Primordial Black Hole formation during the QCD phase transition

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RAGTIME 24

10 October 2022 - Opava

G. Franciolini, IM, P.Pani, A. Urbano - arXiv:2209.05959
IM, K. Jedamzik, S. Young - in preparation

From inflation to black hole mergers and back again: Gravitational-wave data-driven constraints on inflationary scenarios with a first-principle model of primordial black holes across the QCD epoch

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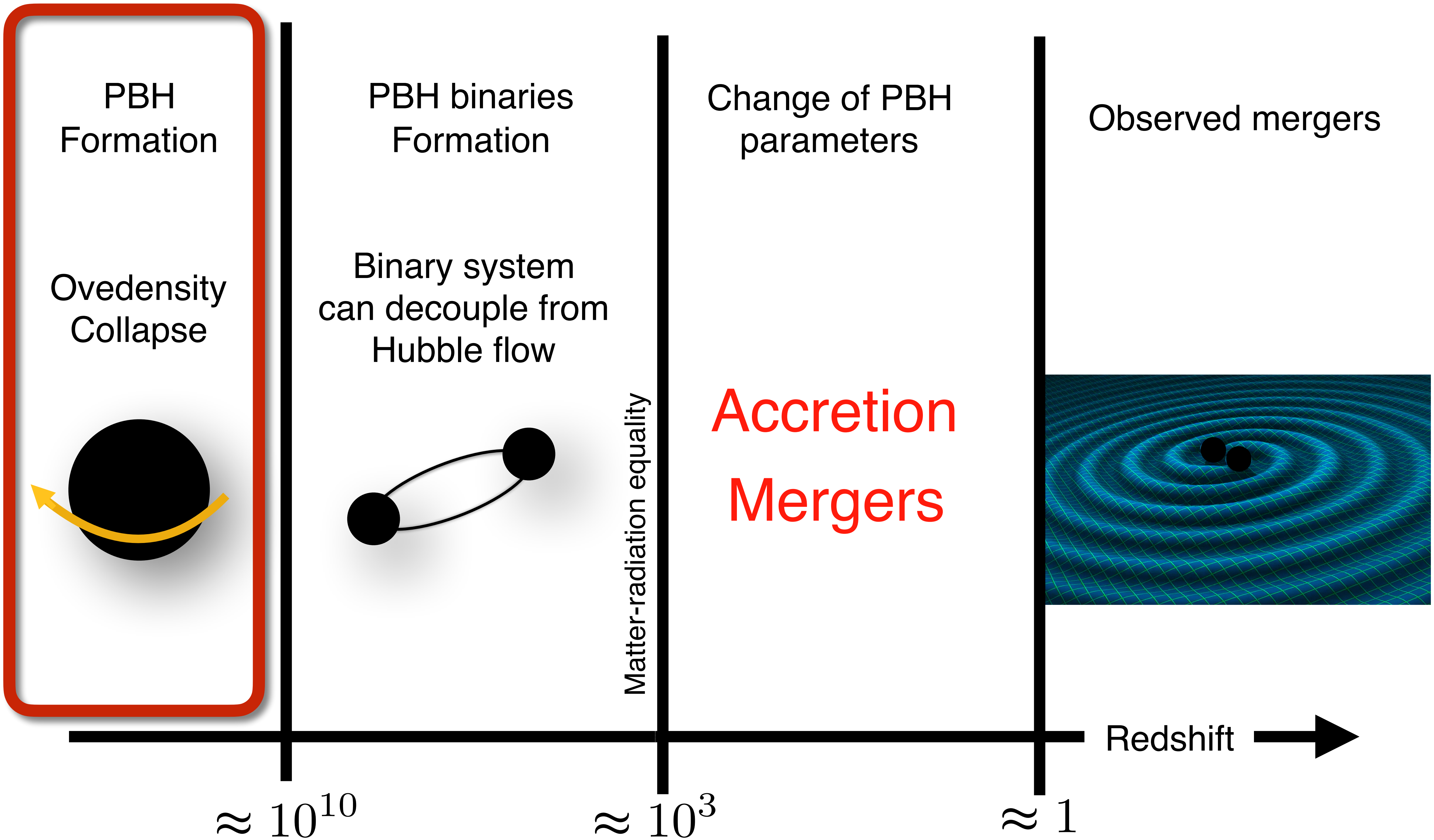
Recent population studies have searched for a subpopulation of primordial black holes (PBHs) in the gravitational-wave (GW) events so far detected by LIGO/Virgo/KAGRA (LVK), in most cases adopting a phenomenological PBH mass distribution. When deriving such population from first principles in the standard scenario, however, the equation of state of the Universe at the time of PBH formation may strongly affect the PBH abundance and mass distribution, which ultimately depend on the power spectrum of cosmological perturbations. Here we improve on previous population studies on several aspects: (i) we adopt state-of-the-art PBH formation models describing the collapse of cosmological perturbations across the QCD epoch; (ii) we perform the first Bayesian multi-population inference on GW data including PBHs and directly using power spectrum parameters instead of phenomenological distributions; (iii) we critically confront the PBH scenario with LVK phenomenological models describing the GWTC-3 catalog both in the neutron-star and in the BH mass ranges, also considering PBHs as subpopulation of the total events. Our results confirm that LVK observations prevent the majority of the dark matter to be in the form of stellar mass PBHs. We find that the best fit PBH model can comprise a small fraction of the total events, in particular it can naturally explain events in the mass gaps. If the lower mass-gap event GW190814 is interpreted as a PBH binary, we predict that LVK should detect up to a few subsolar mergers and one to ≈ 30 lower mass gap events during the upcoming O4 and O5 runs. Finally, mapping back the best-fit power spectrum into an ultra slow-roll inflationary scenario, we show that the latter predicts detectable PBH mergers in the LVK band, a stochastic GW background detectable by current and future instruments, and may include the entirety of dark matter in asteroid-mass PBHs.

CONTENTS

I. INTRODUCTION

Primordial black holes (PBHs) [1–4] might have formed in the early universe after inflation from the collapse of large amplitude cosmological perturbations [5–8] or by other mechanisms. In the standard formation scenario,

PBH evolution

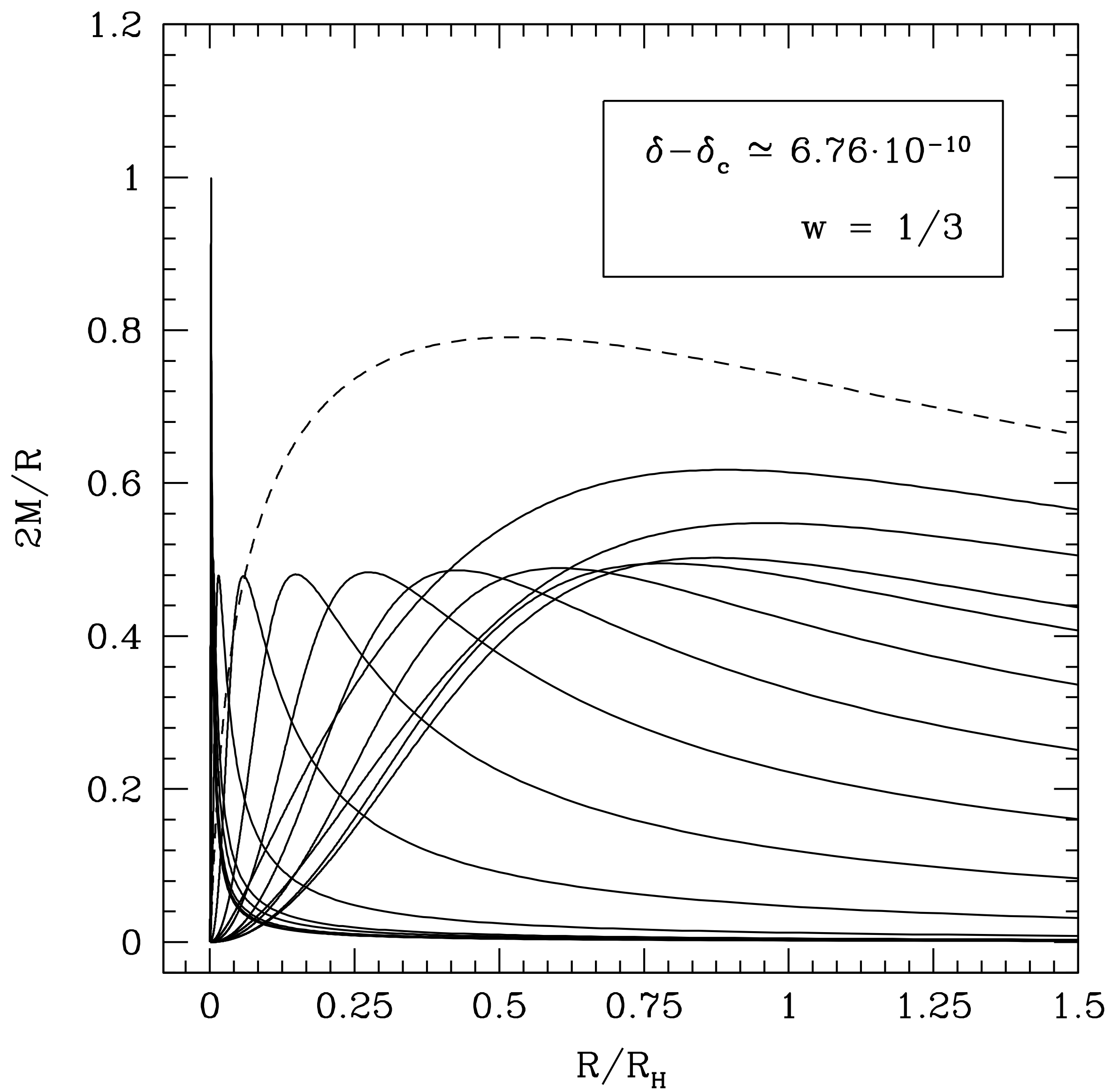


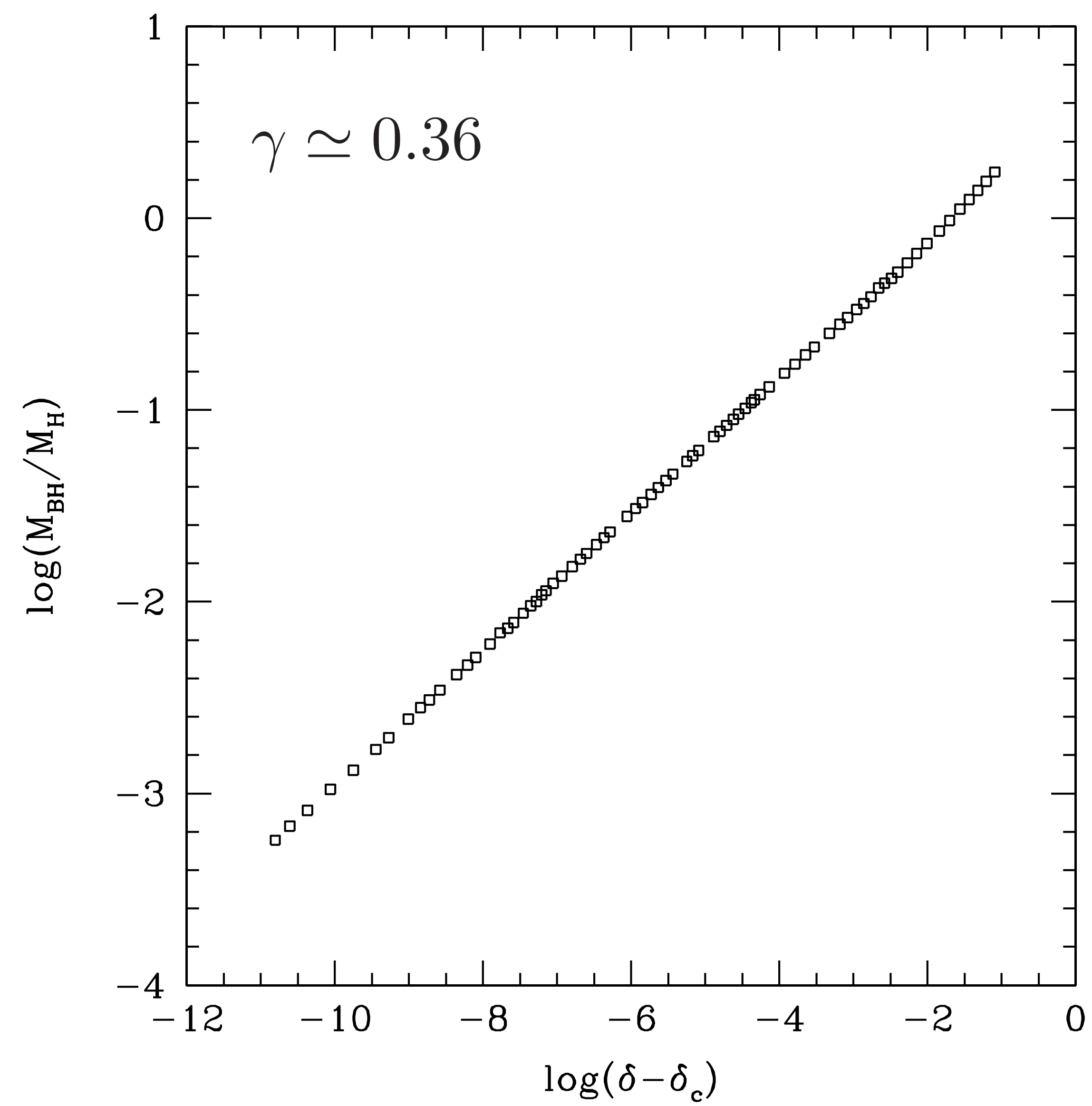
Courtesy of *Antonio Riotto*

Numerical Results: PBH formation / mass spectrum

$$R(r,t) = 2M(r,t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H$$





$\mathcal{K}, \delta_c$ — shape dependent

IM, J. Miller, Polnarev, Rezzolla - CQG (2005 - 2013)

Initial conditions: curvature profile

- The asymptotic metric ($t \longrightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9}\left[\boxed{\nabla^2\zeta(r)} + \boxed{\frac{1}{2}\left(\nabla\zeta(r)\right)^2}\right] e^{-2\zeta(r)}$$

- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

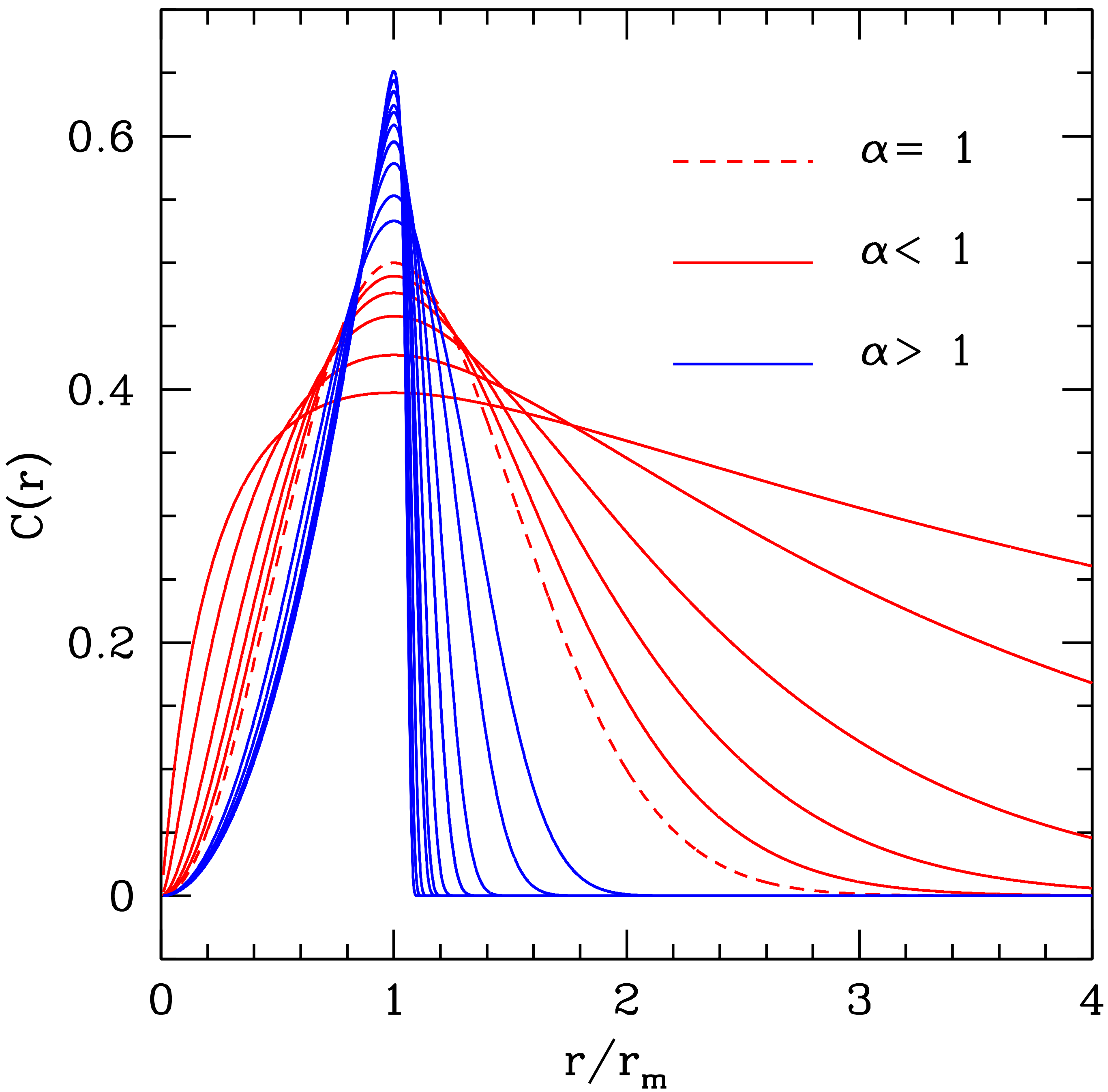
$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = \boxed{-\frac{4}{3}\tilde{r}\zeta'(r)}\left[1 + \boxed{\frac{1}{2}\tilde{r}\zeta'(r)}\right] \quad \Rightarrow \quad \delta = \boxed{\delta_G}\left[1 - \boxed{\frac{3}{8}\delta_G}\right]$$

Shape parameter

I. Musco - PRD (2019)

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\alpha \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)} = \frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$



$$0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}$$

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

- **PDF of δ follows a Gaussian distribution:** $P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (k r_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$
- **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$
- **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3 \times 10^{-3}$
- **Non linear effects:** $\delta = \delta_G \left[1 - \frac{3}{8} \delta_G\right] \;\Rightarrow\; 1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1 - \sqrt{1 - \frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$

S.*Young, IM, C.Byrnes* JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_*\hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_*\hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2\hat{r}_m^2 - 1)\frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos(k\hat{r}_m)\right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1+F(\alpha)\right]\alpha = -\frac{1}{2}\left[1+\hat{r}_m\frac{\int dk k^4 \cos(k\hat{r}_m)P_\zeta(k,\eta)}{\int dk k^3 \sin(k\hat{r}_m)P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1-\frac{2}{5}e^{-1/\alpha}\frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right)-\Gamma\left(\frac{5}{2\alpha},\frac{1}{\alpha}\right)}}\,.$$

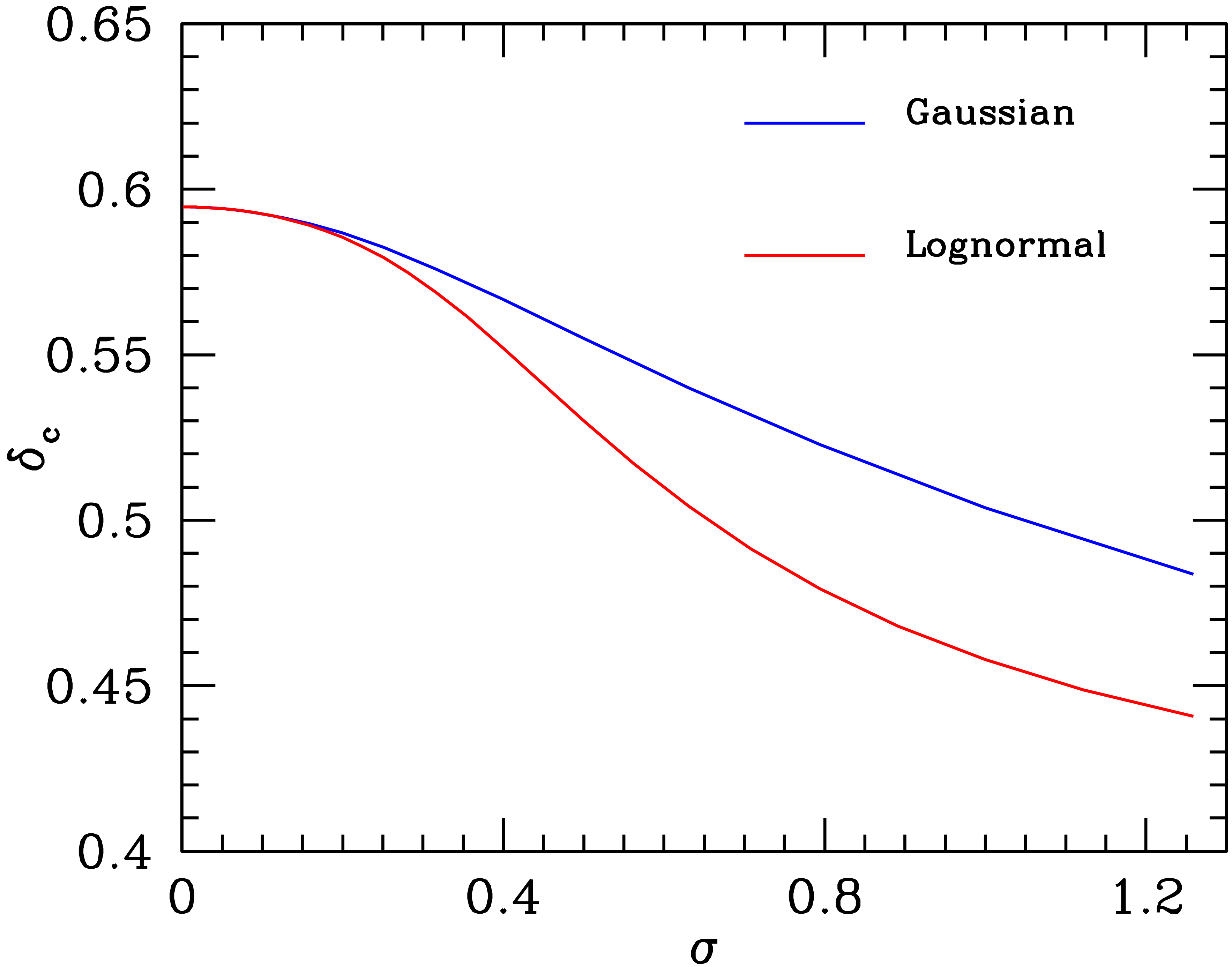
4. **The threshold** δ_c**:** compute the threshold as function of α, fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

Power Spectrum:

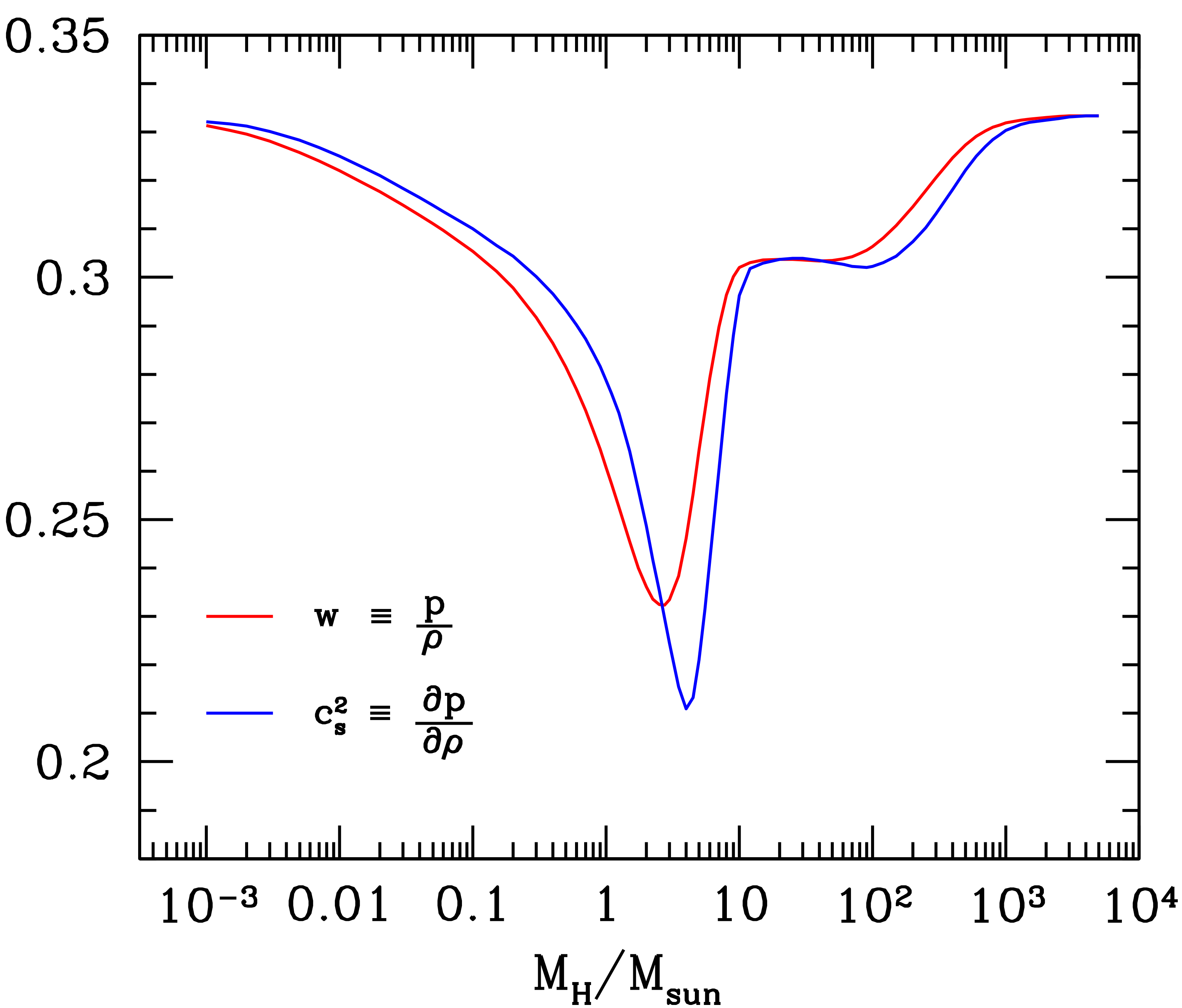
Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$

Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$



QCD Phase-transition

- Significant softening of the equation of state (lattice QCD simulations)
- Introducing an intrinsic scale



$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$$

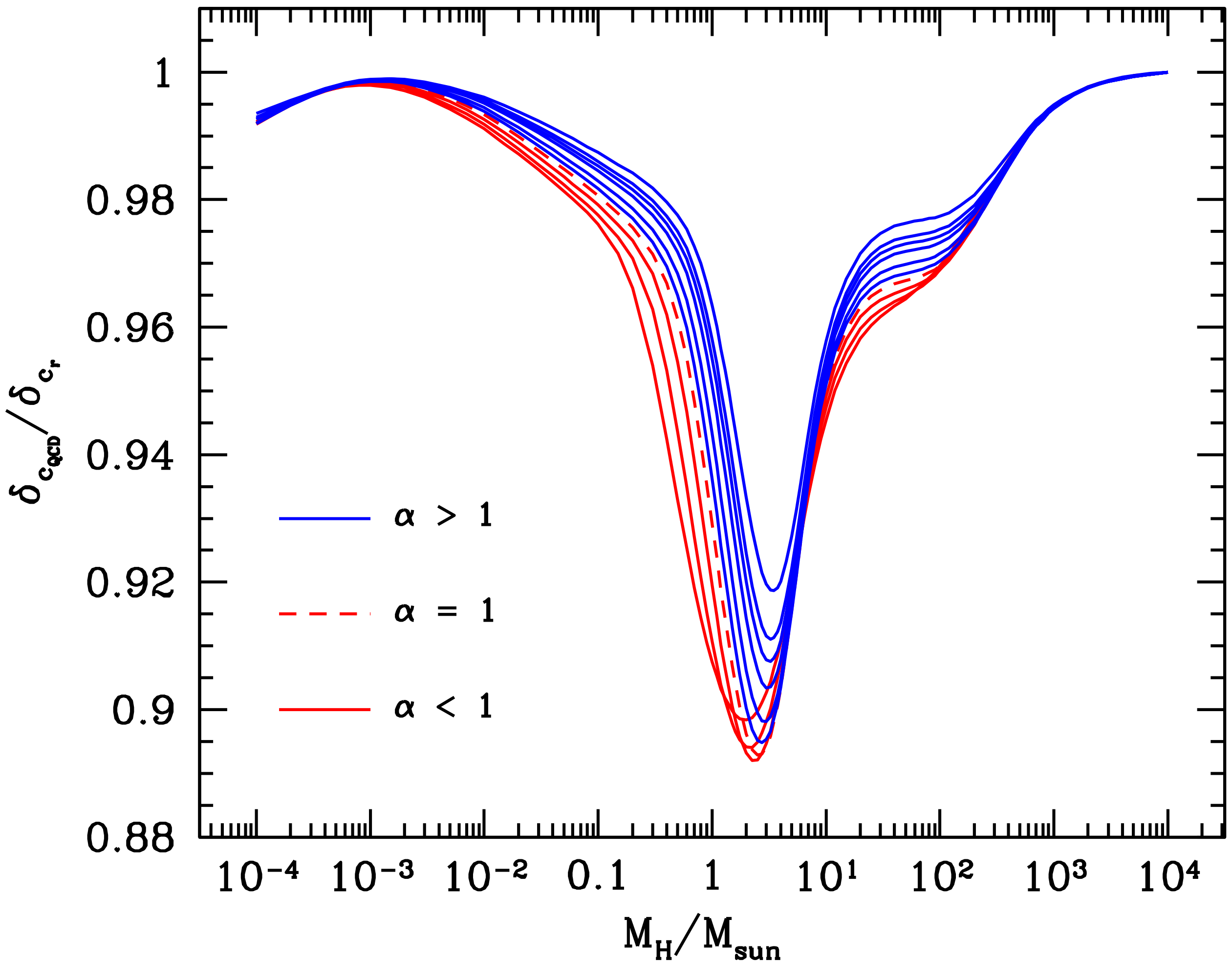
$$s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

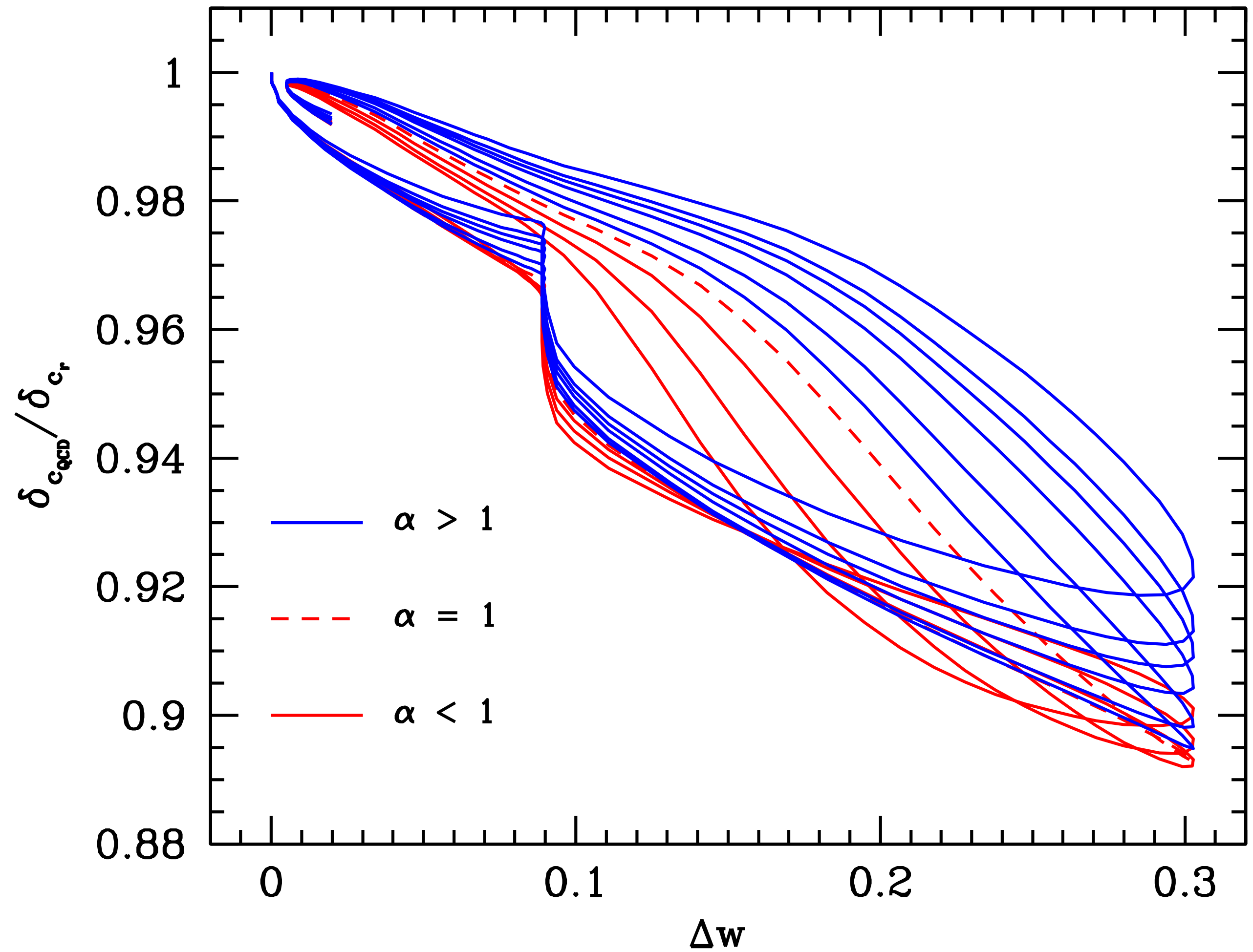
$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1$$

$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + Th'_{\rm eff})}{3(4g_{\rm eff} + Tg'_{\rm eff})} - 1$$

PBH Threshold during the QCD





Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of ***w(T)***.

Significant enhancement of PBH formation around the solar mass scale!

$$\Delta w = \frac{1/3 - w}{1/3} = 1 - 3w$$

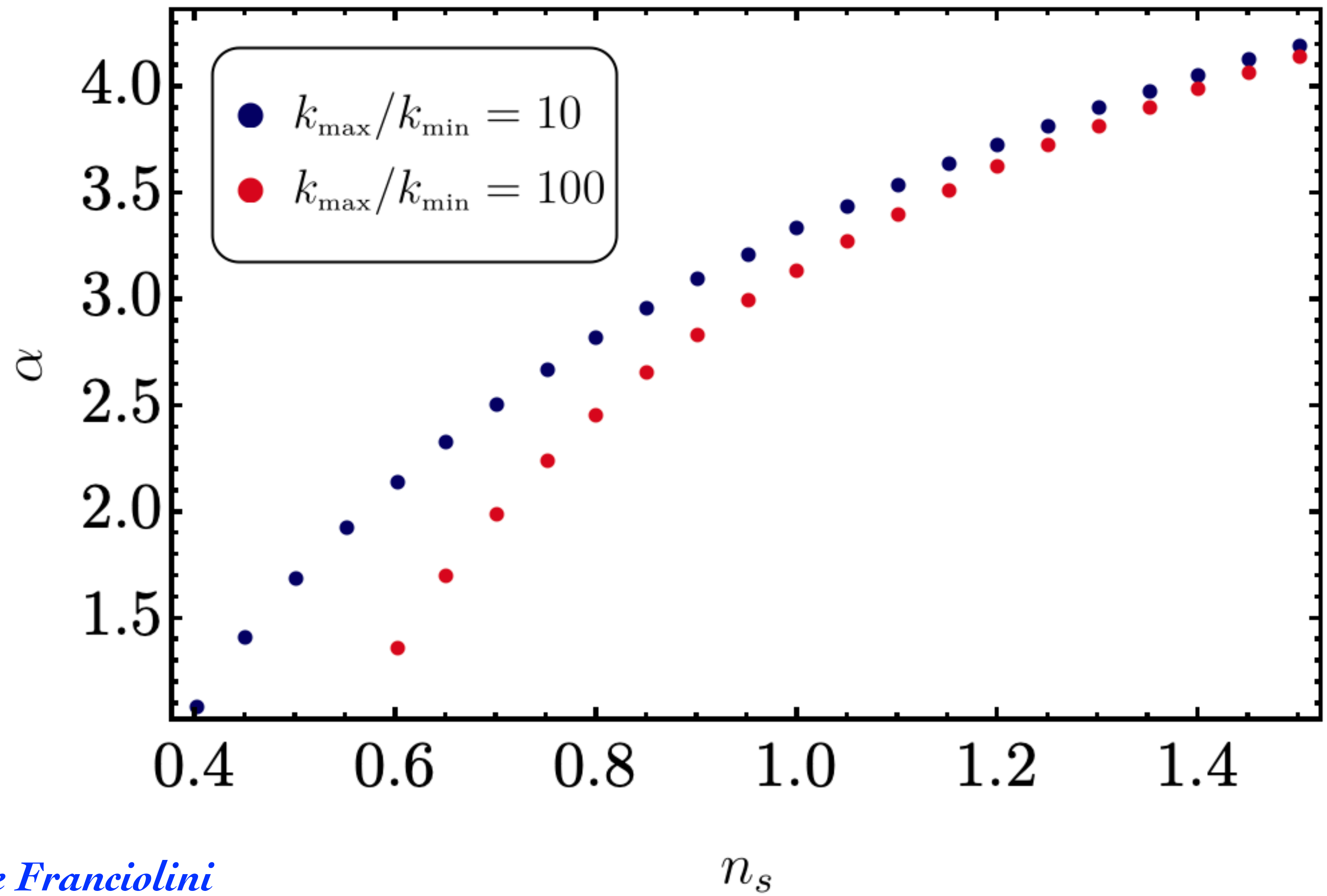
IM, K. Jedamzik, Sam Young - in progress…

From the Power Spectrum to the shape parameter

$$P_\zeta(k) = A\,(k/k_{\rm min})^{n_s-1}\,\Theta(k-k_{\rm min})\Theta(k_{\rm max}-k)$$

n_s — spectrum tilt

$k_{\rm max}/k_{\rm min}$ — cut-off scale



Courtesy of *Gabriele Franciolini*

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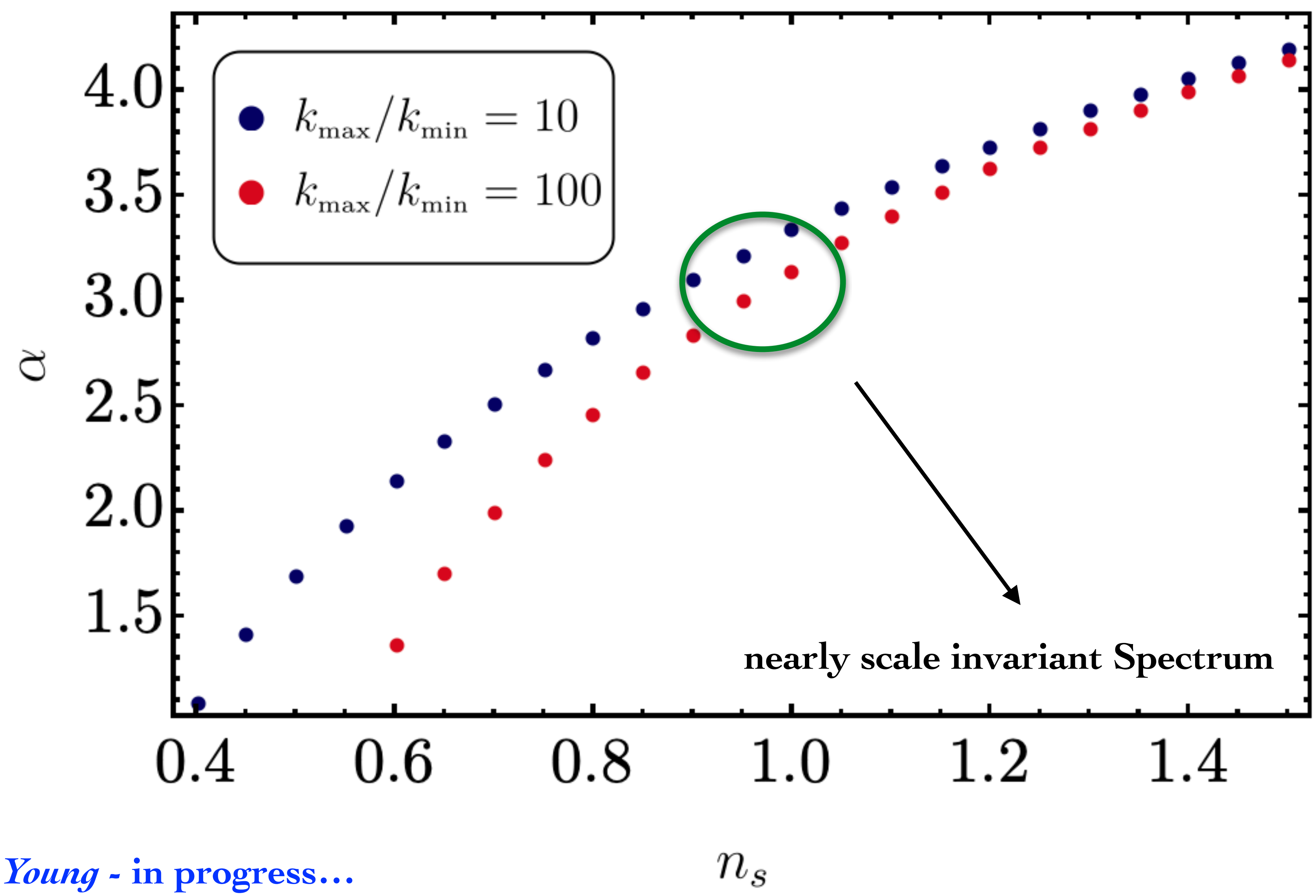


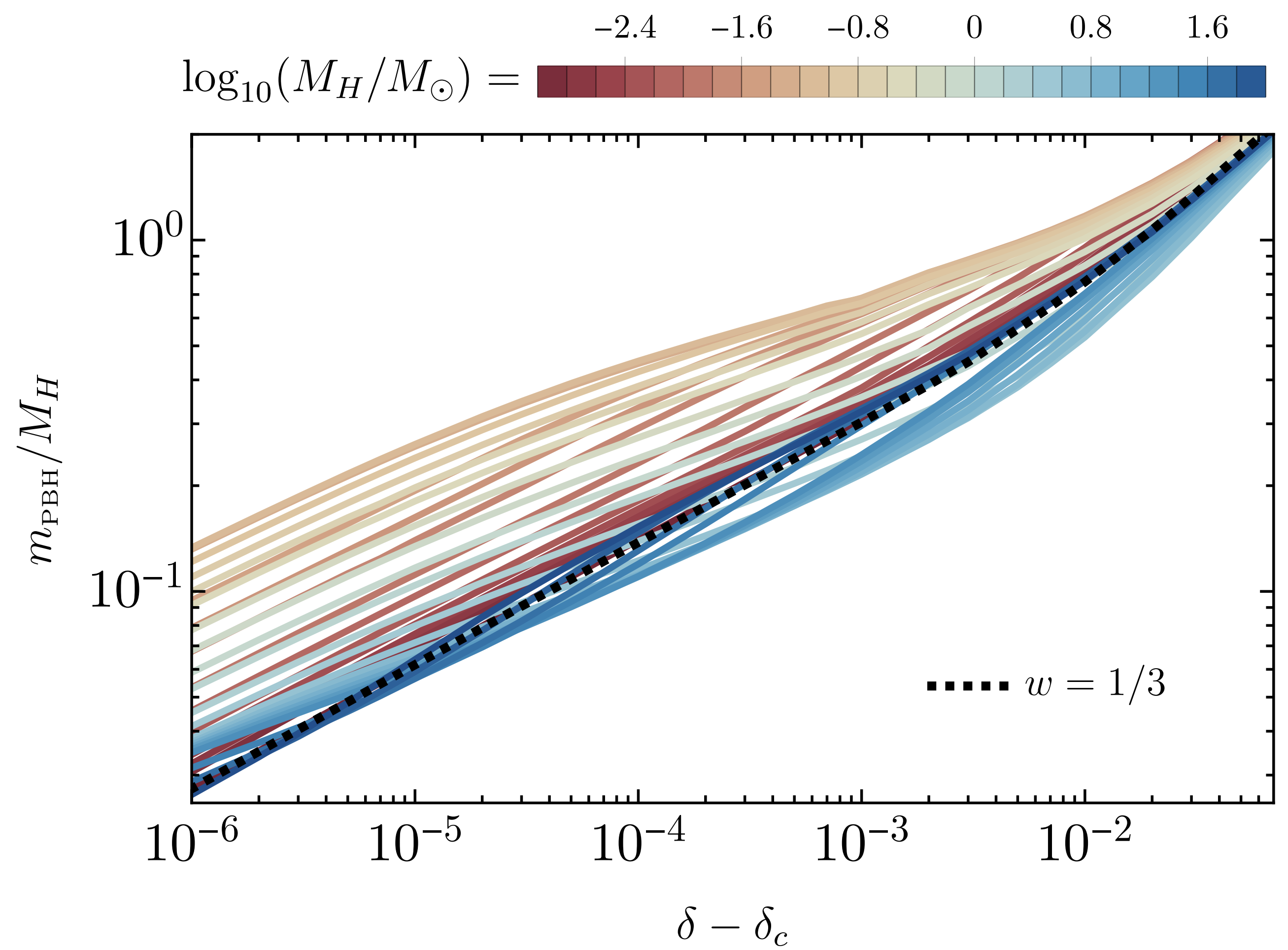
IM, K. Jedamzik, Sam Young - in progress…

PBH Mass Spectrum - QCD

For a nearly scale invariant Power Spectrum

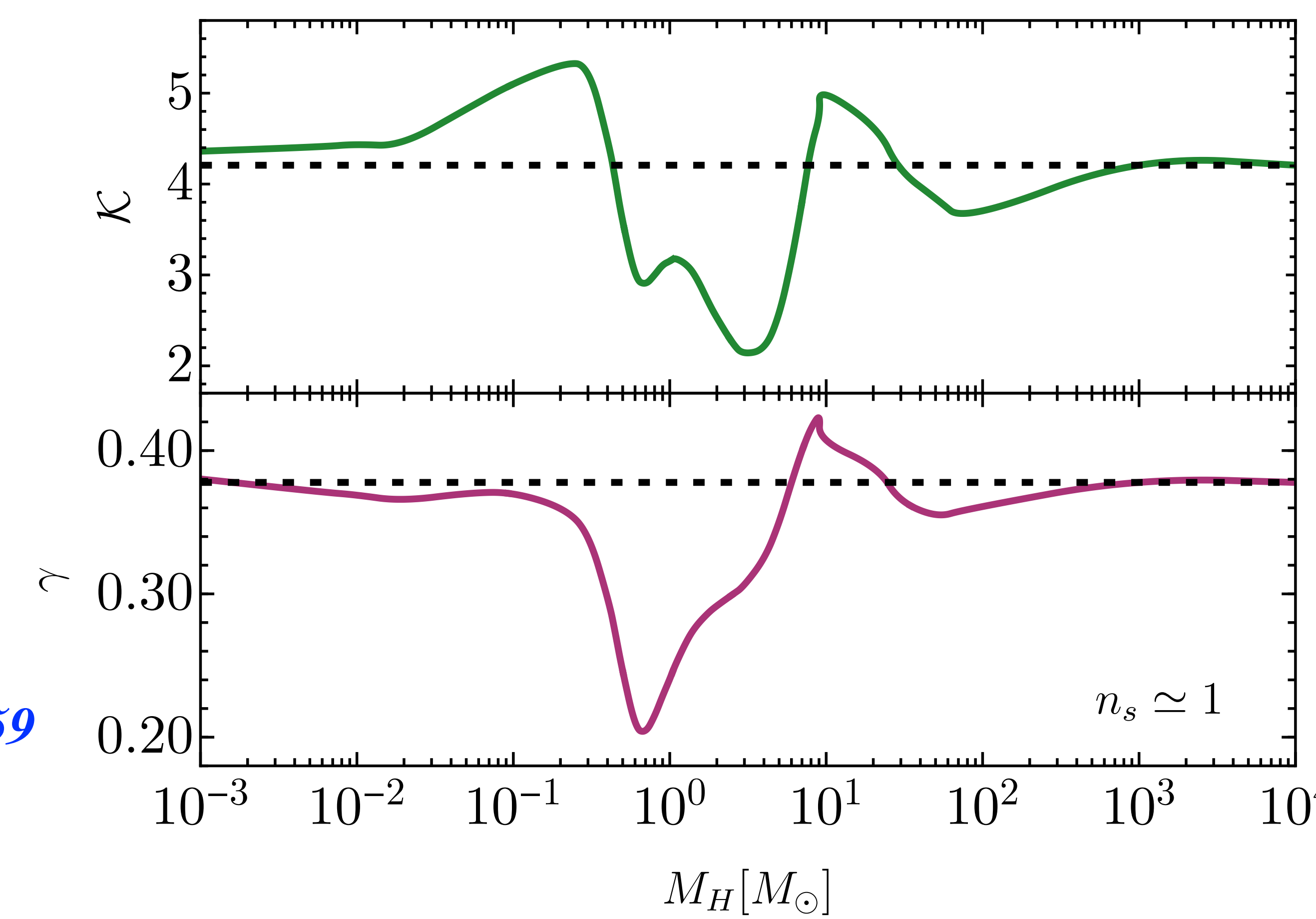
↓

$$\alpha \simeq 3$$



$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$



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PBH mass distribution - QCD

Mass Function $\psi(m_{PBH})$: fraction of PBHs with mass in the infinitesimal interval of m_{PBH}

$$\psi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}M_{\rm PBH}}$$

$$\int {\rm d}m_{\rm PBH}\ \psi(m_{\rm PBH}) = 1$$

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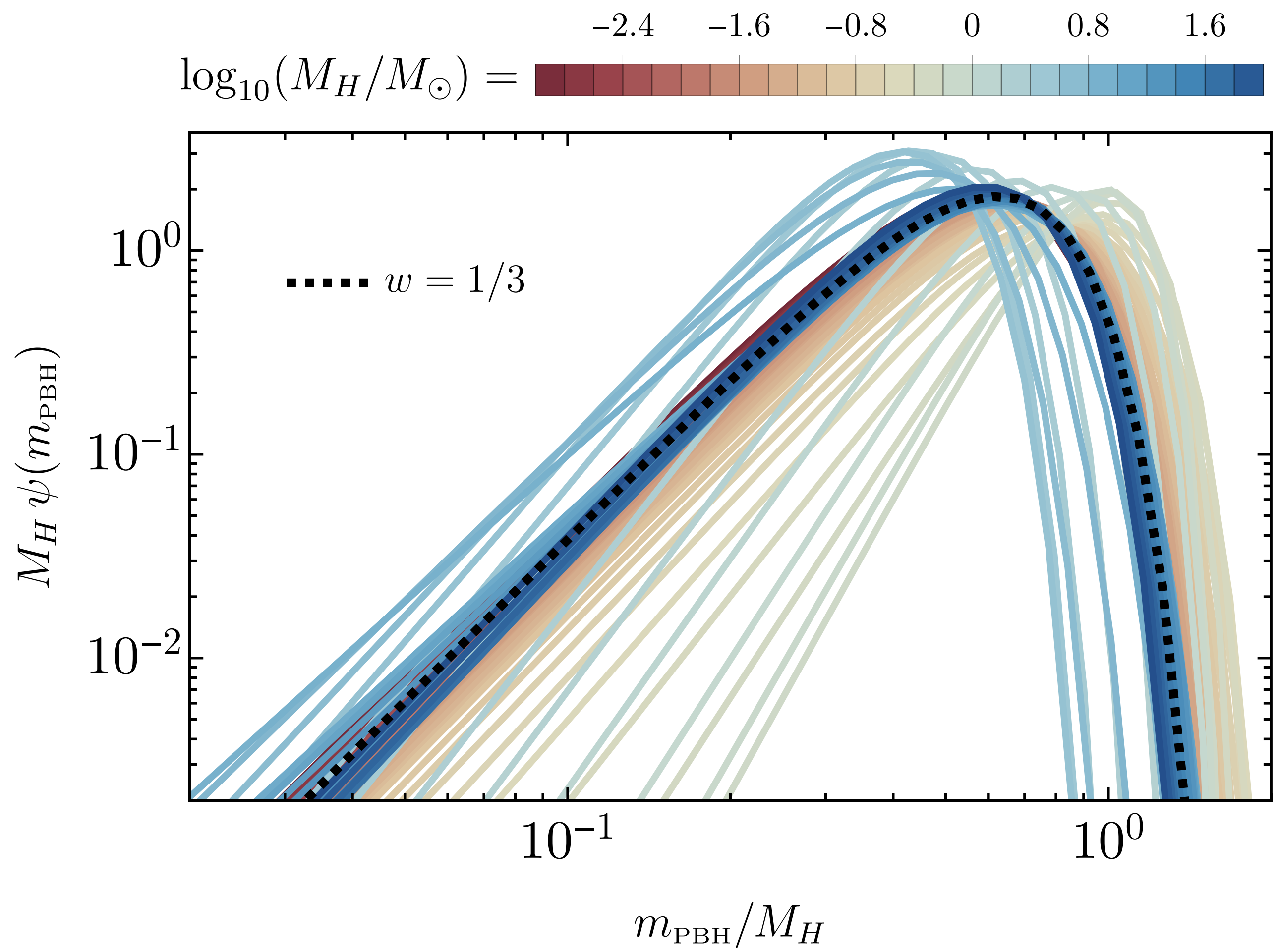
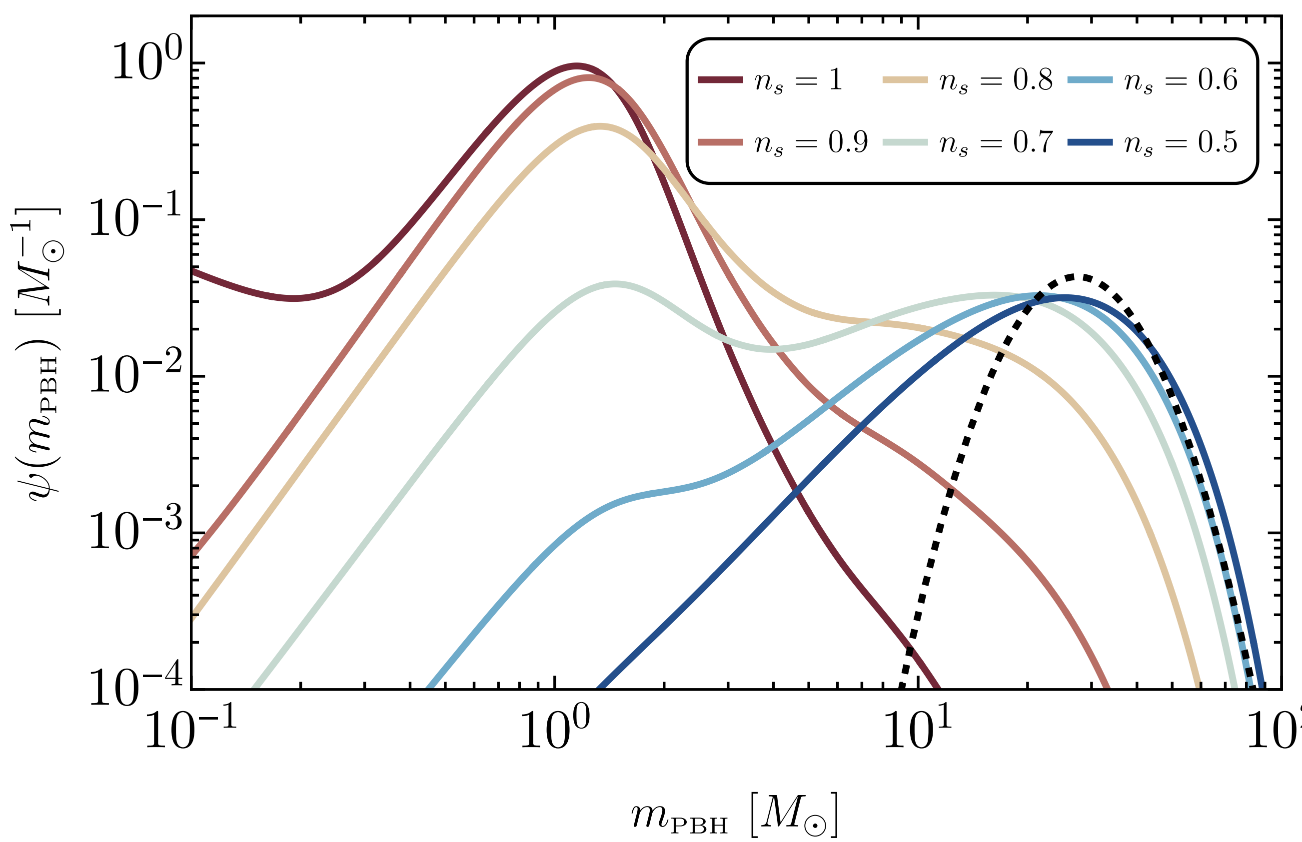
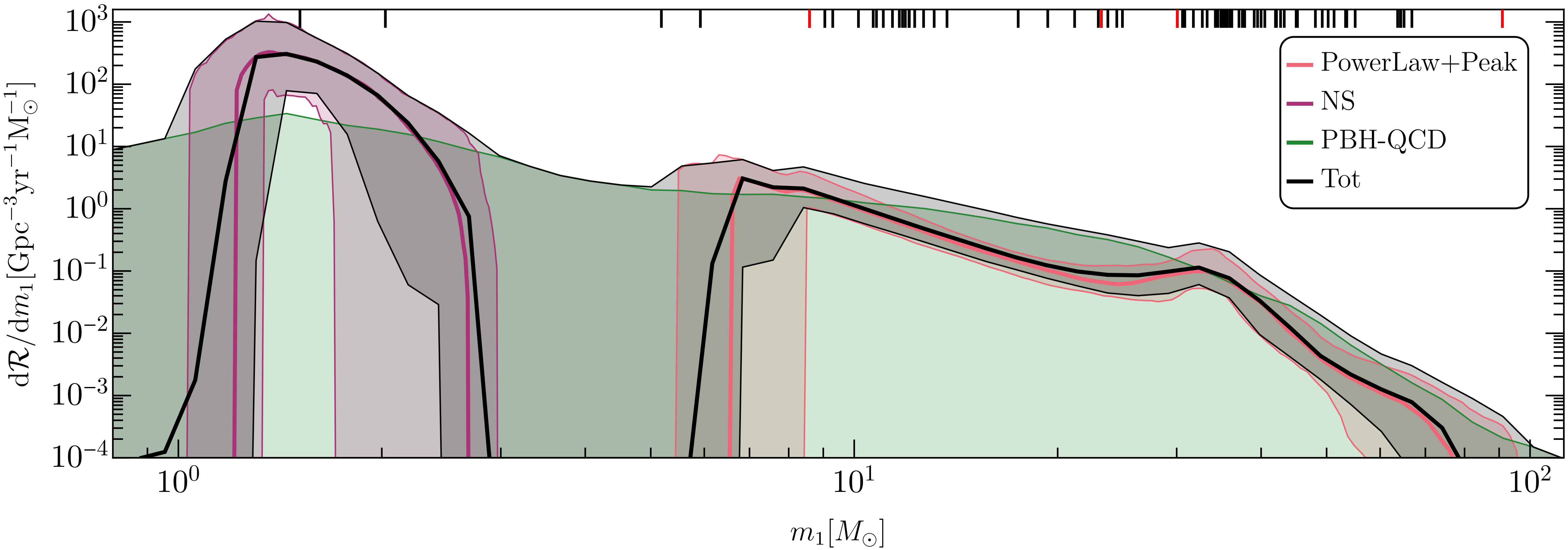




BH merger rate

G. Franciolini, IM, P.Pani, A urbano - arXiv:2209.05959

- Making **Bayesian inference analysis** we found that a sub-population of PBHs is compatible with the LVK catalog.
- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).



PBH constraints

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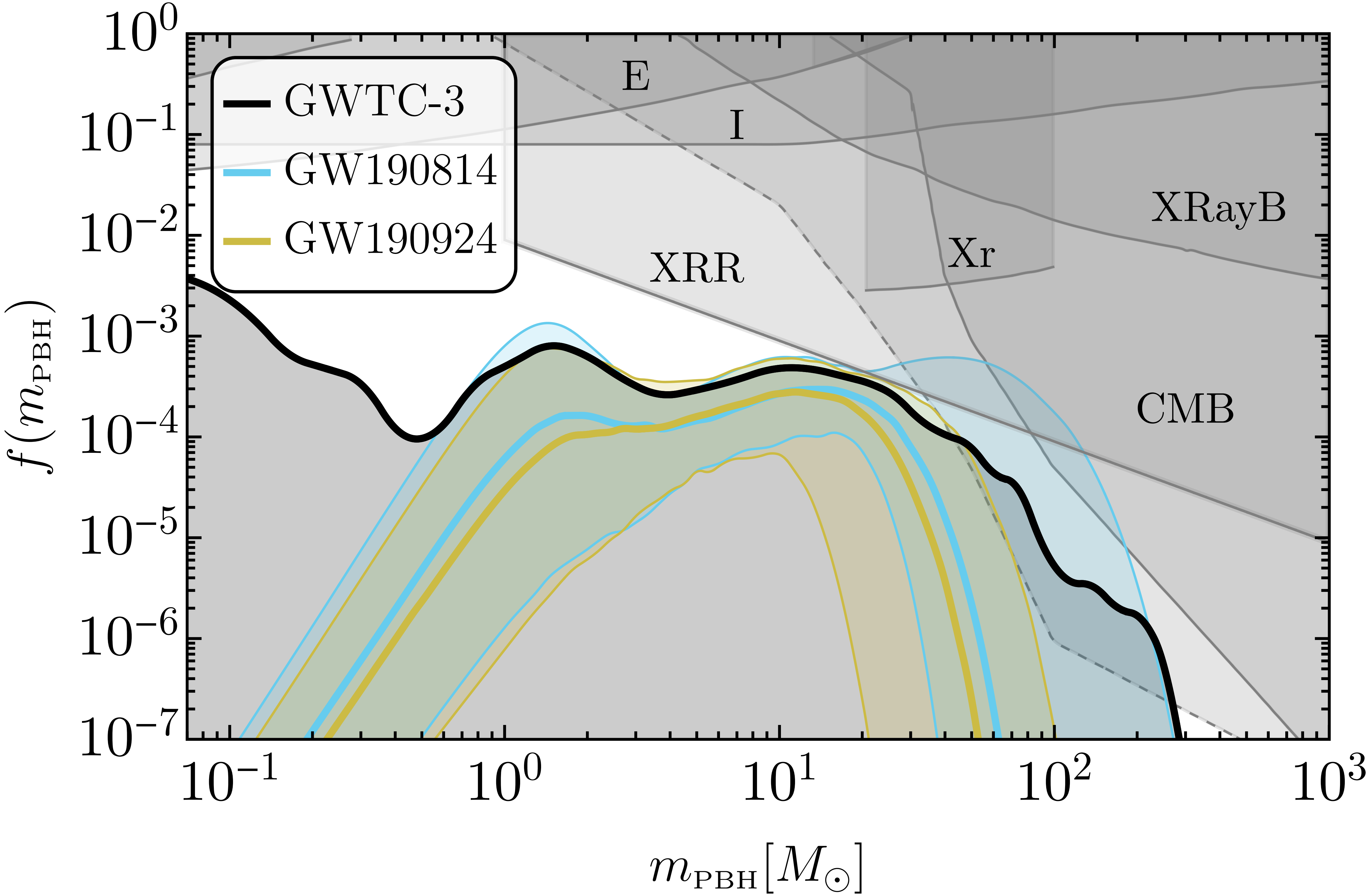

Conclusions

- The **non linear threshold for PBH** and **the mass spectrum** could be fully computed from the **shape of the power spectrum of cosmological perturbations**, making relativistic numerical simulations.

- A softening of the equation of state **(QCD) significantly enhances the formation of PBHs**, with a **mass distribution peaked between 1 and 2 solar masses** (the range of heavy NSs and light BHs).

- This could gives **a sub-population of BH mergers compatible with the LVK catalog**, explaining mass gap events as **GW190814**.

- Our analysis predicts a **constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%)**, compatible with the current observational constraints.

- **A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range (USR inflation models)**.