String loop around black hole: stability, vibrations and frequencies

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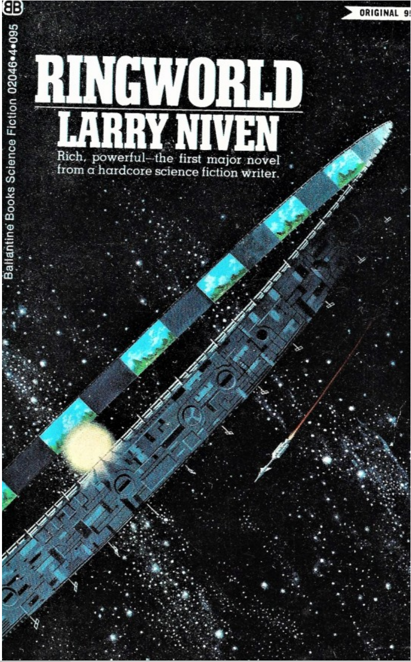
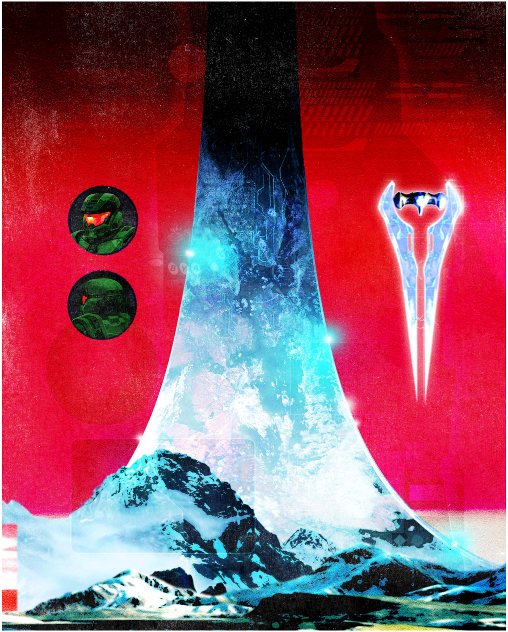
Ringworld - circular megastructure

- science fiction novel by Larry Niven (1970)
- Ringworld - gigantic structure shaped as a ring with a radius of around 1 AU, rotating around a star in the center of the ring
- 1971 World Science Fiction Convention: MIT students chanting:
  "The Ringworld is unstable!"
- The Ringworld
  - stable against axial displacements - bob back and forth around the star
  - unstable for transverse perturbation - near-side grav. attraction $>$ far-side
- 1980 sequel: The Ringworld Engineers - attitude jets fix



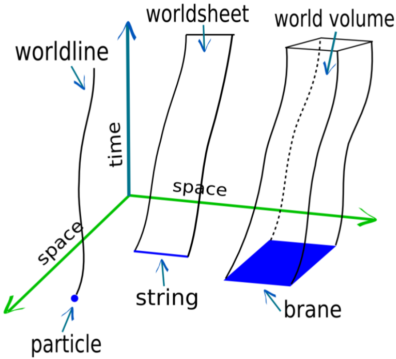


Thin structure around black hole - string loop - GR description

1D string immersion into 4D spacetime

$$X^{\alpha}(\tau,\sigma) = (t,r,\theta,\phi);$$

worldsheet w: $a,b \in \{\tau,\sigma\}$, $\alpha \in \{t,r,\theta,\phi\}$
τ - string evolution / σ - along string



w should be minimal (Nambu–Goto action)

$$S = -\mu \int_w \mathrm{d}w = -\mu \int \sqrt{-h}\, \mathrm{d}\sigma \mathrm{d}\tau$$

$$h_{ab} = g_{\alpha\beta} X^{\alpha}_{|a} X^{\beta}_{|b}, \qquad \Sigma^{ab} = -\mu\sqrt{-h}\, h^{ab}$$

induced metric on the worldsheet h_{ab} / worldsheet stress-energy tensor Σ^{ab}

take $\delta S = 0$ and (after some algebra) NG string equation of motion are obtained

$$(\Sigma^{ab} g_{\mu\lambda} X^{\mu}_{,a})_{,b} - {}^{1}\!/_{2} \Sigma^{ab} g_{\mu\nu,\lambda} X^{\mu}_{,a} X^{\nu}_{,b} = 0$$

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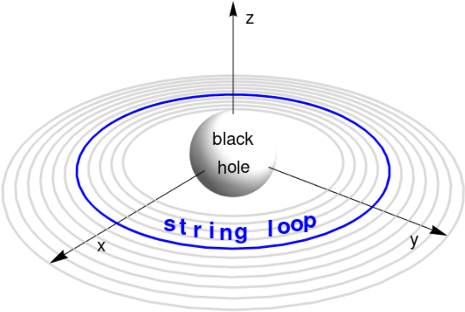

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$$-\frac{\partial}{\partial\tau} X^\alpha_{|\tau} - \Gamma^\alpha_{\beta\gamma} X^\beta_{|\tau} X^\gamma_{|\tau} + \frac{\partial}{\partial\sigma} X^\alpha_{|\sigma} + \Gamma^\alpha_{\beta\gamma} X^\beta_{|\sigma} X^\gamma_{|\sigma} = 0$$

Motivation II: magnetic flux tube (plasma) = relativistic string

- string with tension μ vs. magnetic flux tube (GRMHD)

$$-\frac{\partial}{\partial\tau}X^{\alpha}_{|\tau} - \Gamma^{\alpha}_{\beta\gamma}X^{\beta}_{|\tau}X^{\gamma}_{|\tau} + \frac{\partial}{\partial\sigma}X^{\alpha}_{|\sigma} + \Gamma^{\alpha}_{\beta\gamma}X^{\beta}_{|\sigma}X^{\gamma}_{|\sigma} = 0$$

$$-\frac{\partial}{\partial\tau}\left(\frac{Qq}{\rho}x^{\alpha}_{|\tau}\right) - \frac{Qq}{\rho}\Gamma^{\alpha}_{\beta\gamma}x^{\beta}_{|\tau}x^{\gamma}_{|\tau} + \frac{\partial}{\partial\sigma}\left(\frac{\rho}{4\pi q}x^{\alpha}_{|\sigma}\right) + \frac{\rho}{4\pi q}\Gamma^{\alpha}_{\beta\gamma}x^{\beta}_{|\sigma}x^{\gamma}_{|\sigma} \simeq 0$$

$$\nabla_{\alpha}T^{\alpha\beta} = 0, \quad \nabla_{\alpha}\rho u^{\alpha} = 0, \quad \nabla_{\alpha}(b^{\alpha}u^{\beta} - b^{\beta}u^{\alpha}) = 0 \quad x^{\alpha}_{\tau} = \frac{u^{\alpha}}{q}, \quad x^{\alpha}_{\sigma} = \frac{b^{\alpha}}{\rho}$$

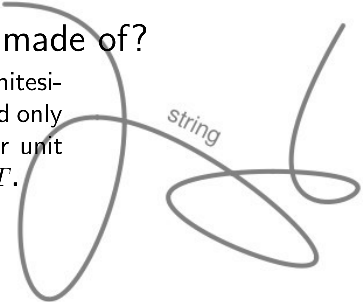
$$T^{\alpha\beta} = Qu^{\alpha}u^{\beta} - Pg^{\alpha\beta} - 1/(4\pi)b^{\alpha}b^{\beta} \quad P = p - \frac{1}{8\pi}b^{\alpha}b_{\alpha}, \quad Q = p + \epsilon - \frac{1}{4\pi}b^{\alpha}b_{\alpha}$$

V. S. Semenov and L. V. Bernikov, *Magnetic flux tubes - nonlinear strings in relativistic magnetohydrodynamics*, Astro. and Space Sci. **184**, 157-166 (1990)

- kinetic dynamo model (magnetic tension)

C. Cremaschini and Z. Stuchlík, *Magnetic loops generation by collisionless gravitationally-bound plasmas in axisymmetric tori*, Phys. Rev. E **87**, 043113 (2013)

Equation of state: what are the strings made of?



particle (0D object) is specified by its rest mass; all infinitesimally thin structures (=strings, 1D objects) are described only by two numbers **energy density** U and **tension** T per unit length: **equation of state is relation between** U **&** T**.**

$$S = \int \mathcal{L}\, \mathrm{d}\sigma \mathrm{d}\tau \quad \rightarrow \quad c_{\mathrm{T}}^2 = \frac{T}{U}, \quad c_{\mathrm{L}}^2 = -\frac{\mathrm{d}T}{\mathrm{d}U}$$

.

perturbation velocity: transverse (normal) c_{T} / logitunidal (along) c_{L}

"dust" string

non-interacting test particles

$$T = 0$$

$$c_{\mathrm{T}}^2 = c_{\mathrm{L}}^2 = 0$$

Nambu–Goto string

simplest case for string with tension $T = \mu$

$$T = U$$

$$c_{\mathrm{T}}^2 = 1,\ c_{\mathrm{L}}^2 = -1$$

current-carrying string

scalar field φ added on the string $\rightarrow$ currents $\varphi_{|a}$

$$\mathcal{L} = -\sqrt{-h}(\mu + h^{ab}\varphi_{|a}\varphi_{|b})$$

$$c_{\mathrm{T}}^2 = 1,\ c_{\mathrm{L}}^2 \in [0,1)$$

more models

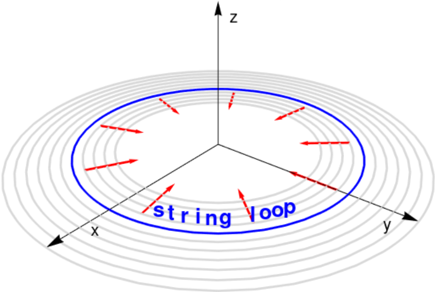
. . .

Problem: radial instability for Nambu–Goto string loop

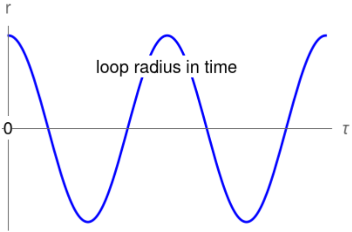
take sting loop in equatorial plane
$X^\alpha(\tau,\sigma) = (t(\tau), r(\tau), \pi/2, \sigma)$

"geodesic" string equation
$X^\alpha_{|\tau\tau} + \Gamma^\alpha_{\beta\gamma} X^\beta_{|\tau} X^\gamma_{|\tau}$
$- X^\alpha_{|\sigma\sigma} - \Gamma^\alpha_{\beta\gamma} X^\beta_{|\sigma} X^\gamma_{|\sigma} = 0$

$$\boxed{\ddot{r} + \mu^2 r = 0}$$



- harmonic oscillator ($\sim \cos\tau$); string loop radius is going to ZERO!
- even in flat spacetime the NG loop is unstable $r \to 0$
- zero string loop radius is catastrophe we would like to avoid.



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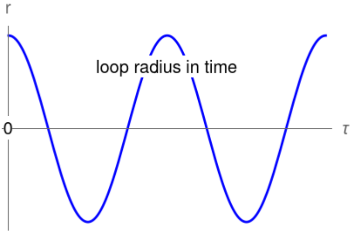
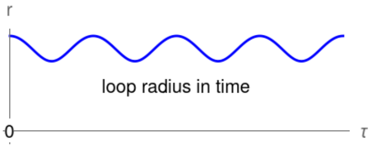


Solution: string loop stabilization by angular momentum

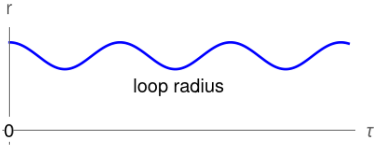
A) current-carrying string

scalar field φ $\rightarrow$ currents $j_a = \varphi_{|a}$ prevents collapse due to angular momentum $J^2 \equiv j_\sigma^2 + j_\tau^2 > 0$

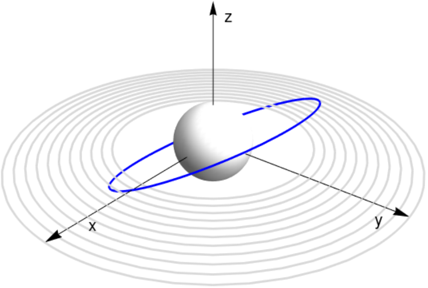
$$\boxed{\ddot{r} + \mu^2 r - J^4 r^{-3} = 0}$$



B) rotation off eq. plane



allowing the sting loop to "wiggle" can create angular momentum and prevent radial collapse



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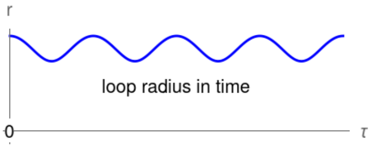

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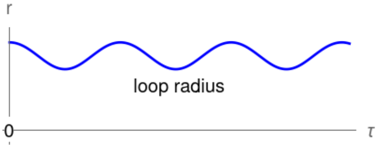


allowing the sting loop to "wiggle" can create angular momentum and prevent radial collapse

String loop perturbation - frequencies of oscillations is Schw.

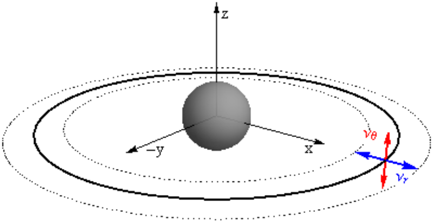
- instability? - loop linear perturbation

$$r = r_0 + \delta r(\tau), \quad \theta = \pi/2 + \delta\theta(\tau)$$

- harmonic oscillator with frequencies:
  $\omega^2 > 0$ (stable) $\parallel$ $\omega^2 < 0$ (unstable)

$$\ddot{x} + \omega^2 x = 0$$

- vertical oscillations have Keplerian frequency in all cases $\sim r^{-3/2}$



$$\Omega_{\rm r}^2 = \frac{c_{\rm L}^2(1-r)^2(3-r) + \left(r^2 - 8r + 12\right) c_{\rm T}^4 + (r-3)^3 c_{\rm T}^2 + 3(r-2)^2}{(r-2)r^4\left(c_{\rm T}^4 - 1\right)}, \qquad \Omega_\theta^2 = \frac{1}{r^3},$$

$$\text{"dust" string (particle)} \quad \Omega_{\rm r}^2 = \frac{r-6}{r^4}, \qquad \text{c. c. string} \quad \Omega_{\rm r}^2 = \frac{r^2 - 5r + 3}{r^4}$$

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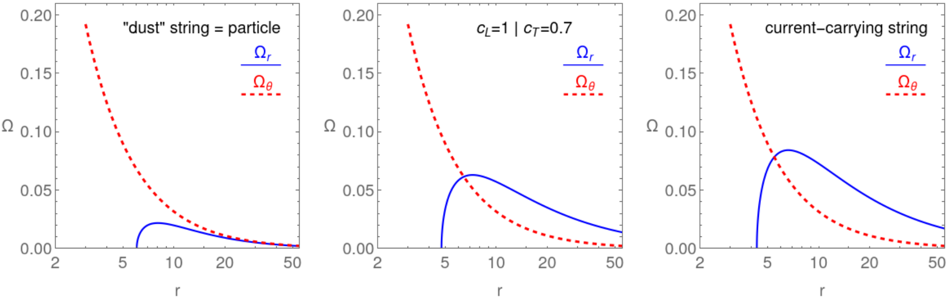

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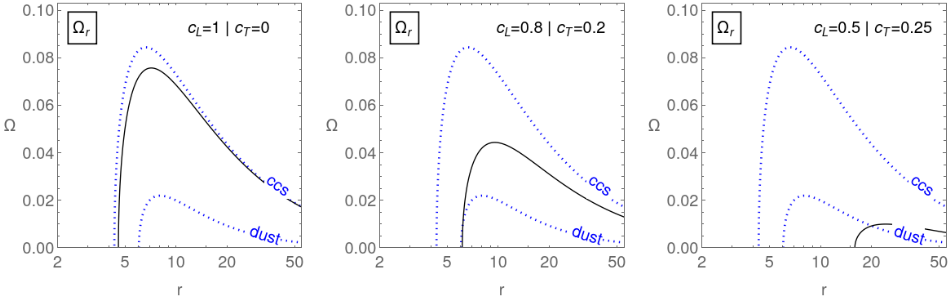

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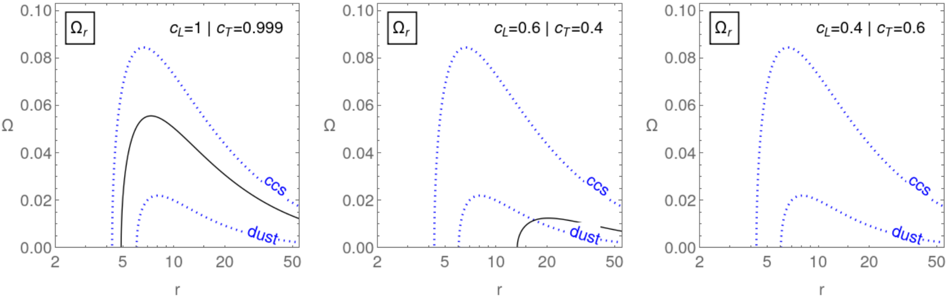

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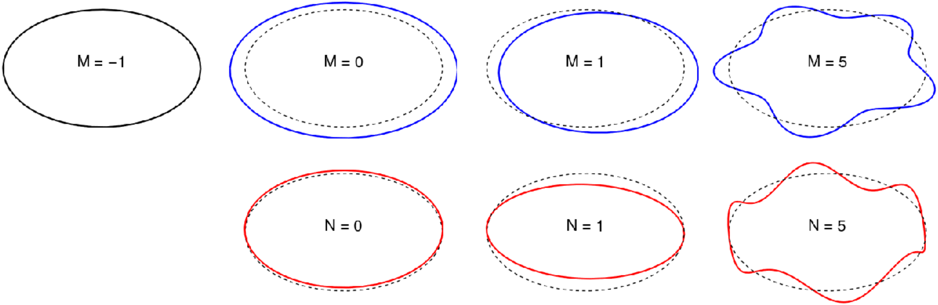
There can be also higher vibrational modes on string loop...



- so far we have explored instability in radial M=0 mode only
- radial (horizontal) vs. vertical modes
- how to find higher mode instability? - loop linear perturbation - frequencies!

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Summary & we are working on...

- Nambu-Goto string loop is radially unstable.
- We can make string loop stable by:
  equation of state (current carrying loop) $\leftrightarrow$ let loop "wiggle" (rotating loop).
- Frequencies of fundamental modes - astrophysical applicability? QPOs,...

- String loop model and its plasma motivations, limits of applicability.
- Vibrations of loop (1D) vs vibrations of branes (2D).

Thank you for your attention

• J. Natario, L. Queimada, R. Vicente: *Rotating elastic string loops in flat and black hole spacetimes: stability, cosmic censorship and the Penrose process*, Classical and Quantum Gravity, 35 (7) 075003 (2018) [arXiv:1712.05416]