Eccentric accretion flows:

a simple model of the stationary structure and dynamics

Jiří Horák

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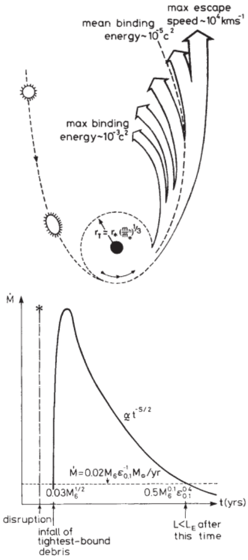
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

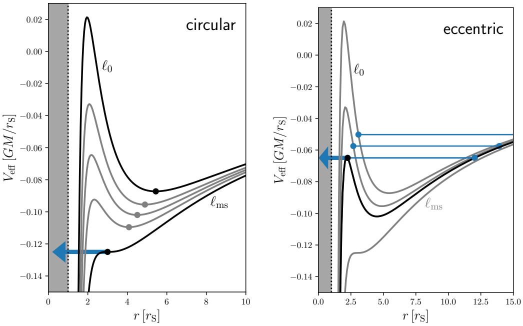
Motivation: Eccentric flows and TDEs



Rees (1988), Nature 333, 523

Problems of the classical TDE model:

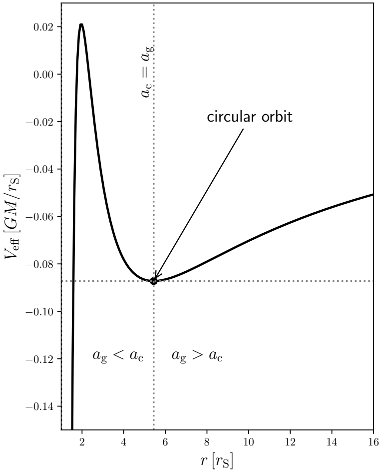
- Many TDEs show much less power
- Different decay than $\propto t^{-5/3}$
- Circularization (simulations)??



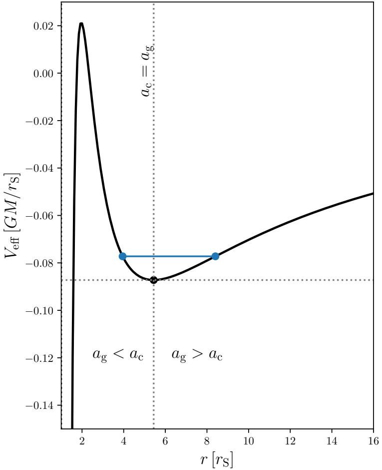
... Piran+2015, Liu+2021

Eccentric accretion?

Motivation: Circular and eccentric flows



Motivation: Circular and eccentric flows

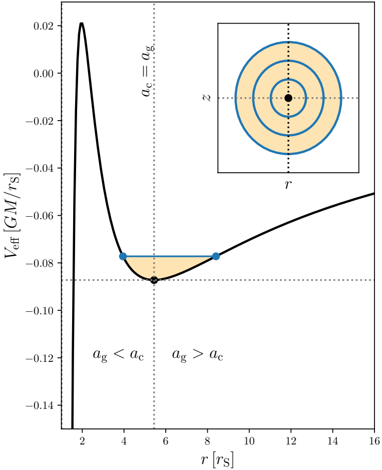


Force balance:

$$a_c^r - a_g^r \approx -\kappa^2 (r - r_0)$$

$$a_g^z \approx -\Omega_\perp^2 z$$

Motivation: Circular and eccentric flows



Force balance:

$$a_{\rm c}^r - a_{\rm g}^r \approx -\kappa^2 (r - r_0) = \frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$a_{\rm g}^z \approx -\Omega_\perp^2 z = \frac{1}{\rho}\frac{\partial p}{\partial z}$$

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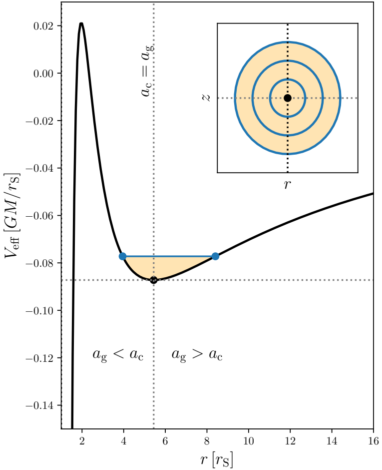

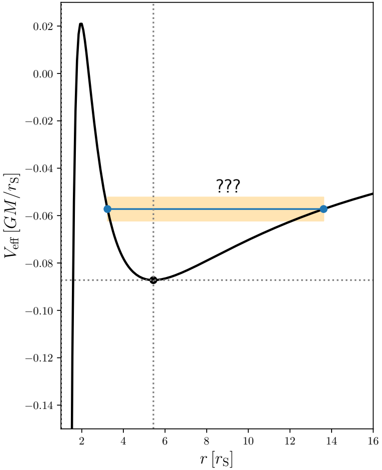

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Eccentric tori around

eliptical orbits?

SEMI-ANALYTIC HYDRODYNAMICAL MODEL OF AN ECCENTRIC ACCRETION FLOW

Equations for geometrically thin flow

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$
$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\, \boldsymbol{v} = -\frac{1}{\rho} \boldsymbol{\nabla} p - \boldsymbol{\nabla} \Phi$$

Dimensionless equations for dimensionless quantities: $\bar{q} \equiv q/q_*$

Dynamics: Given a typical angular momentum ℓ_* and the mass M:

$$v_* \equiv \frac{GM}{\ell_*}, \quad r_* \equiv \frac{\ell_*^2}{GM}, \quad t_* = \frac{\ell_*^3}{(GM)^2}$$

Termodynamics: Given a typical density and pressure p_*, ρ_*:

$$c_{s*} = \sqrt{\frac{p_*}{\rho_*}} \qquad \boxed{\epsilon \equiv \frac{c_{s*}}{v_*} \ll 1}$$

Thin disks: $\epsilon \sim H/r$, slender tori: $\epsilon = \beta \sim c_s/r\Omega$

Equations with small parameter

In cylindrical coordinates the rescaled equations read

$$\frac{\partial \bar{v}^r}{\partial \bar{t}} + \bar{v}^r \frac{\partial \bar{v}^r}{\partial \bar{r}} + \bar{v}^\phi \frac{\partial \bar{v}^r}{\partial \phi} + \bar{v}^z \frac{\partial \bar{v}^r}{\partial \bar{z}} - \bar{r}\left(\bar{v}^\phi\right)^2 + \frac{\epsilon^2}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{\bar{r}}{\bar{R}^3} = 0,$$

$$\frac{\partial \bar{v}^\phi}{\partial \bar{t}} + \bar{v}^r \frac{\partial \bar{v}^\phi}{\partial \bar{r}} + \bar{v}^\phi \frac{\partial \bar{v}^\phi}{\partial \phi} + \bar{v}^z \frac{\partial \bar{v}^\phi}{\partial \bar{z}} + \frac{2}{\bar{r}}\bar{v}^r \bar{v}^\phi + \frac{\epsilon^2}{\bar{r}^2\bar{\rho}}\frac{\partial \bar{p}}{\partial \phi} = 0,$$

$$\frac{\partial \bar{v}^z}{\partial \bar{t}} + \bar{v}^r \frac{\partial \bar{v}^z}{\partial \bar{r}} + \bar{v}^\phi \frac{\partial \bar{v}^z}{\partial \phi} + \bar{v}^z \frac{\partial \bar{v}^z}{\partial \bar{z}} + \frac{\epsilon^2}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\bar{z}}{\bar{R}^3} = 0,$$

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{\rho}\bar{v}^r\right) + \frac{\partial}{\partial \phi}\left(\bar{\rho}\bar{v}^\phi\right) + \frac{\partial}{\partial \bar{z}}\left(\bar{\rho}\bar{v}^z\right) = 0,$$

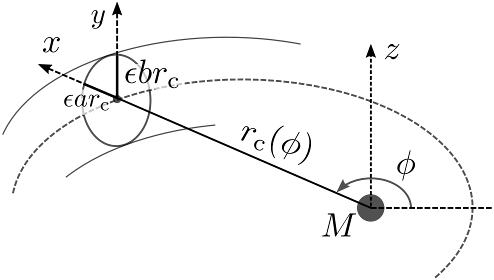
Polytropic equation of state:

$$\bar{p} = \bar{K}\bar{\rho}^{1+1/n}, \quad \bar{K} \equiv \frac{K}{c_{s*}^2}\rho_*^{1/n}.$$

All the scaled quantities are of the order unity even when $\epsilon \to 0$.

Perturbative solution

Scaled coordinates (x, y):



$$x \equiv \frac{r - r_c}{\epsilon a r_c}$$

$$y \equiv \frac{z}{\epsilon b r_c}$$

$r_c(\phi)$ – central streamline, $a(\phi)$, $b(\phi)$ – rad/vert thickness

Expansions (stationary configuration):

$$\bar{v}^r = \underbrace{\bar{v}_0^r(\phi)}_{\text{centr. stream}} + \epsilon \underbrace{\bar{v}_1^r(x, \phi, y)}_{\text{correction}} + \ldots,$$

$$\bar{v}^\phi = \bar{v}_0^\phi(\phi) + \epsilon \bar{v}_1^\phi(x, \phi, y) + \ldots,$$

$$\bar{v}^z = \epsilon \bar{v}_1^z(x, \phi, y) + \ldots$$

$$\bar{\rho} = \bar{\rho}_0(x, \phi, y) + \epsilon \bar{\rho}_1(x, \phi, y) + \ldots,$$

$$\bar{p} = \bar{p}_0(x, \phi, y) + \epsilon \bar{p}_1(x, \phi, y) + \ldots.$$

Zero-th order: (central streamline) = (test-particle orbit)

$$\bar{v}_0^\phi \frac{d\bar{v}_0^r}{d\phi} - \bar{r}_c \left(\bar{v}_0^\phi\right)^2 + \frac{1}{\bar{r}_c{}^2} = 0,$$

$$\bar{v}_0^\phi \frac{d\bar{v}_0^\phi}{d\phi} + \frac{2}{\bar{r}_c} \bar{v}_0^r \bar{v}_0^\phi = 0$$

Two integrals of motion:

$$\left(\frac{\ell_c}{\ell_*}\right) = \bar{r}_c{}^2 \bar{v}_0^\phi = \text{const} \equiv 1, \qquad \text{(angular momentum)}$$

$$\left(\frac{E_c}{E_*}\right) = \frac{1}{2}\left[\left(\bar{v}_0^r\right)^2 + \left(\bar{r}_c \bar{v}_0^\phi\right)^2\right] - \frac{1}{\bar{r}_c} = \text{const} \qquad \text{(energy)}$$

Solution:

$$\bar{r}_c = \frac{1}{1 + e\cos(\phi - \phi_0)}, \quad \bar{v}_0^r = e\sin(\phi - \phi_0), \quad \bar{v}_0^\phi = [1 + e\cos(\phi - \phi_0)]^2$$

and

$$E = -\frac{1}{2}\left(1 - e^2\right), \quad T = 2\pi(1 - e^2)^{3/2}$$

First-order: torus shape and angular momentum

Solution:

$$\bar{v}_1^r = \left[\frac{\mathrm{d}\tilde{a}}{\mathrm{d}\phi} + 2\lambda\bar{v}_0^r\right]x, \quad \bar{v}_1^\phi = 2\left[\lambda\bar{v}_0^\phi - \frac{\tilde{a}}{\bar{r}_\mathrm{c}}\right]x, \quad \bar{v}_1^z = \bar{v}_0^\phi \frac{\mathrm{d}(br_\mathrm{c})}{\mathrm{d}\phi}y$$

Here $\tilde{a}$, b and λ are solutions of set of ODEs

$$\frac{\mathrm{d}^2\tilde{a}}{\mathrm{d}\phi^2} + \tilde{a} - \bar{\beta}^{2+2/n}\left(1 + e^2 + 2e\cos\phi\right)(1 + e\cos\phi)^{-2+2/n}\,\tilde{a}^{-1-1/n}b^{-1/n} = 4\lambda$$

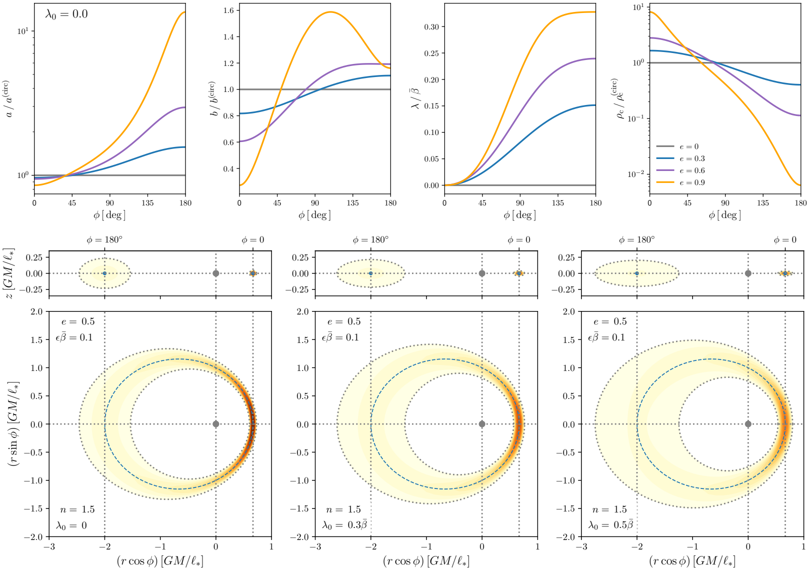
$$\frac{\mathrm{d}^2 b}{\mathrm{d}\phi^2} + b - \bar{\beta}^{2+2/n}(1 + e\cos\phi)^{-2+2/n}\,\tilde{a}^{-1/n}b^{-1-1/n} = 0$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\phi} + \frac{1}{2}\bar{\beta}^{2+2/n}e\sin\phi\,(1 + e\cos\phi)^{2/n}\,\tilde{a}^{-1-1/n}b^{-1/n} = 0$$

Meaning of the parameters:

- Thickness paramter: $\bar{\beta} = [2(n+1)\bar{K}]^{n/(2n+2)}\dot{m}_\phi^{1/(2n+2)}$
- Radial thickness: $\Delta r \sim \epsilon\tilde{a}/\bar{v}_0^\phi$
- Vertical thickness: $\Delta z \sim \epsilon r_\mathrm{c} b$
- Angular momentum gradient: $(\mathrm{d}\ell/\mathrm{d}r) \sim (2\lambda/\tilde{a}\bar{r}_\mathrm{c}{}^2)(\ell_*/r_*)$
  Kepler: $\lambda \to \infty$

Solutions



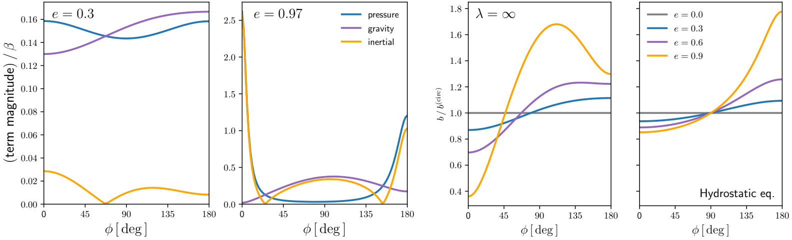
$\lambda_0 = 0.0$
$a\,/\,a^{(\mathrm{circ})}$
$b\,/\,b^{(\mathrm{circ})}$
$\lambda\,/\,\bar{\beta}$
$\rho_c\,/\,\rho_c^{(\mathrm{circ})}$
ϕ [deg]
$e = 0$
$e = 0.3$
$e = 0.6$
$e = 0.9$
$\phi = 180°$
$\phi = 0$
$z\ [GM/\ell_*]$
$(r \sin\phi)\ [GM/\ell_*]$
$(r \cos\phi)\ [GM/\ell_*]$
$e = 0.5$
$\epsilon\bar{\beta} = 0.1$
$n = 1.5$
$\lambda_0 = 0$
$\lambda_0 = 0.3\bar{\beta}$
$\lambda_0 = 0.5\bar{\beta}$

Vertical structure

In circular disks given by hydrostatic equilibrium

Here

$$\underbrace{v^k \frac{\partial v^z}{\partial x^k}}_{\text{inertial}} + \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial z}}_{\text{pressure}} + \underbrace{\frac{GMz}{R^3}}_{\text{gravity}} = 0$$

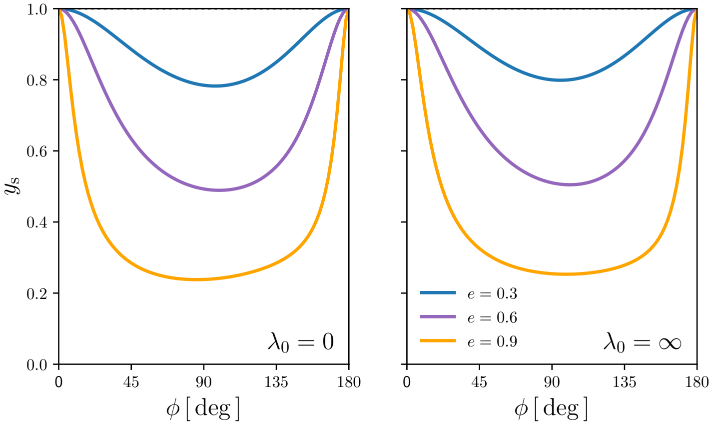


For high eccentricities, the intertial term is always dominant.

Supersonic compression at pericenter

Sonic height:

$$v_1^z\,(y_s) = c_s\,(y_s)$$



Nozzle shocks?

Oscillations

Perturbations:

$$\boldsymbol{v} = \boldsymbol{v}_{\rm b} + \boldsymbol{u},$$
$$\rho = \rho_{\rm b}\left(1 + \frac{h}{c_{\rm s}^2}\right),$$
$$p = p_{\rm b} + \rho_{\rm b} h$$

Governing equations

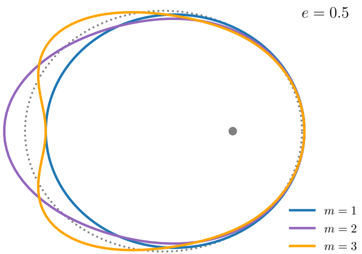
$$\frac{\mathrm{D}(\tilde{a}\tilde{u})}{\mathrm{D}t} - \frac{2}{r_{\rm c}}\tilde{a}u^{\phi} + 2v_0^2\frac{\partial(\epsilon h)}{\partial x} = 0,$$
$$\frac{\mathrm{D}(r_{\rm c}^2 u^{\phi})}{\mathrm{D}t} + \frac{2a_0 v_0^{\phi}}{\tilde{a}}\tilde{u} - \frac{v_0^r}{\tilde{a}}\frac{\partial(\epsilon h)}{\partial x} = 0,$$
$$\frac{\mathrm{D}}{\mathrm{D}t}(b r_{\rm c} u^z) + \frac{\partial(\epsilon h)}{\partial y} = 0,$$
$$f^{n-1}\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\epsilon h}{c_{\rm sc}^2}\right) + \frac{1}{a r_{\rm c}}\frac{\partial}{\partial x}(f^n\tilde{u}) + \frac{1}{b r_{\rm c}}\frac{\partial}{\partial y}(f^n u^z) = 0.$$

Variability at multimples of $2\pi/T$

Radial and vertical mode:

$$\epsilon h = \mathcal{A}\frac{c_{\rm sc}^2 x}{b}\cos\psi, \quad \epsilon h = \mathcal{A}\frac{c_{\rm sc}^2 y}{b}\cos\psi$$

$$\omega = \frac{2\pi}{T}(m-1) = \frac{m-1}{(1-e^2)^{3/2}}$$
$$\psi \equiv \phi + \omega(\tau - t) + \gamma,$$
$$\tau \equiv \int_0^{\phi}\frac{\mathrm{d}\phi}{v_0^{\phi}}$$



Discussion & Conlcusions

- Simple model of 3D structure of slender tori of arbitrary eccentricity.
- Radial and vertical structure is very different from the circular disks.
- Epicyclic oscillations with frequencies of multiples of $2\pi/T$.
- Most of the oscillations at apocenter.
- Supersonic compression at the pericenter for high $e \Rightarrow$ nozzle shocks?

- Including relativity, schocks and slow evolution in future:
- Relativity can be included as perturbation to Newtonian flow.
- Relativistic precession may cause oblique shocks at apocenters.
- Shocks may be efficient mechanism for angular momentum transport.
- “Composite” solutions.
- Slow (secular) evolution of e, ϕ_0, $\bar{\beta}$, λ_0.