are no horizons at all so the metric describes a naked singularity.

Radial dependence



Radial dependence of Reissner–Nordström metric for black hole and naked singularity.

Kerr-Newman metric

Kerr-Newman metric is generalization of Reissner–Nordström metric toward spinning objects and generalization of usually used Kerr metric to charged objects.

$$
\begin{aligned}
g = &-\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)\otimes\left(dt - a\sin^2\theta d\phi\right) \\
&+ \frac{\sin^2\theta}{\rho^2}\left((r^2+a^2)d\phi - adt\right)\otimes\left((r^2+a^2)d\phi - adt\right) + \frac{\rho^2}{\Delta}dr\otimes dr \\
&+ \rho^2 d\theta\otimes d\theta
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta &= r^2 - 2mr + a^2 + Q^2, \\
\rho^2 &= r^2 + a^2\cos^2\theta.
\end{aligned}
$$

Orbits around charged objects

- Radius of last stable circular orbit (LSCO) in Reissner–Nordström is a solution of equation[1]

$$mr^3 - 6m^2r^2 + 9mQ^2r - 4Q^4 = 0.$$

- Radius of marginally bound orbits (MBO) in Reissner-Nordström metric is solution of following equation[2]

$$mr^3 - 4m^2r^2 + 4mQ^2r - Q^4 = 0.$$

1. Pugliese, D. *et al. Phys. Rev. D* **83,** 024021. arXiv: `1012.5411 [astro-ph.HE]` (2011).
2. Beheshti, S. & Gasperin, E. *Phys. Rev. D* **94,** 024015. arXiv: `1512.08707 [gr-qc]` (2016).

Zero-gravity radius

Let us define pseudo-potential for Reissner–Nordström metric:[3]

$$\Phi = \frac{1}{2} \log \left[1 - \frac{2m}{r} + \frac{Q^2}{r^2} \right].$$

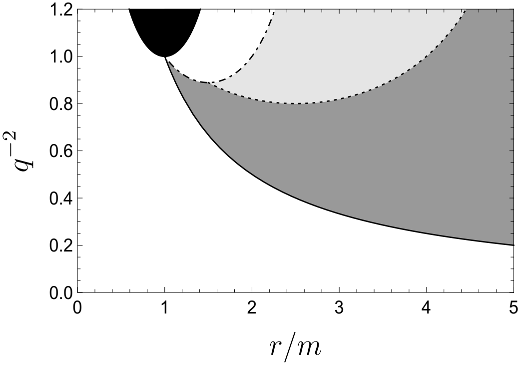
Then the 4-acceleration of the static observer can be expressed as[0]

$$a_\mu = \partial_\mu \Phi.$$

For Reissner–Nordström metric, the 4-derivative of pseudo-potential has zero for zero-gravity radius $r_0 = q^2$. At zero gravity sphere of radius z_0 test particle can stay at rest.

4. Kluzniak, W. & Kita, D. *arXiv e-prints*, astro–ph/0006266. arXiv: `astro-ph/0006266 [astro-ph]` (June 2000).

Stability of orbits



Stability diagram for Reissner-Nordström spacetime with $q \equiv Q/m$.[3]

3. Vieira, R. S. S. & Kluźniak, W. *Mon. Not. Roy. Astron. Soc.* **523,** 4615–4623. arXiv: 2304.05932 [astro-ph.HE] (2023).

Kerr-Newman metric in spherical Kerr-Shild coordinates

In order to perform numerical simulations of accretion onto black holes the coordinate system which is non-singular at horizon has to be used. In the past the spherical Kerr-Shild coordinates were used in case of Kerr metric. They can be easily generalized to Kerr-Newman metric.

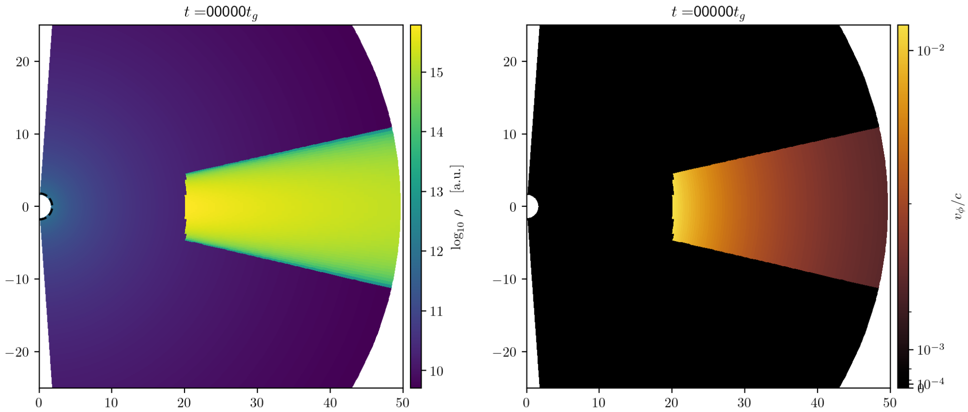
$$
\begin{aligned}
g = &-(1-B)dt\otimes dt + (1+B)dr\otimes dr + \Sigma d\theta\otimes d\theta \\
&+\left(r^2+a^2+Ba^2\sin^2\theta\right)\sin^2\theta d\phi\otimes d\phi + B(dt\otimes dr + dr\otimes dt) \\
&- aB\sin^2\theta(dt\otimes d\phi + d\phi\otimes dt) - a(1+B)\sin^2\theta(dr\otimes d\phi + d\phi\otimes dr)
\end{aligned}
$$

where

$$
\Sigma = r^2 + a^2\cos^2\theta,
$$

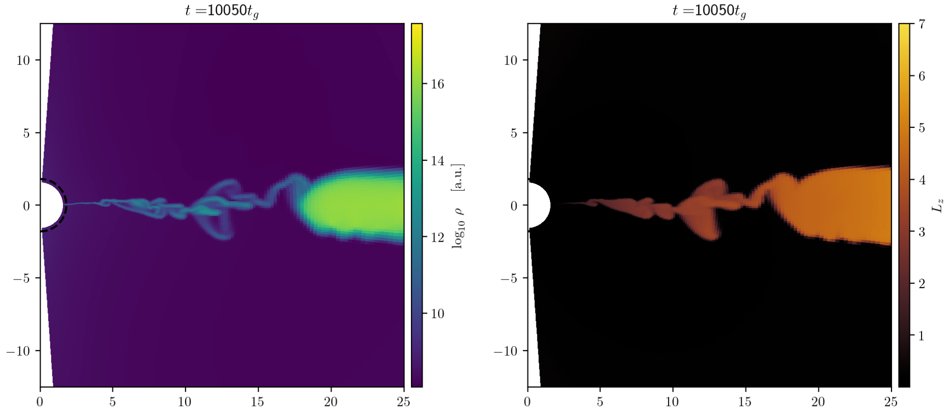
$$
B = \frac{2mr - Q^2}{\Sigma}.
$$

Initial setup



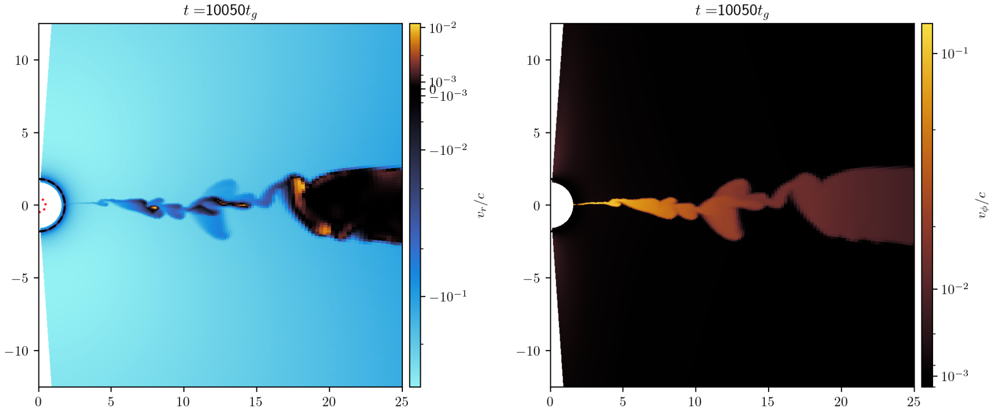
Energy density ρ (left panel) and azimuthal velocity v_ϕ (right panel) of initial setup consisting of Kluźniak-Kita disc.[4]

Accretion onto $Q = 0.6m$ black hole



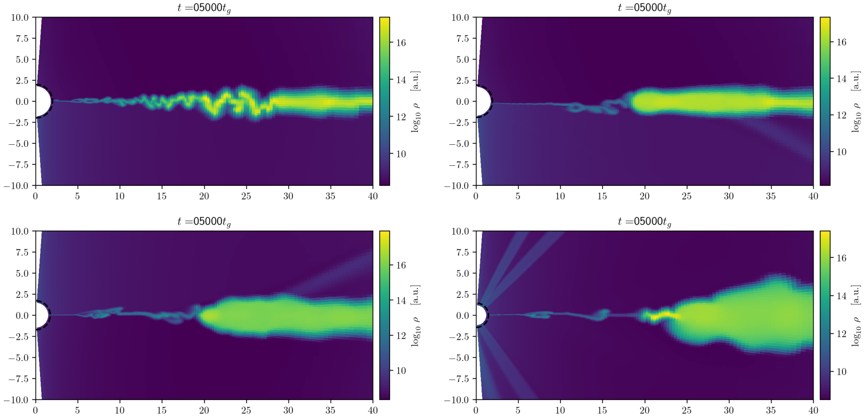
Energy density ρ (left panel) and z component of angular momentum L_z (right panel).

Back-flow



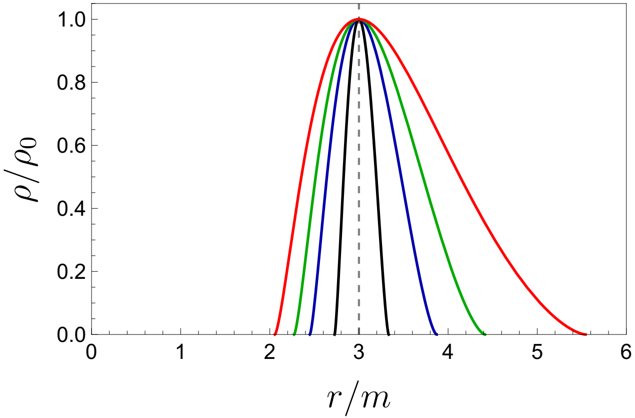
Radial velocity v_r (left panel) and azimuthal velocity v_ϕ (right panel).

Dependence on charge



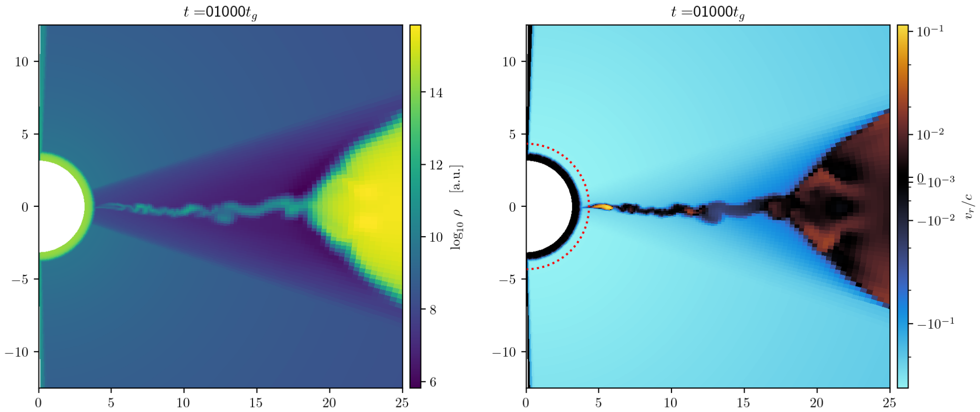
Energy density ρ for various charges: $Q = 0.2m$ (top left), $Q = 0.4m$ (top right), $Q = 0.7m$ (bottom left), $Q = 0.9m$ (bottom right).

Levitating atmosphere around naked singularity



Profile of levitating atmosphere.[3]

Accretion onto naked singularity $Q = 1.8m$



Energy density ρ (left panel) and radial velocity v_r (right panel).

Summary

1. General relativity predicts that black holes can not only have mass and spin, but also electric charge.
2. It is worth studying numerically if and how accretion onto charged compact objects differs from neutral ones.
3. When the charge or spin of the object is large enough the horizons disappear and the object is the naked singularity.
4. Extension of legacy KORAL code was developed which allow performing radiative GRMHD simulations in nearly general background metrics.

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Thank you for your attention.