Maximum mass of the rotating dense stellar cores with strong magnetic fields: Strange Quark Stars

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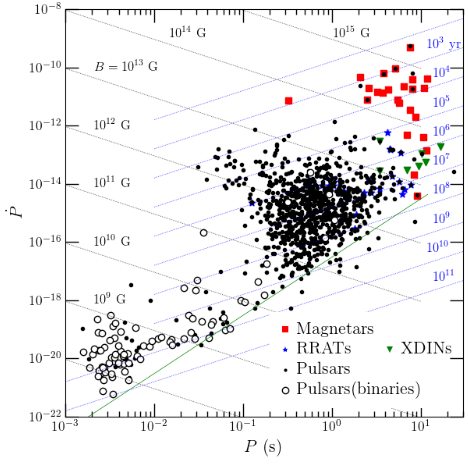
A brief story about strange quark stars

1. Hybrid star: Neutron star with a quark core
2. Pure quark star (**SQS**)
   - SQSs are supposed to be completely made up of up, down, and strange quarks with a small fraction of electrons
   - They might exist after a super luminous explosion (**Quark-Novae**)
     a) an increase in the core density due to **spin-down** of the proto-neutron star or the neutron star (NS)
     b) an increase in the core density following the **accretion** from the companion in binary NS
   - There are a few observational signatures of **Quark-Novae**, a promising candidate is **Cassiopeia A** (Ouyed, et al, RAA, 483, 2015)



Strong magnetic field and rotation

1. The second gravitational collapse might lead to the SQS with orders of magnitude stronger magnetic field.
2. According to the theoretical studies and observations the magnetic field in the core of strange stars might reach 10^{18} **G**.
3. Investigation of rotating compact objects confirm that they are **stable with larger maximum gravitational masses** compared to the non-rotating.
4. Because large magnetic field quickly spins down the nascent magnetars, at present they are rotating slowly.

Equation of state

Our EOS model

- The density dependent MIT bag model
- Pure strange quark matter (**SQM**) contains **up, down and strange** quarks.
- Strong magnetic field up to 5×10^{18}**G**
- Landau quantization effect

Equation of state

Total energy density of SQM:

$$\varepsilon_{tot} = \sum_{i,\pm} \varepsilon_i^{\pm} + \mathcal{B}_{bag} \tag{1}$$

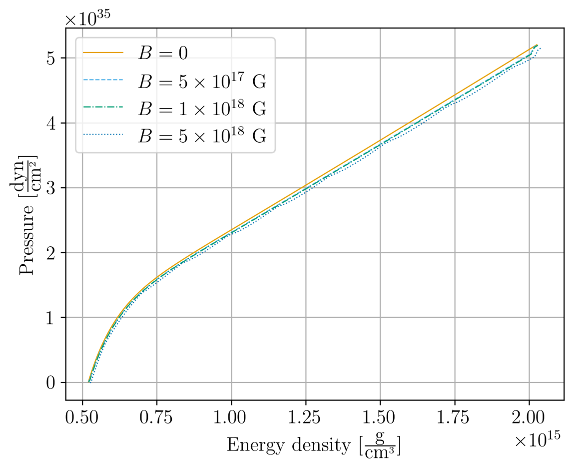
- $\varepsilon_i^{\pm}$ is the kinetic energy density, i represent quarks and $\pm$ is the spin of quarks. $\varepsilon_i^{\pm}$ is computed using the **Fermi relations considering the Landau quantization effect**.
- $\mathcal{B}_{bag}$ is the bag constant:

$$\mathcal{B}_{bag}(\rho) = \mathcal{B}_\infty + (\mathcal{B}_0 - \mathcal{B}_\infty) \exp\left(-\alpha(\frac{\rho}{\rho_0})^2\right) \tag{2}$$

  $\alpha = 0.17$, and $\mathcal{B}_0 = \mathcal{B}_{bag}(0) = 400\ \mathrm{MeV/fm^3}$. We should define $\mathcal{B}_\infty$ in a way that bag constant be compatible with experimental data (CERN-SPS). We determine $\mathcal{B}_\infty = 8.99$ $\mathrm{MeV/fm^3}$ by putting quark energy density = hadronic energy density.
- **Equation of state:**

$$P(\rho) = \rho \left(\frac{\partial \varepsilon_{tot}}{\partial \rho}\right) - \varepsilon_{tot} \tag{3}$$

Equation of state



- Pressure as a function of the total energy density of SQM.
- Although the effect of the magnetic field on EOS is not too significant, we will show the magnetic field effects on the structure of the SQS in the following sections.

Structure

The structure differential equations of star are derived by solving the Einstein equations,

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi T^{\mu\nu} \tag{4}$$

$R^{\mu\nu}$ and R are is the Ricci tensor the Ricci scalar, $g^{\mu\nu}$ is the metric coefficient and $T^{\mu\nu}$ **the energy momentum tensor**,

$$T^{\mu\nu} = (\varepsilon+P)u^{\mu}u^{\nu}+Pg^{\mu\nu}+\frac{\mathcal{M}}{B}\left[b^{\mu}b^{\nu}-(b.b)(u^{\mu}u^{\nu}+g^{\mu\nu})\right]+\frac{1}{\mu_0}\left[-b^{\mu}b^{\nu}+(b.b)(u^{\mu}u^{\nu}+\frac{1}{2}g^{\mu\nu})\right] \tag{5}$$

- First and second terms: Perfect fluid energy-momentum tensor
- Third term: Magnetization, $\mathcal{M}$ represents the interaction of the electromagnetic field with matter that it is given by the coupling between the electric current j^{ϕ} and the magnetic vector potential A^{ϕ}, where $j^{\phi} = \Omega j^{t} + (\varepsilon + P)k_0$ (Ω is rotation velocity of the star and k_0 is current function).
- Last term: The pure electromagnetic contribution to $T^{\mu\nu}$

The metric in axisymmetric space-time,

$$ds^2 = -N^2 dt^2 + A^2(dr^2 + r^2 d\theta^2) + \lambda^2 r^2 \sin^2(\theta)(d\phi - N^\phi dt)^2 \tag{6}$$

N, A, λ, and N^ϕ are function of (r, θ).

Four elliptic partial differential equations using 3+1 formalism,

$$\Delta_3 \;=\; 4\pi A^2(E^T + S_r^r + S_\theta^\theta + S_\phi^\phi) + \frac{\lambda^2 r^2 \sin^2(\theta)}{2N^2}\delta N^\phi \delta N^\phi - \delta\nu\delta(\nu + \beta) \tag{7}$$

$$\Delta_2[\alpha + \nu] = 8\pi A^2 S_\phi^\phi + \frac{3\lambda^2 r^2 \sin^2(\theta)}{4N^2}\delta N^\phi \delta N^\phi - \delta\nu\delta\nu \tag{8}$$

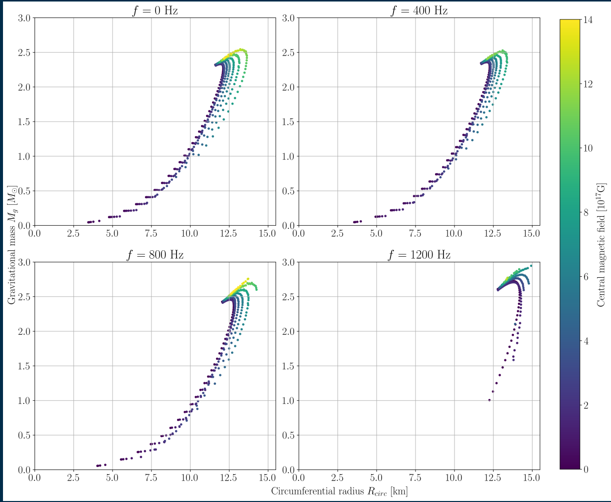
$$\Delta_2[(N\lambda - 1) r \sin(\theta)] = 8\pi N A^2 \lambda r \sin(\theta)(S_r^r + S_\theta^\theta) \tag{9}$$

$$\left[\Delta_3 - \frac{1}{r^2 \sin^2(\theta)}\right](N^\phi r \sin(\theta)) \;=\; -16\pi \frac{NA^2}{\lambda^2}\frac{J^\phi}{r\sin(\theta)} + r\sin(\theta)\delta N^\phi \delta(\nu - 3\beta) \tag{10}$$

$\nu = \ln N$, $\alpha = \ln A$, $\beta = \ln\lambda$, and J^ϕ is electromagnetic current. In the above equations, E^T, and S_j^i are total energy and stress, respectively.

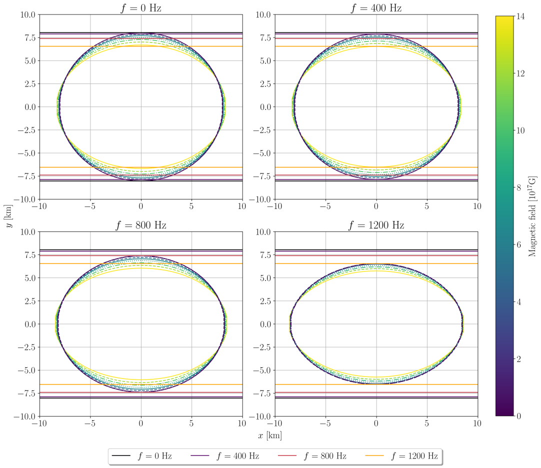
Numerical method

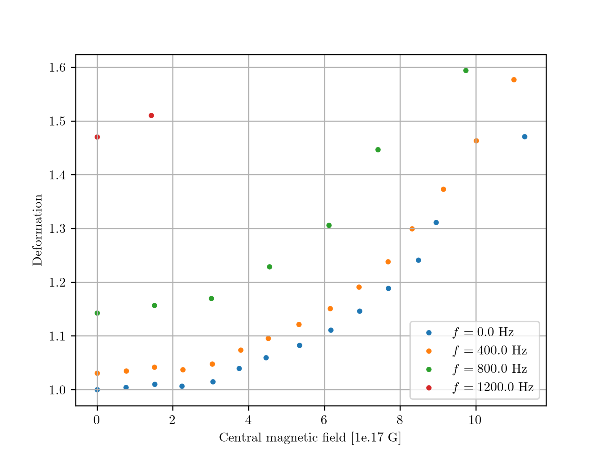
- We solve a set of four elliptic partial differential equations presented in the previous section using **LORENE library** http://www.lorene.obspm.fr.
- LORENE uses state of **spectral methods** to solve partial differential equations which makes calculations much more accurate than grid-based methods.
- We use **Et _ magnetization class** (*Lorene/Codes/Mag_eos_star*) to calculate hydrostatic configurations for uniformly rotating magnetized stars.
- We made 51 stellar configuration with specified the **central entalphy** value (in the range from $0.01c^2$ up to $0.51c^2$ with spacing of $0.01c^2$), for different **magnetic fields** by changing current function from 0 to 35000 with spacing of 5000, and the **rotational frequencies** of 0 Hz, 400, 800, and 1200 Hz.



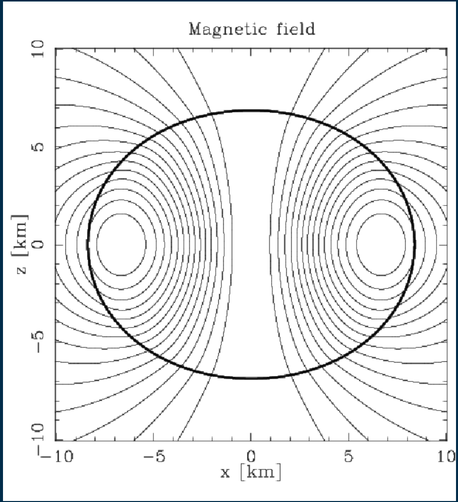
- $\boldsymbol{R_{circ}}$ is circumferential radius and $\boldsymbol{M_g}$ is the gravitational mass
- $M_g^{max} = 2.3 M_\odot$ for the non-magnetized, non-rotating star
- $M_g^{max} = 2.38 M_\odot$ for the rapidly rotating (800 Hz), non-magnetized star
  "Comparable with the recent detection of millisecond *"black widow" pulsar PSR J09520607*, $M_g \simeq 2.35 \pm 0.17$"
- $M_g^{max} \simeq 2.6 M_\odot$ for the rapidly rotating (1200 Hz), $B_c = 1.4 \times 10^{17}$ G star
- At rotational frequencies of 800 and 1200 Hz with $B_c \geq 6 \times 10^{17}$ G and $B_c \geq 1.4 \times 10^{17}$ G, respectively, there is no configuration.

Magnetic and rotational deformation



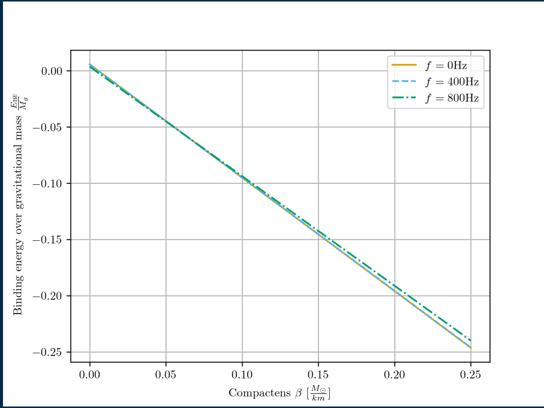


Magnetic and rotational deformation



- **Color** of the ellipses indicates the **magnitudes of the central magnetic field**.
- **Straight lines**, which are at the same positions in all panels, indicate the **polar radius** of the non-magnetized configuration in each rotational frequency
- The **polar radius** is **decreasing** with increasing the magnetic field and rotational frequency.
- Deformation parameter $a = R_{eq}/R_{pol}$ changes from **1** ($f = 0$, $B = 0$) to **1.58** ($f = 800$ Hz, $B_c \sim 10^{18}$ G)

Binding energy and compactness



- We compute the total binding energy $E_{BE} = (M_g - M_b)c^2 + E_{ext}$ (M_b is the baryon mass and E_{ext} is external energy of star) and compactness $\beta = M_b/R_{circ}$ **($M_\odot$/km)** in the presented models.
- Relation between E_{BE} and β (with negligible effect of different magnetic field strength and rotational frequency) in our models is given by,

$$\frac{E_{BE}}{M_g} = -\beta \qquad (11)$$

- We found that our model gives less binding energy compared to CDDM (Confined density-dependent mass) model Jiang et al. APS, 2019

Conclusion

- The last stable configurations of SQS gives the following gravitational masses
  A) $M_g^{max} = 2.3M_\odot$ for the non-magnetized, non-rotating star
  B) $M_g^{max} = 2.38M_\odot$ for the rapidly rotating, non-magnetized star (Comparable with the recent detection of millisecond *"black widow" pulsar PSR J09520607*, $M_g \simeq 2.35 \pm 0.17$)
  C) $M_g^{max} \simeq 2.6M_\odot$ for the rapidly rotating, strongly magnetized star ($f = 1200$ Hz, $B_c \simeq 1.4 \times 10^{17}$ G)
- There is no stable configuration for very fast rotating strongly magnetized SQS (**e.g. at** $f = 1200$**Hz and** $B_c \geq 1.4 \times 10^{17}$ **G**)
- Deformation parameter $a = R_{eq}/R_{pol}$ changes from 1 **(**$f = 0$**,** $B = 0$**) to** 1.58 **(**$f = 400$ **Hz,** $B_c \sim 10^{18}$ **G)**
- There is a linear relation between total binding energy and compactness of SQS (with a negligible effect of magnetic field strength and rotational frequency).
- We find the maximum $|E_{BE}/M_g| \simeq 0.25$ that is less than the value in CDDM model