Energy dissipation in astrophysical simulations: results of the Orszag-Tang test

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- Dissipation of magnetic energy in the still poorly understood phenomena of solar flares, magnetic substorms in the Earth magnetosphere, jets, and relativistic ejections from the accretion discs of compact objects.
- In such systems, **magnetic reconnection** is the accepted mechanism which **heats and accelerates plasmoids** and affects the dynamics of plasma.
- In this work we aim at studying energy dissipation in **MHD** framework through the **Orszag-Tang (OT) vortex** that is a popular model problem for numerical MHD codes.
  * OT test problem has **simple initial conditions** and **periodic boundary condition**.
  * This problem tests the power of the code to capture the MHD shocks and shock-shock interactions.
- We quantitatively compare results obtained between relativistic (**Rel-MHD**) and non-relativistic (**MHD**), resistive (**Res-MHD**) and non-resistive, as well as 2D and 3D setups.
- We use MHD codes, **PLUTO4.4** (A modular code for computational astrophysics in Newtonian framework) and **KORAL** (A General Relativistic Radiation MHD **GRRMHD**) that is similar to HARM code.

Magnetohydrodynamic equations/Res-Rel-MHD module

- Magneto-fluid dynamics can be described using the equations of **conservation of mass** (continuity equation), **energy-momentum**, and **Maxwell equations**.
- For a fluid propagating in the laboratory reference frame with relativistic bulk velocity $\boldsymbol{v}$, the bulk Lorentz factor is defined with Γ, and four velocity $\boldsymbol{u}^{\boldsymbol{\mu}}$, the conservation equations are,

$$\frac{\partial D}{\partial t} + \nabla(Dv) = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{m}}{\partial t} + \nabla(\omega \boldsymbol{u}^{\mu} \boldsymbol{u}^{\nu} + \boldsymbol{p} \boldsymbol{g}^{\mu\nu} + \boldsymbol{T}^{\mu\nu}) = 0 \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \boldsymbol{m} = 0, \tag{3}$$

- $g^{\mu\nu}$ is the metric tensor in Minkowski coordinates, $D = \Gamma\rho$ is the mass density, $\boldsymbol{m} = \omega\Gamma\boldsymbol{u}^{\boldsymbol{\mu}} + \boldsymbol{E} \times \boldsymbol{B}$ is the total momentum density, $\varepsilon = \omega\Gamma^2 - p + P_{\mathrm{EM}}$ is the total energy density. The specific enthalpy is denoted by $\omega = \varepsilon + p$ and $P_{\mathrm{EM}} = (E^2 + B^2)/2$ denotes the electromagnetic energy density. $\boldsymbol{T}^{\mu\nu}$ is the energy-momentum tensor.

non-relativistic and Ideal MHD limits

Non-relativistic limit: When $\beta = v/c << 1$ hence $\Gamma \approx 1 + \beta^2/2$, the total momentum density $\boldsymbol{m}$ and the total energy density ε change to the non-relativistic forms.

$$\boldsymbol{m} = \rho \boldsymbol{v}, \qquad \varepsilon = \rho e + \frac{1}{2}\left(\frac{m^2}{\rho} + B^2\right) \tag{4}$$

Ideal MHD limit: In the **current density** $\boldsymbol{J}$ definition, η is diffusivity,

$$J = \frac{\Gamma}{\eta}[E + v \times B - (E \cdot v)v] \tag{5}$$

The diffusive time scale $\tau_\eta = L^2/\eta$ can be compared with the dynamical time scale $\tau_v = L/v$, where L is the length scale of system and v is a characteristic velocity scale. The ratio of this time scales is known as the magnetic Reynolds number or the Lundquist number,

$$S = \frac{\tau_\eta}{\tau_v} = \frac{vL}{\eta} \tag{6}$$

Some astrophysical systems satisfy $S >> 1$ that is equivalent to $\eta << 1$.

Orszag-Tang vortex test problem

- Simulations are set up in a **2D box $0 < x, y < 2\pi$** with the following velocity and magnetic fields:

$$v = v_0(-\sin y, \sin x, 0) \quad (7)$$

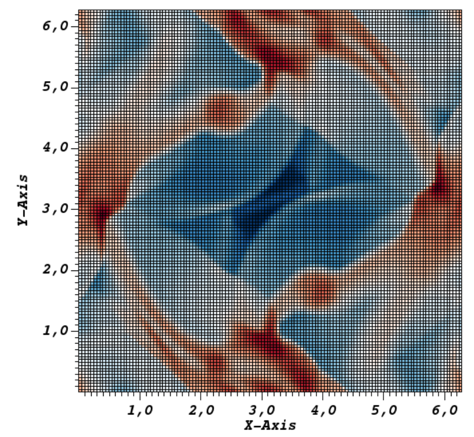
$$B = (-\sin y, \sin 2x, 0) \quad (8)$$

- We define the initial velocity by the speed of light considering $v_0 = 0.99c/\sqrt{2}$.
- The initial value of density and pressure are $\rho = 1$ and $p = 10$, respectively.
- We consider the ideal polytropic equation of state with the adiabatic index **4/3**.

Numerical setup

- We perform Orszag-Tang test problem for two MHD codes: **PLUTO 4.4** (made by Andrea Mignone) and **KORAL** (made by Aleksander Sadowski).
- We use approximate Riemann solver: Harten-Lax-van Leer **(HLLD)** solver
- We use **second order (linear) reconstruction** in space and second order Runge–Kutta **(RK2)** time stepping method

Mass-density plot in resolution of **64 × 64**



- We study the energy dissipation, reconnection and plasmoid formation, and estimate the numerical resistivity with the time evolution of magnetic energy $\overline{E_B} = B^2/2$.
- Furthermore, we investigate the energy transformation by comparing the time evolution of the energy components.

  Kinetic energy $\overline{K} = \rho(\Gamma - 1)$,

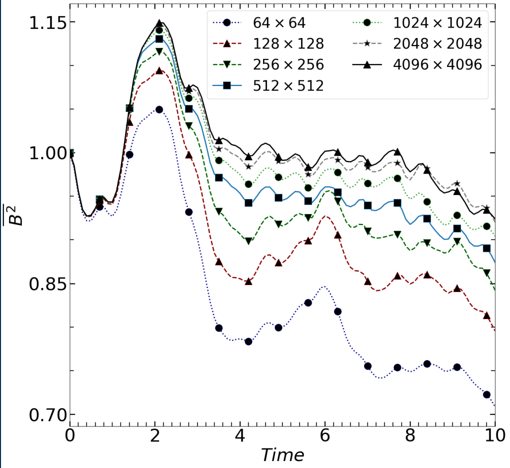
  (non-relativistic limit $\overline{K} = \rho v^2/2$)

  Electric energy $\overline{E_E} = E^2/2$

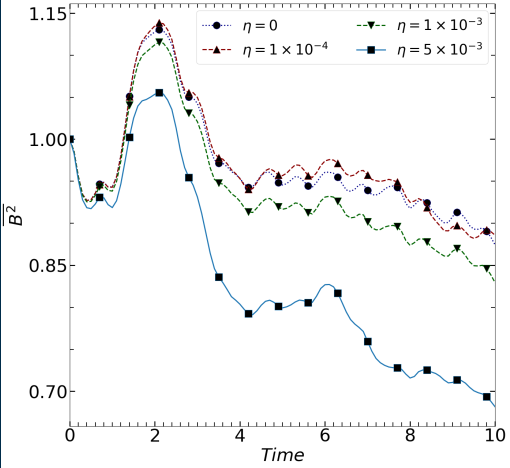
  Internal energy $\overline{U_{int}} = p/(\gamma - 1)$
- We compute the averaged density of energy components by integrating the value over grid cells in the simulation box.

PLUTO

MHD

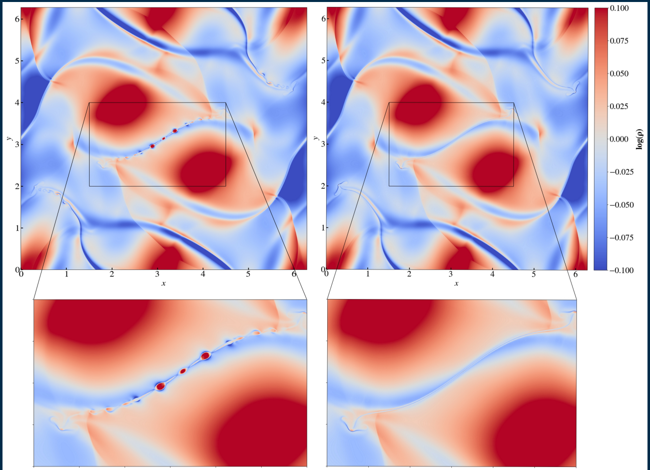


Res-MHD 512 × 512



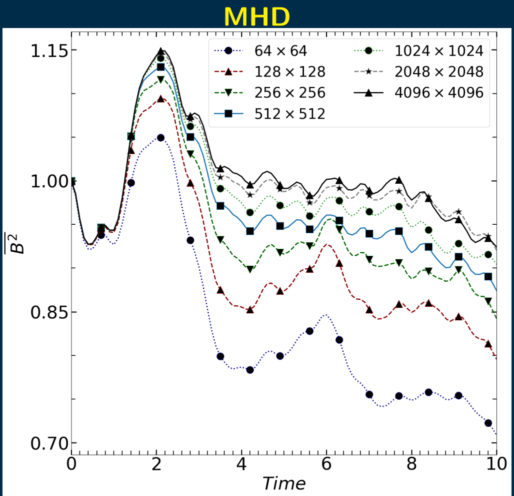
PLUTO

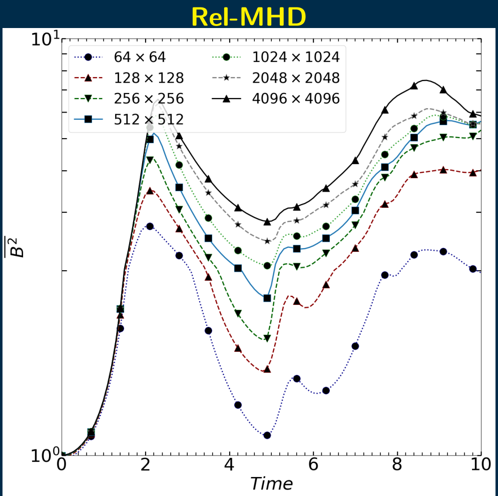
$\eta = 0$ $\eta = 10^{-4}$



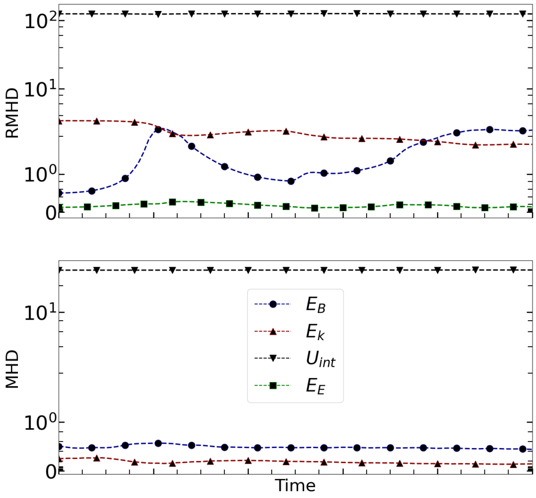
- The mass-density plots at $t = 2.5$ in the ideal MHD and Res-MHD with the physical resistivity of 10^{-4}
- In the MHD simulation: The plasmoid unstable current sheet in the center of the simulation box is shown,
- In Res-MHD simulation: (with Lundquist number $S = 10^4$) the current sheet does not exist.
- The theoretical studies confirm that the current sheet is plasmoid unstable at the Lundquist number $S > 10^4$.

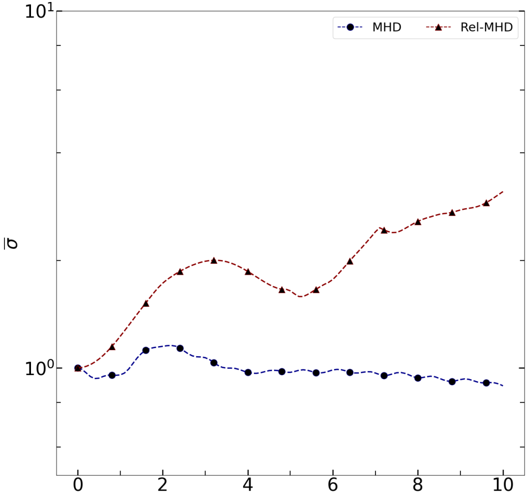
KORAL


MHD
1.15
1.00
0.85
0.70
$\overline{B^2}$
0
2
4
6
8
10
Time
64 × 64
128 × 128
256 × 256
512 × 512
1024 × 1024
2048 × 2048
4096 × 4096


Rel-MHD
10^1
10^0
$\overline{B^2}$
0
2
4
6
8
10
Time
64 × 64
128 × 128
256 × 256
512 × 512
1024 × 1024
2048 × 2048
4096 × 4096

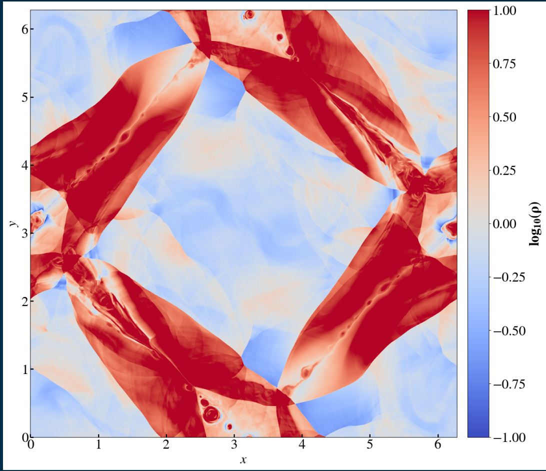
KORAL


RMHD
MHD
10^2
10^1
10^0
0
E_B
E_k
U_{int}
E_E
Time

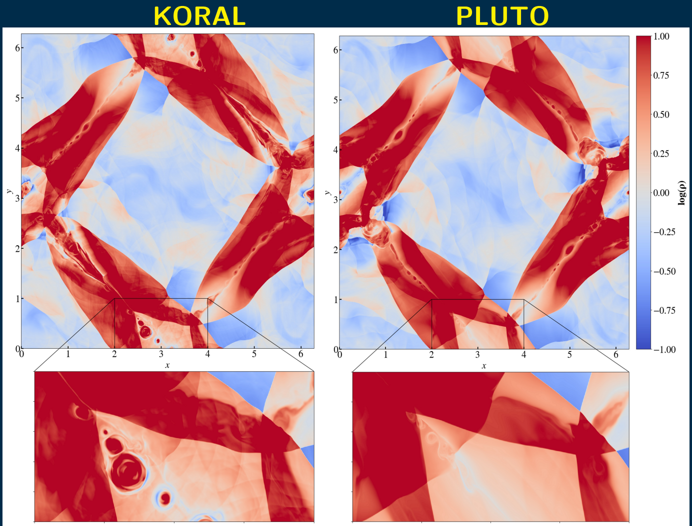

MHD
Rel-MHD
$\overline{\sigma}$
10^1
10^0
0
2
4
6
8
10

KORAL

Rel-MHD 4096 × 4096 $t = 9$

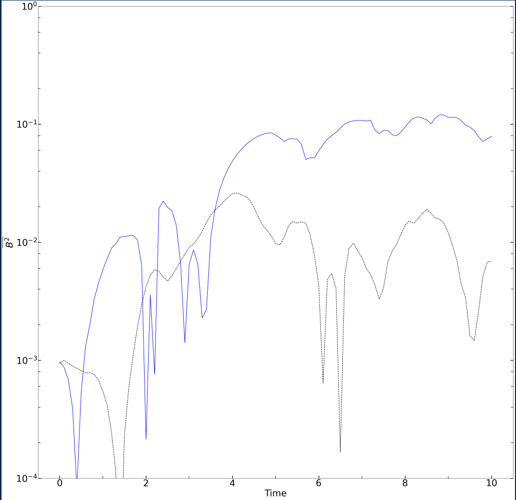


- Magnetic energy in MHD simulation dissipates after $t \simeq 2$ while in Rel-MHD it is arising from $t \simeq 5$.
- In both modules energy converts between kinetic energy $\overline{K}$ and magnetic energy $\overline{E_B}$.
- In Rel-MHD the current sheets is resolved at $t \simeq 2$, there is a **plasmoid instability** until the final simulation time $t = 10$. In MHD a current sheet is resolved at $t \simeq 2$.
- In Rel-MHD magnetization $\boldsymbol{\overline{\sigma} = B^2/\rho}$ increases with time due to the magnetic reconnection and plasmoids formation. In MHD magnetization is the highest at $t \simeq 2$ when the current sheet and chain of plasmoids appears.



- Mass-density plot of the simulations at $t = 9$ in the resolution of 4096×4096.
- KORAL has less numerical dissipation and resolves the current sheet and plasmoids.

Residual of B^2



- $|B^2_{koral} - B^2_{pluto}|/B^2_{koral}$.
- Blue-solid curves: Rel-MHD, black-dashed curves: MHD.
- **Numerical dissipation is lower in KORAL**.
- IN MHD module the difference between KORAL and PLUTO is negligible where in Rel-MHD it is more important, particularly in the resolution of 4096×4096.

Conclusion

- In simulations there is a numerical dissipation (Resistivity) that decreases with increasing the resolution.
- We estimate the numerical resistivity in the resolution of **512×512** in PLUTO code **less than 10^{-4}** that is corresponding the Lundquist number greater than **10^4**.
- The current sheet becomes plasmoid unstable in the **Lundquist number $S > 10^4$**.
- In Rel-MHD the **current sheets become resolve at** $t \simeq 2$, there is a **plasmoid instability** until the final simulation time $t = 10$.
- KORAL is more precise in order to capture substructures and it has less numerical resistivity.
- The difference between PLUTO and KORAL in **MHD** module is order of magnitude **less than** the difference in **Rel-MHD** module.