Black hole encircled by a thin disc

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joint work with D. Kofroň

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CHARLES UNIVERSITY
Faculty of mathematics
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Dedicated to O. Semerák on the occasion of his 60th orbit around the Sun.

What is it **not** about?

Usual simplification: disc as a test field on a fixed background.

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Can we include disc's own self-gravity?

What **is** it about?

An explicit **exact** superposition of the **Schwarzschild black hole** encircled by a **disc**.

- exact (vacuum) solution to Einstein equations

Assumptions:

- axially symmetric thin disc $\Longrightarrow$ distributional source $\propto \delta(z)$
- zero overall net rotation $\Longrightarrow$ static metric

Disc stretches from the horizon to infinity, but the density is falling off quickly enough $\Longrightarrow$ the field is regular everywhere (outside of the horizon) and asymptotically flat

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Weyl metric

Gravitational field of **static** and **axially symmetric** vacuum spacetimes is described by

$$\mathrm{d}s^2 = -e^{2\nu}\,\mathrm{d}t^2 + \rho^2 e^{-2\nu}\,\mathrm{d}\phi^2 + e^{2\lambda-2\nu}(\mathrm{d}\rho^2 + \mathrm{d}z^2)\ , \tag{1}$$

where t, ρ, ϕ, z are the Weyl cylindrical coordinates and $\nu(\rho, z), \lambda(\rho, z)$.

Vacuum Einstein equations then

$$\Delta\nu = 0\ , \qquad \lambda = \int_{\text{axis}}^{\rho,z} \rho(\nu_{,\rho}^2 - \nu_{,z}^2)\,\mathrm{d}\rho + 2\rho\nu_{,\rho}\nu_{,z}\,\mathrm{d}z\ , \tag{2}$$

where

- Δ is 3D Laplace operator in cylindrical coordinates (ρ, z),
- the integration for λ is evaluated along some path through the vacuum region.

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Kuzmin-Toomre family of thin discs

Kuzmin (1956); Toomre (1963) obtained a family of thin discs (λ function found by Bičák et al. (1993)) which Newtonian densities are

$$w_n = \frac{(2n+1)b^{2n+1}}{2\pi} \frac{\mathcal{M}}{(\rho^2 + b^2)^{n+3/2}} , \tag{3}$$

where $\mathcal{M}$ is the discs total mass and b is a constant, described by the gravitational potential

$$\nu_n = -\frac{\mathcal{M}}{(2n-1)!!} \sum_{k=0}^{n} \frac{(2n-k)!}{2^{n-k}(n-k)!} \frac{b^k}{r_b^{k+1}} P_k \left(|\cos\theta_b|\right) , \tag{4}$$

where P_l are the Legendre polynomials, and

$$r_b^2 := \rho^2 + (|z| + b)^2 , \qquad |\cos\theta_b| := \frac{|z| + b}{r_b} . \tag{5}$$

Inversion of the Kuzmin-Toomre dics

Performing a coordinate transformation (Kelvin transformation)

$$\rho \to \frac{b^2\rho}{\rho^2+z^2}\,, \qquad z \to \frac{b^2 z}{\rho^2+z^2}\,, \tag{6}$$

and the respective potential transform

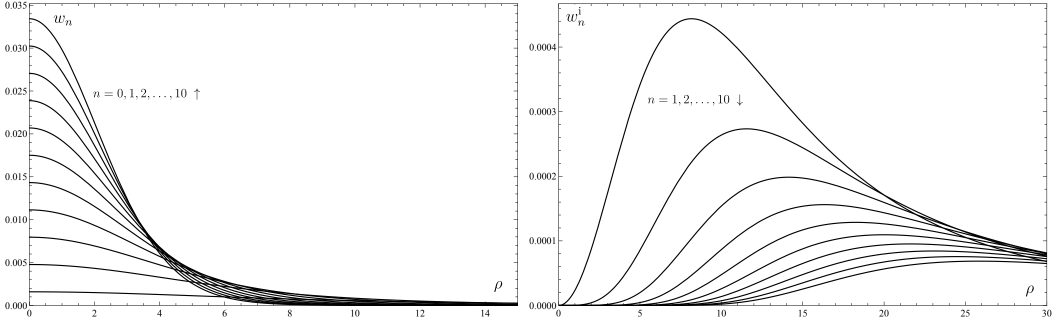
$$\nu_n(\rho,z) \longrightarrow \frac{b}{\sqrt{\rho^2+z^2}}\nu_n\left(\frac{b^2\rho}{\rho^2+z^2},\frac{b^2 z}{\rho^2+z^2}\right) \quad \Longrightarrow \quad w_n(\rho) \longrightarrow \frac{b^3}{\rho^3}w_n(b^2/\rho)\,, \tag{7}$$

results in "annular" disc of a density

$$w_n^{\mathrm{i}} = \binom{n+1/2}{n}\frac{\mathcal{M}b}{2\pi}\frac{\rho^{2n}}{(\rho^2+b^2)^{n+3/2}}\,, \tag{8}$$

where the multiplication constant is chosen in a way that $\mathcal{M}$ is still the total mass.

Inversion of the Kuzmin-Toomre dics



The resulting potential (after a suitable resummation) then

$$\nu_n^{\mathrm{i}} = -\binom{n+1/2}{n} \frac{\mathcal{M}}{(1+2n)!!} \sum_{k=0}^{n} \frac{(2n-k)!}{2^{n-k}(n-k)!} \, {}_2F_1(1+k, k-n; k-2n; 2) \frac{(-b)^k}{r_b^{k+1}} P_k\left(|\cos\theta_b|\right) ,$$

where ${}_2F_1$ is the hypergeometric function.

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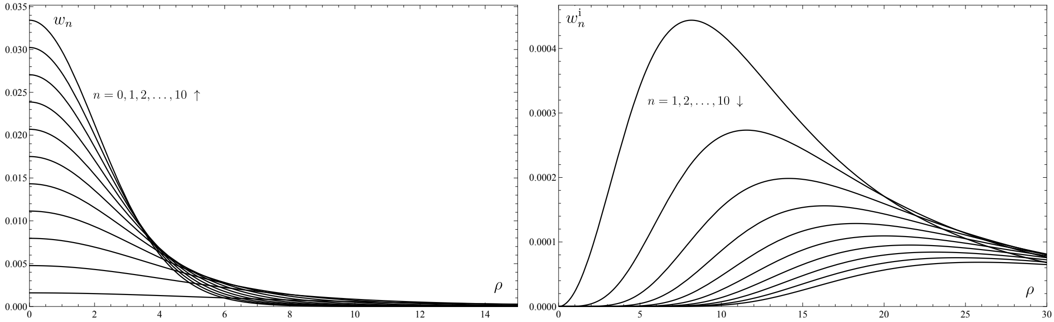

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The second metric function λ

Just like for the original Kuzmin-Toomre solution (Bičák et al., 1993), the second metric function can be analytially obtained integrating (2)

$$\lambda_n^{\mathrm{i}} = -\binom{n+1/2}{n}^2 \frac{\mathcal{M}^2 \sin^2\theta_b}{\left[(1+2n)!!\right]^2} \sum_{k,l=0}^{n} \mathcal{B}_{k,l} \frac{(-b)^{k+l}}{r_b^{k+l+2}} \mathcal{P}_{k,l}(\theta_b) \; , \tag{9}$$

where

$$\mathcal{B}_{k,l} \equiv \frac{(2n-k)!(2n-l)!}{2^{2n-k-l}(n-k)!(n-l)!(k+l+2)} \, {}_2F_1(1+k, k-n; k-2n; 2) \, {}_2F_1(1+l, l-n; l-2n; 2) \; ,$$

$$\mathcal{P}_{k,l} \equiv (k+1)(l+1)P_k P_l + 2(k+1)|\cos\theta_b| P_k P_l' - \sin^2\theta_b P_k' P_l' \; ,$$

$$P_k' \equiv \frac{\mathrm{d}}{\mathrm{d}|\cos\theta_b|} P_k(|\cos\theta_b|) \; .$$

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Vogt-Letelier discs

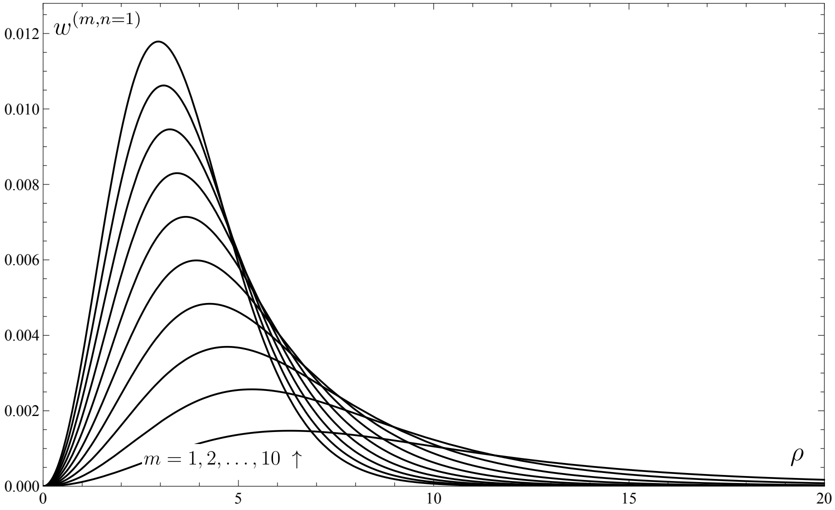
Vogt & Letelier (2009) did a superposition of the original Kuzmin-Toomre discs

$$\nu^{(m,n)} = W^{(m,n)} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{\nu_{m+k}}{2m+2k+1} \,, \tag{10}$$

where $W^{(m,n)}$ is a constant. The disc's Newtonian density

$$w^{(m,n)} = W^{(m,n)} \frac{\mathcal{M} b^{2m+1}}{2\pi} \frac{\rho^{2n}}{(\rho^2+b^2)^{m+n+3/2}} \,. \tag{11}$$

The inverted Kuzmin-Toomre discs are the special case of this family when $m = 0$.


$w^{(m,n=1)}$
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
5
10
15
20
ρ
$m = 1, 2, \ldots, 10 \uparrow$

Fixing the constant $W^{(m,n)}$, so the total mass of the disc is $\mathcal{M}$, the Vogt-Letelier potential can be recast into a form

$$\nu^{(m,n)} = -(2m+1)\mathcal{M}\binom{m+n+1/2}{n}\sum_{j=0}^{m+n}\mathcal{Q}_j^{(m,n)}\frac{b^j}{r_b^{j+1}}P_j(\cos\theta_b)\ , \qquad (12)$$

where

$$\mathcal{Q}_j^{(m,n)} = \begin{cases} \frac{2^{j-m}(2m-j-1)!!}{(2m+1)(m-j)!}\,{}_3F_2\left(j-n, \frac{j+m+1}{2}, \frac{j+m+2}{2}; j+1, \frac{2j+2m+3}{2}; 1\right) & \text{if } j \le m \\ \frac{(-1)^{j-m}j!}{(2j+1)!!}\binom{n}{j-m}\,{}_3F_2\left(\frac{j+1}{2}, \frac{j+2}{2}, j-m-n; \frac{2j+3}{2}, j-m+1; 1\right) & \text{if } j > m\ , \end{cases}$$

and ${}_3F_2$ are the generalized hypergeometric functions.

The second metric function λ

The second metric function can be again obtained

$$\lambda^{(m,n)} = -(2m+1)^2 \binom{m+n+1/2}{n}^2 \mathcal{M}^2 \sin^2\theta_b \sum_{k,l=0}^{m+n} \mathcal{B}_{k,l}^{(m,n)} \frac{b^{k+l}}{r_b^{k+l+2}} \mathcal{P}_{k,l}(\theta_b) \,, \tag{13}$$

$$\mathcal{B}_{k.l}^{(m,n)} = \frac{\mathcal{Q}_l^{(m,n)} \mathcal{Q}_k^{(m,n)}}{k+l+2} \,, \tag{14}$$

while the polynomials $\mathcal{P}_{k,l}$ are the same as for the inverted Kuzmin-Toomre discs (9).

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The total gravitational potential

The discs have an "annular" character $\Longrightarrow$ suitable for a superposition with a central black hole.

The Schwarzschild black hole in Weyl coordinates (a solid finite rod of the mass M and lenght $2M$)

$$\nu_{\text{Schw}} = \frac{1}{2} \ln \frac{d_1 + d_2 - 2M}{d_1 + d_2 + 2M}, \qquad \lambda_{\text{Schw}} = \frac{1}{2} \ln \frac{(d_1 + d_2)^2 - 4M^2}{4 d_1 d_2}. \tag{15}$$

where $d_{1,2} \equiv \sqrt{\rho^2 + (|z| \mp M)^2}$.

The Laplace equation is linear $\Longrightarrow$ the total gravitational potential is a simple sum

$$\nu = \nu_{\text{Schw}} + \nu_{\text{disc}}. \tag{16}$$

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$$\nu = \nu_{\text{Schw}} + \nu_{\text{disc}}\,. \tag{16}$$

The second metric function λ

The metric function λ does not superpose that simply. In fact, we denoted

$$\lambda = \lambda_{\text{Schw}} + \lambda_{\text{disc}} + \lambda_{\text{int}} \ , \tag{17}$$

where $\lambda_{\text{Schw}}, \lambda_{\text{disc}}$ are contributions from the black hole and the disc, and

$$\lambda_{\text{int},\rho} = 2\rho\left(\nu_{\text{Schw},\rho}\nu_{\text{disc},\rho} - \nu_{\text{Schw},z}\nu_{\text{disc},z}\right) \ , \tag{18}$$

$$\lambda_{\text{int},z} = 2\rho\left(\nu_{\text{Schw},\rho}\nu_{\text{disc},z} + \nu_{\text{Schw},z}\nu_{\text{disc},\rho}\right) \ . \tag{19}$$

Notice that (18), (19) are linear in ν_{disc}, hence λ_{int} satisfies the same recurrence relations as ν_{disc}. Namely

$$\lambda_{\text{int}}^{(0,n+1)} = \lambda_{\text{int}}^{(0,n)} + \frac{b}{2(n+1)}\frac{\partial}{\partial b}\lambda_{\text{int}}^{(0,n)} \ , \qquad \lambda_{\text{int}}^{(0,0)} = -\frac{\mathcal{M}}{r_b}\left(\frac{d_1}{b+M} - \frac{d_2}{b-M}\right) - \frac{2\mathcal{M}M}{b^2-M^2} \ ,$$

$$\frac{(2m+1)(2n+3)}{2m+2n+3}\lambda_{\text{int}}^{(m+1,n)} = \lambda_{\text{int}}^{(m,n)} + \frac{4m(n+1)}{2m+2n+3}\lambda_{\text{int}}^{(m,n+1)} - b\frac{\partial}{\partial b}\lambda_{\text{int}}^{(m,n)} \ .$$

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Two simple physical interpretations of the disc are possible

- a single component ideal fluid with a certain surface density ($e^{\nu-\lambda}\sigma$) and azimuthal pressure ($e^{\nu-\lambda}P$) which keeps the orbits at their radius
- two identical counter-orbiting dust components with proper surface densities ($\sigma_+ = \sigma_- \equiv \sigma/2$) following circular geodesics

Both characteristics σ and P are encoded in the jump of the normal derivative of the gravitational potential

$$\sigma + P = \frac{\nu_{,z}(z=0^+)}{2\pi} = w(\rho)\,, \qquad P = \frac{\nu_{,z}(z=0^+)}{2\pi}\rho\nu_{,\rho} = w(\rho)\rho\nu_{,\rho}\,. \tag{20}$$

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The two-streams interpretation is possible where the physical speed v of freely rotating particles with respect to a *local* static observer acquires time-like values $(0 \leq v < 1)$.

This time-like condition also covers the energy conditions and the non-negativity of the relativistic densities and azimuthal pressures.

For such speed, in the Weyl-type spacetimes, it is

$$v_+^2 \equiv v_-^2 \equiv v^2 = \frac{\sigma}{P} = \frac{\rho\nu_{,\rho}}{1 - \rho\nu_{,\rho}} \,. \tag{21}$$

Circular velocity

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The coordinate statements have to be taken with some caution (although Weyl coordinates represented some spacetime features adequately).

We should check the physical properties of the discs also in terms of some invariant measures like

circumferential radius $r_{\mathrm{cf}} := \rho e^{-\nu} \Longrightarrow$ azimuthal circumference $= \int_0^{2\pi} \sqrt{g_{\phi\phi}}\,\mathrm{d}\phi = 2\pi r_{\mathrm{cf}}$,

proper radial distance from the axis $r_{\mathrm{prop}} := \int_0^{\rho} \sqrt{g_{\rho\rho}}\,\mathrm{d}\rho = \int_0^{\rho} e^{\lambda-\nu}\,\mathrm{d}\rho$.

Coordinate vs geometrical measures

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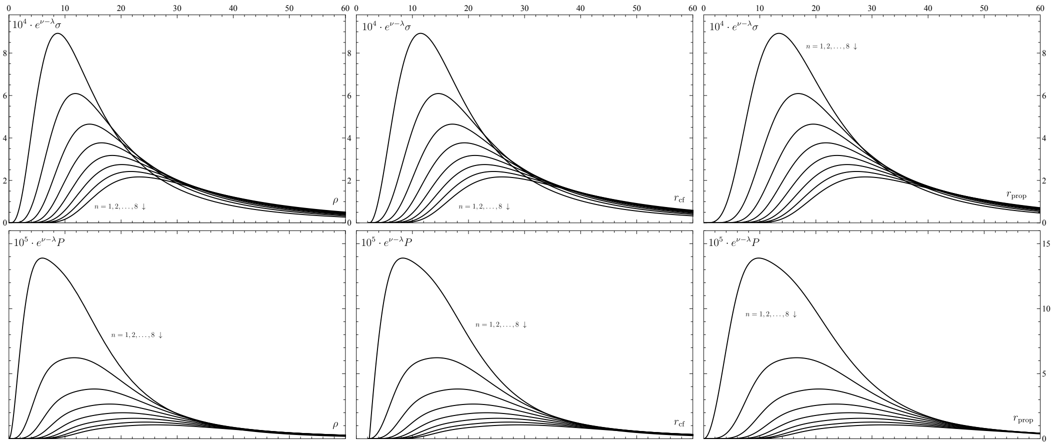

Fig. 1: Inverted Kuzmin-Toomre family ($n = 1, 2, \ldots, 8$) is depicted. We chose the mass of the discs $\mathcal{M} = 3M$ and $b = 10M$.

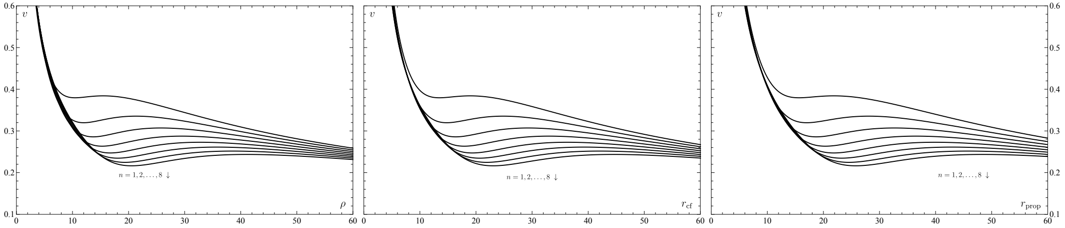

Fig. 2: Inverted Kuzmin-Toomre family ($n = 1, 2, \ldots, 8$) is depicted. We chose the mass of the discs $\mathcal{M} = 3M$ and $b = 10M$.

Similar plots can be obtained for the whole Vogt-Letelier family of discs.

Summary

- We obtained full metric (both non-trivial metric functions) describing the whole family of Vogt-Letelier discs as well as the full metric describing their superposition with a central black hole.
- Both metric functions are given *analytically* and in *closed-form*.
- Some basic physical properties of the discs are checked against the radial coordinate, circumferential radius and the proper distance.
- Soon in ApJ.

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Thank you for your attention.

Question time!

The black hole horizon

The intrinsic geometry of the black-hole horizon changes due to the presence of the disc.

At any given coordinate time $t = \text{const}$, the horizon is a 2D-surface which can be isometrically embedded into Euclidean 3-space (we used method by Smarr (1973)).

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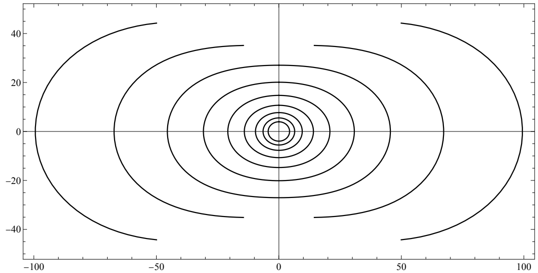

Fig. 3: A section ($\phi = \text{const}$) of the isometric embedding of the black-hole horizon distorted by the 3th order inverted Kuzmin-Toomre disc with $b = 2M$. The disc masses $\mathcal{M} = 5M, 7.5M, 10M, \ldots 25M$. Both axes are in units of M.